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Edmund Price

LOGARITHMOLOGIA:

OR THE WHOLE

DOCTRINE

OF

LOGARITHMS,

Common and Logistical,

IN

THEORY and PRACTICE.

IN THREE PARTS.

PART I. The THEORY of LOGARITHMS;

Shewing their Nature, Origin, Construction, and Properties, demonstrated in various Methods, viz. 1. By Plain Arithmetic. 2. By the Logarithmic Curve. 3. By Dr. HALLEY's Infinite Series. 4. By Fluxions. 5. By the Properties of the Hyperbola. 6. By the Equiangular Spiral. 7. By a Logarithmic Inspectional Scale of twenty-two Inches length. With the Construction of the artificial Lines of Numbers, Sines, and Tangents. Also the Nature and Construction of Logistical Logarithms. The whole illustrated and made easy by many and suitable Examples.

PART II. The PRAXIS of LOGARITHMS;

Wherein all the Rules and Operations of Logarithmical Arithmetic, both Common and Logistical, by Numbers and Instruments, are copiously exemplified. Together with the Application thereof to the several Branches of Mathematical Learning.

PART III. A Three-fold CANON of LOGARITHMS;

In a new and more compendious Method than any extant;

- Viz. { 1. A Canon of Logarithms of NATURAL NUMBERS.
2. A Canon of Logarithms of SINES and TANGENTS.
3. A Table of LOGISTICAL LOGARITHMS.

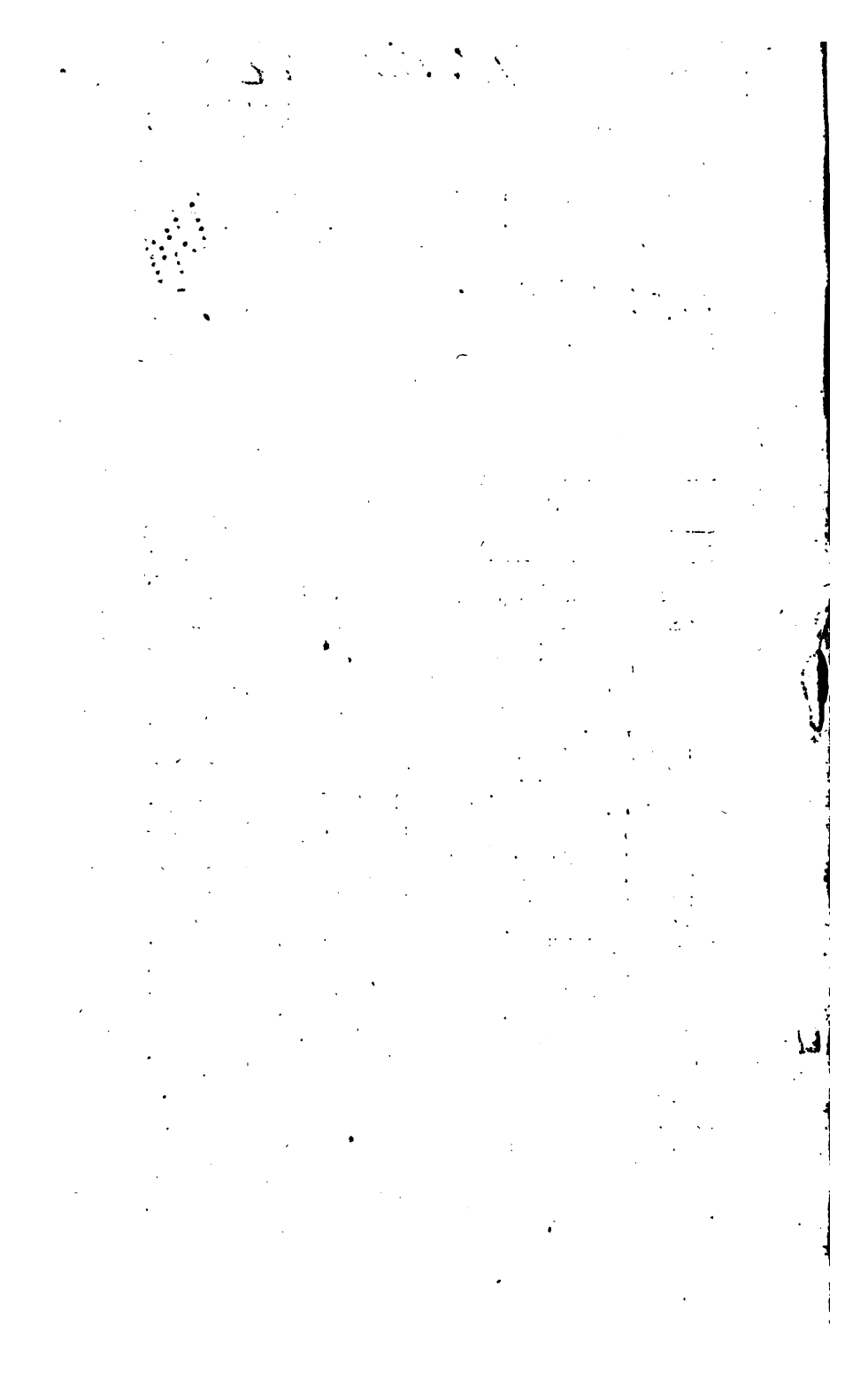
The whole being a *Compleat System* of this most useful Art; and enrich'd with all the Improvements therein from its Original to the Present Time.

By BENJAMIN MARTIN,
Author of the *Philological Library of Literary Arts and Sciences, &c.*

L O N D O N:

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THE
P R E F A C E.

THO' Logarithms *may justly be esteem'd*
the principal Invention of modern Ages,
on account of their excellent and most
extensive Use in Mathematical Literature, yet
it may with equal Truth be said, that little
more is known of them, generally speaking,
than their practical Use in some Rules of com-
mon Arithmetic and trigonometrical Calcula-
tions; (and how few are perfect in this!)
For (saith the great Improver of this Art, the
learned Dr. Halley) " I find very few of
" those, who make constant use of Logarithms,
" have attain'd an adequate Notion of them;
" to know how to make or examine them, or
" to understand the Extent of the Use of
" them: contenting themselves with the Ta-
" bles of them, as they find them, without
" daring to question them, or caring to know
" how to rectify them, should they be found
" amiss; being, I suppose, under the Appre-
" hension of some great Difficulty therein, &c."

03-30-34 MEN

For the sake of such Persons the following Treatise is principally intended; wherein they will find every thing necessary relating to the Theory and Practice of this admirable Art.

For, as to the Theory, (the principal Part, and so very rarely known) I have exhibited all the Methods, whereby it has been taught and explain'd by the Inventor, and several Improvers thereof, since his time; as by the Extraction of Roots; by the Logarithmic Curve from Dr. Keil; by an Infinite Series, from Dr. Halley; by the Method of Fluxions, from Mr. Ditton; by the Hyperbola, from Mr. Domky; by the Equiangular Spiral, from Halley, Wallis, &c. by the large Logarithmic Scale, of my own constructing: the like to which, for Largeness, was never before published; for a compleat Account of which see Chap. X. of the Theory.

I say, by all these various Methods I have endeavour'd to explain, illustrate, and facilitate the Knowledge of the Nature, Properties, and Construction of those excellent Numbers, called Logarithms. I have also exemplified the Manner of making Logarithms for the prime Numbers, by many and different Examples, and in several ways; and have taken all possible Care to render this most abstruse and difficult Part, as easy and intelligible, as the Nature of the Subject will admit.

Having

P R E F A C E.

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Having thus explain'd the Nature of Logarithms, I then shew how they are laid on Instruments, and thereby the Construction of the Artificial Lines of Numbers, Sines, and Tangents; first contriv'd by the famous Geometer, Mr. Gunter of Gresham-College; and for that reason they are still called Gunter's Line, and all together Gunter's Scale.

Lastly, I have largely explain'd the Nature, and shewn the Manner of making or constructing the Logistical Logarithms, according to Shakerley's and Street's Form thereof; and which I have not seen done by any other Hand.

These things together will, I hope, be allow'd to make a regular, universal, and compleat Theory of Logarithms, common and logistical, for Integers and Fractions, numerical and instrumental; and such, as for Brevity, yet Copiousness and Variety, has not been before extant.

As touching the Praxis, or Use of Logarithms, which makes the second Part of this Work, I have made it as compleat and perfect as possible; having illustrated all the Rules of Practice with all the Variety of Examples I could devise, that were necessary. And that none may be unappriz'd of the most extensive Service of Logarithms in the Mathematical Disciplines, I have applied them to the
Arithmetic

Arithmetic of all kinds of Numbers; to Trigonometry, in the Solution of all Cases of Plain and Spherical Triangles; to Mercator's Sailing particularly, showing how all its Cases may be resolv'd solely by the Canon of Logarithmic Tangents; to the Mensuration of Superficies and Solids, &c. All which are fundamental Operations, and may each of them be extended or branched out into particular Sciences; but that would have been too tedious a Task, and not absolutely necessary to my Design. I have therefore only applied the Doctrine of Mensuration to the Arts of Gauging, Timber-Measure, and Surveying; because they are the most common and necessary Arts in Life; and because the Use of Gunter's Scale, and Sliding-Rule (though before fully taught, and all along applied; yet) in them is more extensive and various than in any other Arts: and therefore I have taken care not only to shew all the different ways of using those Instruments, but likewise the Rationale of every Operation; a Matter of the greatest Importance, and too often neglected, in Books which treat thereof.

Lastly, I have in the last Chapter given a Variety of Examples of the Use of Logistical Logarithms in the practical Parts of Astronomy; both with respect to Time and Motion, have made it appear that Street's Logistical Logarithms answer all the Ends of Shakerley's; and how they are to be used along with the Common Logarithms of Numbers, Sines, and Tangents.

Tangents. And throughout this second Part, as well as the first, you will find a great Variety of new and useful Particulars, not here to be express'd.

The Third Part of this Work consists of a three-fold Canon of Logarithms, viz. (1.) Of common Numbers from 1 to 10000; and is sufficient for any Number under 10000000 by proper Rules. (2.) Of Sines and Tangents to every Degree and Minute of the Quadrant. (3.) Of Logistical Logarithms of Mr. Street's Form. Concerning which Tables, I shall only observe two things in general, viz.

First, that they are here contrived in a new and most compendious Form, equally easy and useful as those of the common Form; tho' in this they take up but one half the room, as they do in that. An Abbreviation very commodious, and I hope will prove acceptable.

Secondly, the Correctness of these is a matter of the last Concern, and the greatest Argument to recommend them. In order to prove this, I need only say, that those large Tables of Mr. Sherwin's are granted to be the most correct of any extant, from the most careful and exquisite Method he took to make them so, which see in his Preface.

*From these large Tables thus correct, I have made mine, every Figure of the two first Tables with my own Hand; in doing
which*

which, I discover'd several Errors, here and there, as I went along, in them, as exact as they were, which accordingly correcting in mine, I can, I presume, justly pronounce my Tables the most certain and exact, as well as the most compendious of any in being.

Having thus largely declared the several Parts of this Work, and shewn the Reader what an useful Variety he may expect to meet with both in the Theory, Praxis, and Tables, of this most useful and excellent Art; I must leave it to himself, to use or reject it, as he shall judge of the Merits thereof.



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LOGA-

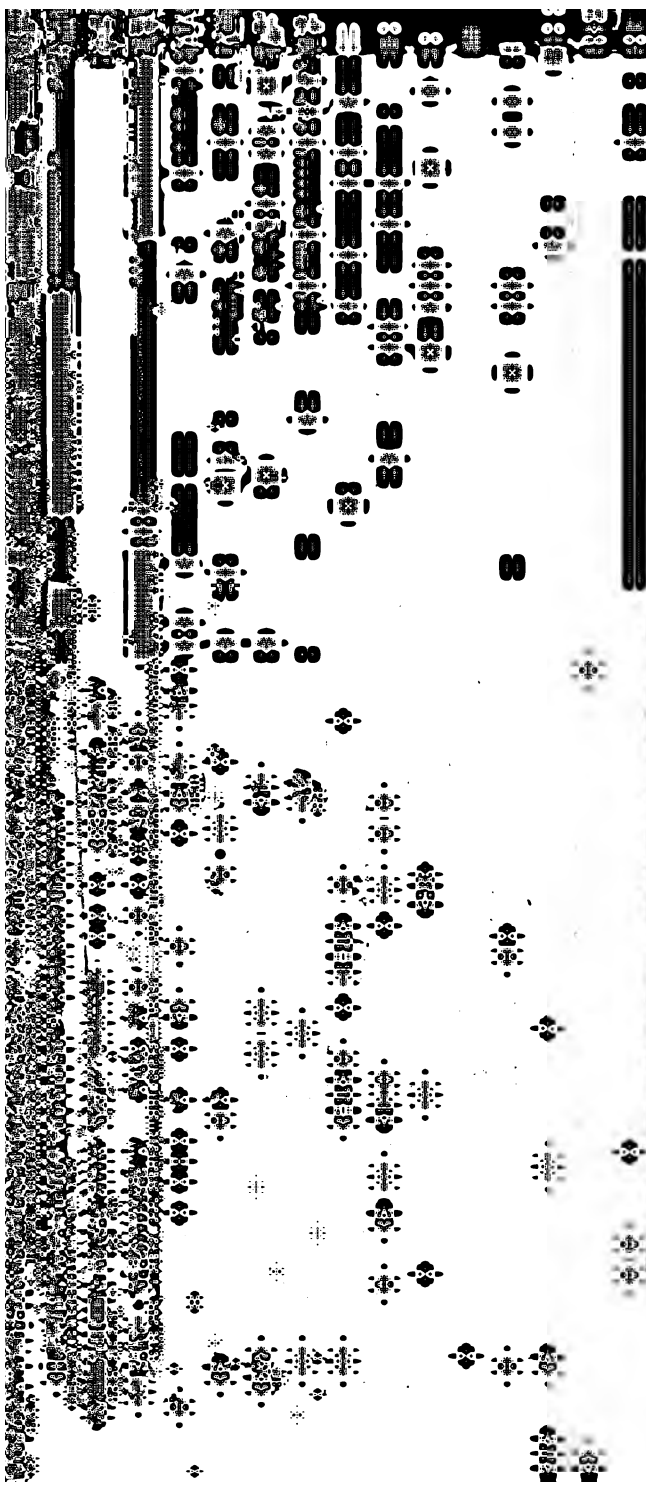
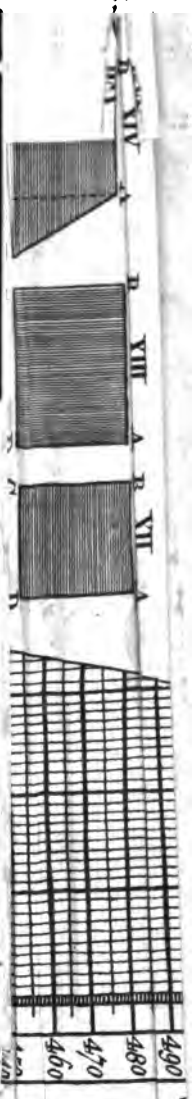
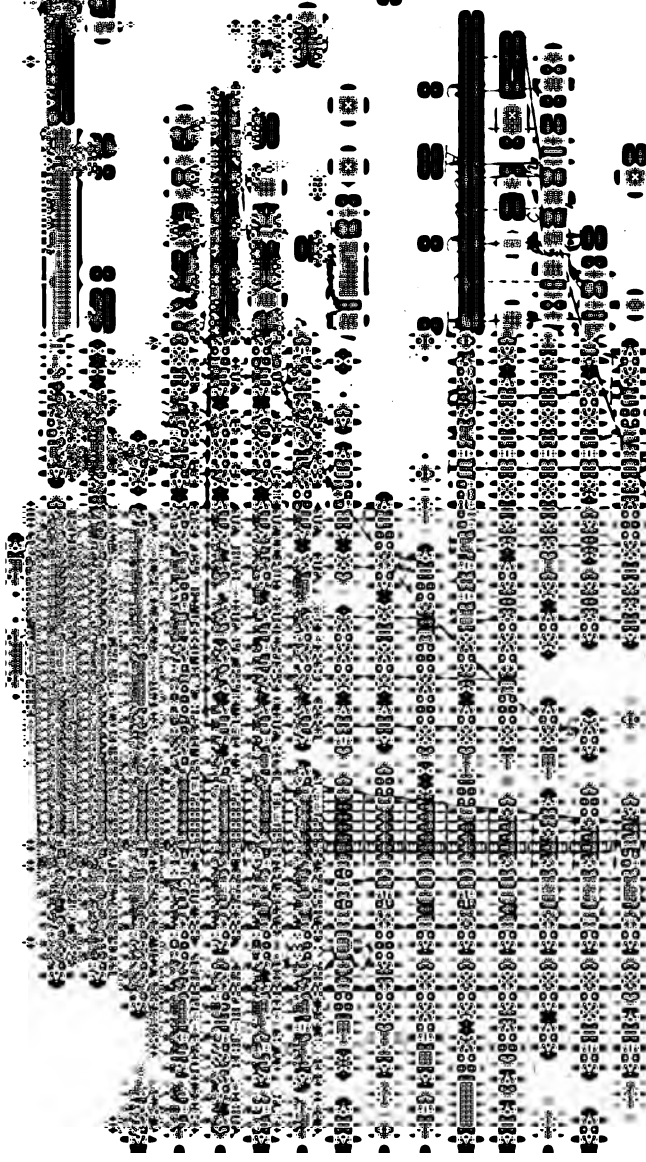


Fig. II.





LOGARITHMOLOGIA.

PART I.

The Theory of LOGARITHMS.

CHAP. I.

Of the Definition, Origin, and Nature of LOGARITHMS.

1. **T**HE best Definition of those Numbers we call *Logarithms*, is contain'd in the very Name or Word (*Logarithm*) itself; for it is compos'd of the two Greek Words λογων ἀριθμὸς, which properly or literally signify, a Number of Ratio's: and a *Logarithm* is no other than a Number, which denotes or shews what Number of Ratio's is contain'd between *Unity*, and that Number of which it is said to be the *Logarithm*.

2. Whence 'tis evident, that in order to have a clear Notion of *Logarithms*, 'tis absolutely necessary to understand first, and that very well, what is meant by the Word *Ratio*, or *Ratio's*, as here used in the Definition of *Logarithms*, and making an essential part thereof.

3. *Ratio*, then, is a certain mutual Habitudo of Magnitudes of the same kind, according to Quantity. This is *Euclid's* Definition: in which four things must be observ'd; as (1.) he saith *Ratio* is a certain mutual Habitudo; by which he means no more than

B

what

what we commonly call the *Proportion of any two things of a like sort to each other*, when by us they are compar'd together. (2.) He useth the general Word *Magnitude* to denote, that all *Subjects of Quantity*, as *Numbers, Lines, Superficies, and Solids*, are capable of such *Ratio, Habitude, or Proportion*, as aforesaid. (3.) He adds this Restriction, *of the same kind*; thereby insinuating there can be no *Ratio or Proportion of Quantity* between Magnitudes of a *different kind*; thus we cannot compare a *Line* to a *Superficies*; because the Quantity of a *Line* is estimated in *Length* only, but the *Quantity* of a *Superficies* ariseth from the joint Consideration of *Length and Breadth*, or the *Product* of each, and so importeth *Space*; which is entirely different from a *Line*, and therefore these two things cannot be the *Terms of Ratio or Comparison*. (4.) Lastly, he says, this *Ratio* is according to *Quantity*; that is, we compare Magnitudes in this Case, only to observe and maintain the *Proportion of Greatness or Bulk* which is between them; or to find *how often, or how many times*, one *lesser Magnitude* is contain'd in another *greater Magnitude*; neglecting all other Considerations and Affections of the said Magnitudes.

4. Having thus consider'd the general Nature of *Ratio's or Proportions*; I shall apply it to *Numbers*, as they are immediately the Subject of *Logarithms*. The *Ratio* therefore, or *Proportion* of a *Number to a Number* is two-fold; for first the *Ratio of a greater Number to a lesser* may consist in the *Addition* of some certain Number to that *lesser Number*; thus the *Ratio of 6 to 2* is made by adding 4 to 2. And if from *Unity* you begin the constant *Addition* of the same Number, suppose 2, you will then have a *Series of Numbers*, whose *Differences* will be the same, as 1, 2, 4, 6, 8, 10, 12, &c. and such Numbers are said to be in *Arithmetical Proportion* or *Progression*;

gression; and this *common Difference* of the Terms, as here 2 is called the *Ratio* of the *Progression*.

5. Secondly, the *Ratio* of a *greater Number* to a *lesser* may consist in a *Multiplication* of the *Lesser Number* by some other Number; thus the *Ratio* of 12 to 4, is made by multiplying 4 by 3; and if from *Unity* you begin a *constant Multiplication* by the *same Number*, suppose 2, you will then have a *Series* of Numbers, as 1, 2, 4, 8, 16, 32, 64, &c. which are said to be in *Geometrical Proportion*, or *Progression*; and the *common Multiplier*, as here 2, is called the *Geometrical Ratio* of this *Progression*.

6. Wherefore in the two *Series* or *Progressions* of Numbers, *viz.*

Arith. 1, 2, 4, 6, 8, 10, 12, 14, &c.

Geom. 1, 2, 4, 8, 16, 32, 64, 128, &c.

'tis easy to observe, that as the second Term exceeds the first by one *Ratio*, so the third Term exceeds the first by two *Ratio*'s, the fourth by three *Ratio*'s, the fifth by four *Ratio*'s; &c. Thus in the *Arithmetical Series*, the *Ratio* of 4 to 1, is *double* of the *Ratio* of 2 to 1; the *Ratio* of 6 to 1, is *triple* the *Ratio* of 2 to 1; the *Ratio* of 8 to 1, is *quadruple* the *Ratio* of 2 to 1, &c. And in the *Geometrical Series*, the *Ratio* of 4 to 1 is the *duplicate* of the *Ratio* of 2 to 1; the *Ratio* of 8 to 1 is *triplicate* of the *Ratio* of 2 to 1, and the *Ratio* of 16 to 1 is *quadruplicate* of the *Ratio* of 2 to 1, and so on. Where 'tis to be observed, that the Words, *double*, *triple*, *quadruple*, &c. are proper to the *Ratio*'s of the *Arithmetical Series*, and imply the *Addition* of them only; but the Terms *duplicate*, *triplicate*, *quadruplicate*, &c. are proper to the *Geometrical Series*, and imply the *Multiplication* of those *Ratio*'s.

7. What has been thus far related of the *Doctrine* of *Ratio*'s, is sufficient for our present purpose, *viz.* the understanding the *Nature* of *Logarithms*. For

4 *Of the Definition, Origin, and*

suppose a Series of Numbers in *Arithmetical Progression*, beginning from 0, and whose *Ratio*, or common Difference, is Unity or 1; and to them be adapted a Series in *Geometrical Progression*, beginning from *Unity*; and whose common Ratio be any assign'd Number, suppose 2, as before; then will those two Series stand as below:

viz. { Arith. 0. 1. 2. 3. 4. 5. 6. 7. 8. 9 &c.
 { Geom. 1. 2. 4. 8. 16. 32. 64. 128. 256. 512. &c.

8. 'Tis evident the Numbers in the first Series shew the Number of Ratio's between their correspondent Numbers and Unity in the second Series. For instance, the Figure 2 in the first Series, shews the Ratio's between its corresponding Number 4 and 1, in the lower Series, are 2; the Numbers 5, 7, 9, in the upper Series, shew the Number of Ratio's between their corresponding Numbers 32, 128, 512, and 1 or Unity, in the lower Series, to be respectively 5, 7, and 9; or that the Ratio is so often repeated from Unity to those Numbers, and consequently so often compounded in them; or farther, that the Ratio of 32 to 1 is compounded of five times the Ratio of 2 to 1; and the Ratio of 128 to 1, of seven times the Ratio of 2 to 1; and the Ratio of 512 to 1, of nines times the Ratio of 2 to 1.

9. Wherefore since the Numbers in the upper Series shew the Number of Ratio's contain'd between their corresponding Numbers and Unity in the lower Series; therefore those Numbers in the upper Series in *Arithmetical Progression* shall be the *Logarithms* of the Numbers in the lower Series of *Geometrical Progression*, and that according to the Definition of *Logarithms* beforegoing.

10. From the said Series, 'tis farther manifest, that the Product of any two Terms in the lower Series corresponds to the Sum of their respective Terms in the upper Series. See the following Examples.

Arith.

Chap. I. *Nature of* LOGARITHMS. 5

Arith. or { $2 + 3 = 5$; $2 + 4 = 6$; $3 + 6 = 9$.
 Logar. {

Geom. or { $4 \times 8 = 32$; $4 \times 16 = 64$; $8 \times 64 = 512$.
 N.Num. {

Also if any two Numbers in the lower Series be divided the one by the other, the Quotient thence arising shall correspond to the Difference of the respective Numbers in the upper Series. Examples in

Arith. $5 - 3 = 2$; $6 - 2 = 4$; $9 - 3 = 6$.

Geom. $32 \div 8 = 4$; $64 \div 4 = 16$; $512 \div 8 = 64$.

11. And universally, if any four contiguous Numbers be taken in the second Series, as the Product of the Extremes is equal to the Product of the Means; so in the first Series of the four corresponding Numbers, the Sum of the Extremes will be equal to the Sum of the Means; as in the Examples below.

Arith. $1 + 4 = 2 + 3 = 5$; $3 + 6 = 4 + 5 = 9$.

Geom. $2 \times 16 = 4 \times 8 = 32$; $8 \times 64 = 16 \times 32 = 512$.

and *vice versa*.

12. Again the *Square*, *Cube*, &c. of any Number in the lower Series of Geometricals will be answer'd by *double*, *triple*, &c. the corresponding Number in the upper Series in Arithmetical Ratio; for Example;

Square { Arith. $2 \times 2 = 4$; $4 \times 2 = 8$; $0 \times 2 = 0$.
 Geom. $4 \times 4 = 16$; $16 \times 16 = 256$; $1 \times 1 = 1$.

The Cube { Arithmet. $1 \times 3 = 3$; $3 \times 3 = 9$.
 Geomet. $2 \times 2 \times 2 = 8$. $8 \times 8 \times 8 = 512$.

And the Converse of this Article is also true; as is evident enough without example.

13. It appears then by these six last Articles, that *Logarithms are a Series of Numbers in Arithmetical Progression, so fitted and adapted to another Series of Numbers in Geometrical Progression, as that each*

6 Of the Definition, Origin, and

Term of the first shall expound (or be the Exponent of) the Ratio of its correspondent Term to Unity in the second Series. And that on this very Principle: For every Addition, Subtraction, Multiplication, or Division of the Logarithmic Numbers there corresponds a mutual Multiplication, Division, Involution, and Extraction of the respective Terms in the Geometrical Series.

14. Now 'tis a Matter entirely arbitrary or indifferent, what Number be made the *first Term* in either Series; for since the *first* are made by *equal Additions*, the *latter* by *equal Multiplication*, be the *Ratio* what it will, the *former* will still be the *Logarithms* of the *latter*; as is evident in the Table adjoin'd.

		Series of Logarithms.					
6	1	1	3	6	7	0	0
18	2	2	5	7	12	1	10
54	4	3	7	8	17	2	20
162	8	4	9	9	22	3	30
486	16	5	11	10	27	4	40
1458	32	6	13	11	32	5	50
4374	64	7	15	12	37	6	60
13122	128	8	17	13	42	7	70
39366	256	9	19	14	47	8	80
118098	512	10	21	15	52	9	90
354294	1024	11	23	16	57	10	100

15. Wherefore to the *same Series of Proportionals* there may be an *infinite Number* of *Series or Scales of Logarithms* contrived; and *vice versa*. Yet of *all* those *infinite* kinds of *Logarithms*, only those whose *first Term* is 0, and the *common Difference* 1, 10, 100, &c. are adapted for use. Because if the *first Term* be a significant Figure, we must necessarily have respect to it in use; and so, in this case,
four

four Terms of the *Logarithmical Series* becomes unavoidable; whereas if 0 be the first Term, three other Terms only suffice in the *Multiplication* of any two *Proportionals* whatever: for the *Sum* of their Logarithms will point out the *Product*, and shew its *Place*, that is, its *Distance* from, or *Ratio* to, the first Term of *that Scale of Proportionals*. But if the *first Term* in the Scale of Logarithms were significant, it must be subducted from the *Sum* of the Mean, in order to find the *Product* of the two *Proportionals*, &c. as before: and so we should perpetually have *double Labour* in every Operation. All this is evident from the different Series of Logarithms in the foregoing Table.

16. This being the Nature, and such the wonderful Properties of those Numbers called Logarithms, 'tis natural to suppose that he who first discover'd them, would make such a *noble Discovery* as subservient as possible to the Uses of Life, for the general Benefit of Mankind, but more immediately of *Artists* or *Mathematicians*.

17. Now to do this, 'twas necessary to calculate and fit a *Scale* or *Canon of Logarithms* to all Numbers which Men commonly make use of in Business, that so the *Fatigue* and *Labour* of *Multiplying*, *Dividing*, &c. large *Sums* or *Numbers* might be

Proport.	Logarith.
1	0.0000000
10	1.0000000
100	2.0000000
1000	3.0000000
10000	4.0000000
100000	5.0000000
1000000	6.0000000

avoided by only the Addition or Substraction, &c. of Logarithms. And of consequence nothing less than a *general Table of Logarithms* for all Numbers between Unity, or 1 and 1000000 or 10000000, could suffice; because no Number must here be wanting, as in the other intermitting Series 1, 2, 4, 8, 16, &c. since general Use requires them all, and therefore their Logarithms.

8 *Of the Definition, Origin, &c.*

18. Towards this 'twas easy to find Logarithms for a Series of Numbers proceeding in a *Decuple Proportion* or *Ratio* from Unity, as 1, 10, 100, 1000, &c. in the Table above. For putting 0 = Logarithm of 1; and 1 = Logarithm of 10; the Logarithm of 100 will be 2; and of 1000, 3; and so on from what I have already said.

19. But the *Difficulty* all consisted in finding the Logarithms for the intermediate Numbers between 1 and 10, 10 and 100, 100 and 1000, &c. or those of them which are called *Prime Numbers*, because they being once found, the Logarithms of all others are easily obtain'd, as we shall see by and by.

20. Now tho' the Numbers 1, 2, 3, 4, 5, 6, 7, 8, 9, are not in *Geometrical Proportion* (but indeed in an *Arithmetical one*) and so their Logarithms not to be obtain'd like the Logarithms of those which are so, yet as the *Ratio* of 10 to 1, shews only what *Distance*, or *how far* 10 is from Unity in the Scale of Proportion, and that the Numbers 2, 3, 4, 5, &c. possess severally a certain part of that Distance, if the said Ratio or Distance of 10 to 1, be supposed to be divided into a vast number of *equal Parts*, suppose 10000000, &c. 'tis evident a certain number of those equal Parts are to be allotted to the Numbers 2, 3, 4, 5, &c. which shall as truly express the *Distances* from, or the *Ratio's* of those several Numbers to Unity, as that of 10 to Unity is express'd by the whole Number 10000000.

21. This indefinite Number (10000000, &c.) of *equal Parts*, into which the whole Ratio of 10 to 1, is divided, may be conceived as a Number of so many *small Ratio's* or *Ratiunculae*, since they are all equal to each other. And so since the whole Number of *Ratiunculae* between 10 and 1, is the Logarithm of 10 to 1; therefore that Number of those *Ratiunculae* which lie between, or express the Distance of 2, 3, &c. to 1, shall also be as much the
Logarithms

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Logarithms of those Numbers 2, 3, &c. or, rather, of their Ratio's to Unity. And thus it appears that even the *Logarithms* of 2, 3, 4, 5, &c. (and for the same Reason) 11, 12, 13, &c. 21, 22, 23, &c. 101, 102, 103, &c. may be found to as great Exactness as is necessary. The Method of doing this by Numbers, shall be the Subject of the next Chapter.



C H A P. II.

Of the Method of making a Table of Logarithms by plain Arithmetic.

1. **H**AVING explain'd the Nature and Properties of *Logarithms*, and shewn not only how they are adapted to Series of Geometrical Proportionals, but also by what means they are to be calculated for all Numbers from Unity to any large Number either above or below it :

2. The *Method* or *Process* it self, then, of doing this, is the next thing to be propos'd ; an *arduous Task* this, to him that first aggress'd it ! At such an Enterprize, he indeed might truly have said, *Hic Labor, hoc Opus est*. And this any but an *inhuman Reader* will be convinced of, by viewing the following *tedious* and *laborious Process* for gaining the *Logarithm* of one single Prime Number, and that only to seven Places of Figures.

3. For, as I said before, 'tis sufficient to produce the *Logarithms* of *Prime Numbers* only, because, by the *Addition*, Substraction, &c. of these, we obtain the *Logarithms* of all *Composit Numbers*, with Ease ; as will appear further on. Now, tho' there are but three Prime Numbers, *viz.* 3, 5, 7, between

1 and 10 ; yet, because the Logarithm of no Number between 1 and 10 can be found by the Logarithms of any of the Proportionals 1, 10, 100, 1000, &c. it follows, that the Logarithm of any one of the nine Digits is found with equal Difficulty for the first. Therefore I shall give an Example in finding the Logarithm for the Number 9.

4. The Number 10, then, being already supposed at such a Distance from Unity as contains 10000000 equal Parts or *Ratiunculae*, which is the Logarithm of 10 ; therefore to find the Logarithm of 9, is to find *how many*, or *what Number* of those *Ratiunculae* are contain'd between 1 and 9, which is done in the following Manner.

5. First, make $A=1$, whose Log. is 0.0000000 ; and $B=10$, which Logarithm 1.0000000 ; as in the Table below.

Secondly, between A and B, or 1 and 10, find a *Mean Geometrical Proportional* $C=3.1622777$; the Logarithm whereof will be half the Logarithm of 10, viz. 5000000.

Thirdly, because C is much less than 9, find another *Mean Proportional* $D=5.623413$ between B and C ; whose Logarithm will be an *Arithmetical Mean* between the Logarithms of B and C, viz. 0.7500000.

Fourthly, because D is still much less than 9, find another *Geometrical Mean* between B and D, viz. $E=7.4989421$; whose Logarithm is an *Arithmetical Mean* between those of B and D, viz. 0.87500000.

Fifthly, since E is yet much less than 9, find yet another *Geometrical Mean* $F=8.6596432$, between B and E ; the Logarithm of which will be an *Arithmetical Mean* 0.93750000, between those of B and E.

6. And thus continue finding *Mean Geometrical Proportionals* between the Numbers *next Greater* and *next Lesser* than 9, till you arrive to the Number 9
itself,

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itself, which shall be clear of all other Figures, but *Cyphers*, to the Number of Places proposed; which will happen at the twenty-sixth Trial, as you see in the following Table.

Proportionals.	Logarithms.
A 1.0000000	0.00000000
C 3.1622777	0.50000000
B 10.0000000	1.00000000
B 10.0000000	1.00000000
D 5.6234132	0.75000000
C 3.1622777	0.50000000
B 10.0000000	1.00000000
E 7.4989421	0.87500000
D 5.6234132	0.75000000
B 10.0000000	1.00000000
F 8.6598432	0.93750000
E 7.4989421	0.87500000
B 10.0000000	1.00000000
G 9.3057204	0.96875000
F 8.6598432	0.93750000
G 9.3057204	0.96875000
H 8.9768713	0.95312500
F 8.6598432	0.93750000
G 9.3057204	0.96875000
I 9.1398170	0.96093750
H 8.9768713	0.95312500
I 9.1398170	0.96093750
K 9.0579777	0.95703125
H 8.9768713	0.95312500
K 9.0579777	0.95703125
L 9.0173333	0.95507812
H 8.9768713	0.95312500
L 9.0173333	0.95507812
M 8.9970796	0.95410156
H 8.9768713	0.95312500

Of the Method of making

Proportionals.	Logarithms.
L 9.0173333	0.95507812
N 9.0072008	0.95458984
M 8.9970796	0.95410156
N 9.0072008	0.95458984
O 9.0021388	0.95434570
M 8.9970796	0.95410156
O 9.0021388	0.95434570
P 8.9996088	0.95422363
M 8.9970796	0.95410156
O 9.0021388	0.95434570
Q 9.0008737	0.95428467
P 8.9996088	0.95422363
Q 9.0008737	0.95428467
R 9.0002412	0.95425415
P 8.9996088	0.95422363
R 9.0002412	0.95425415
S 8.9999250	0.95423889
P 8.9996088	0.95422363
R 9.0002412	0.95425415
T 9.0000831	0.95424652
S 8.9999250	0.95423889
T 9.0000831	0.95424652
V 9.0000041	0.95424271
S 8.9999250	0.95423889
V 9.0000041	0.95424271
X 8.9999650	0.95424080
S 8.9999250	0.95423889
V 9.0000041	0.95424271
Y 8.0000845	0.95424217
X 8.9999650	0.95424080
V 9.0000041	0.95424271
Z 8.9999943	0.95424223
Y 8.0000845	0.95424217

foregoing operose Method: No; for tho' 'tis not impossible, 'tis impracticable by that Method; but it is the Product of modern Invention; an Instance of which will be given in due Place.

8. Having thus found the Log. of $9=0.9542425$ if it be divided by 2, the Quotient or Half, 0.4771212 will be the Logarithm of 3, by the Converse of Article 12. Chap. 1. The Double of the Logarithm of 9 is the Logarithm of $81=1.9084850$ by the same Article. And thus by a continual Multiplication of this one Logarithm by 2, 3, 4, 5, &c. you gain the Logarithms of all the Powers of Nine.

9. 'Tis farther evident, that if the Ratio of 10 to 1, 100 to 10, 1000 to 100, &c. be supposed only Unity or 1, as expressed in the Table of Art. 17. Chap. I. the Logarithm of Nine, viz. 0.95424225 will be no other than a Decimal Fraction, whose Denominator is the said Ratio, or Unity, with Cyphers annex'd, thus $\frac{9542425}{10000000}$.

Therefore also the Log. of any Number, viz.	{	3=0.4771212	} may be express'd Frac- tion wile, as follows, viz.	{	3=0. $\frac{4771212}{10000000}$
		9=0.9542425			9=0. $\frac{9542425}{10000000}$
		10=1.0000000			10=1.0000000
		81=1.9084850			81=1. $\frac{9084850}{10000000}$
		100=2.0000000			100=2.0000000
		729=2.8627275			729=2. $\frac{8627275}{10000000}$
		1000=3.0000000			1000=3.0000000

10. Hence 'tis obvious that the Logarithms are only the Decimal Parts of several Ratio's, 1, 2, 3, &c. of 10, 100, 1000, &c. to Unity; and that those Ratio's themselves make the *integral Part* of the *mixt Decimal* Logarithm, (as I may call it.) These *integral Parts* of those *mixt Decimal Logarithms*, then are what we call the *Indexes*, or rather *Indices*, of those Logarithms, or *fractional Parts*; wherefore, the Ratio's of 10, 100, 1000, 10000, 100000, &c. to Unity, are the *Indices of or belonging to the Logarithms of all intermediate Numbers immediately*

mediately above those Proportionals. So the Ratio of 10 to 1, *viz.* 1, is the *Index* of the Logarithms of all Numbers between 10 and 100. And 2 (the Ratio of 100 to 1) is the *Index* of all Logarithms of Numbers, from 100 to 1000, and so on.

11. From whence 'tis easy to observe, *That the Index of any Logarithm contains a Number of Units less by one than is the Number of Figures in that Number which belongs to the said Logarithm.*

Therefore the Number $\left\{ \begin{array}{c} 9 \\ 81 \\ 729 \\ 1000 \end{array} \right\}$ the Number of whole Figures are $\left\{ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \end{array} \right\}$ the Index of its Log. $\left\{ \begin{array}{c} 0. \\ 1. \\ 2. \\ 3. \end{array} \right\}$

12. Now as this *Index* is thus called, from its *indicating* or shewing how many Places of Figures there are in the Number of the Logarithm; so, on the contrary, the Number of the Logarithm, as plainly shews what the Index of the Logarithm must be. And therefore, since in the Tables of Logarithms, all the Numbers of those Logarithms are appositely placed in proper Columns by them, 'tis entirely needless to print the *Indices*, as being well enough known by the Numbers; and thus they are (for this Reason) omitted in several of the said Tables; and consequently in those we have here, *in so concise a Form*, now first of all made publick.

13. What has been said hitherto, relates altogether to the Logarithms of *Whole Numbers*; but the *same Doctrine* is equally applicable both in *Theory and Practice*, to *fractional Numbers*; for in Effect, the Properties of *Decimal* and *Whole Numbers* are the same. Thus the Series of *decuple Proportionals*, 1, 10, 100, 1000, 10000, &c. may be continued as well *below* Unity, as *above* it; for the following Rank of Numbers, *viz.* $\frac{1}{10000}$, $\frac{1}{1000}$, $\frac{1}{100}$, $\frac{1}{10}$, 1, 10, 100, 1000, 10000, &c. which may be thus express'd, 0,0001, 0,001, 0,01, 0,1, 1, 10, 100, 1000, 10000,

10000, &c. are all in a *Geometrical Decuple Ratio or Proportion*; and consequently equally distant from Unity on each Side, and so their Logarithms equidifferent, and the same. The Ratio's then, in the Scale above Unity, may be called *Positive Ratio's*, and those below Unity, *Negative Ratio's*; and thus the *Index* of a Logarithm of a *Whole Number*, or *Integer*, shall be *positive*, but the *Index* of the Logarithm of *Fraction* shall be *negative*.

14. And since *Vulgar Fractions*, *Duodecimal*, *Sexagesimal*, &c. Fractions, are all reducible to *Decimals*; 'tis evident that all *Numerical Arithmetic* whatsoever is subject to, and manageable by this Art of *Logarithmical Arithmetic*. The only Difficulty being in adapting or readily finding the proper *Index* to the Logarithm of a Decimal Number. For, since the *Index* in the Logarithm of *Integers*, shews only the *Number of Places or Figures* in the said *Integers*, 'tis plain those *Indices decrease*, as the *Places of Figures* in the *Integer* decrease, and intirely vanish when those *Places of Figures* become Unity or one, that is, the *Index* is then 0; and consequently cannot serve for *Decimal Numbers*.

15. Therefore some new kind of *Indices* or *Characteristics* must be invented, which shall be proper only to *Decimal Numbers*, as the other are to *Integers*; and such as shall as readily discover the *Number of Cyphers* to be *prefix'd* to the *significant Figures* in the *Decimal*, as the other determine the *Number of Figures* in the *Integer*, are in the same degree useful for *Decimals*, as they for *Integers*. For as to the *significant Figures* of the *Decimal*, the *Logarithm* it self discovers them; all therefore that is farther necessary, is to procure *such Indices* as shall at all times denote *how far*, or *how many Places* from *Unity*, the *first significant Figure* of the *Decimal* must stand; or, which amounts to the same, *how many Cyphers* must be *prefix'd* to compleat the true Value of the *Decimal*.

16. The *Indices* for this Purpose are therefore judg'd best, *which being subtracted from 9, shall have a Remainder expressing the Number of Cyphers to be prefix'd*; thus the *Index* of a *Decimal* whose first Figure (to the Left,) is significant,

Integers	{	47500=4.6766936
		4750=3.6766936
Decimals	{	475=2.6766936
		47,5=1.6766936
		4,75=0.6766936
		,475=9.6766936
		,0475=8.6766936
		,00475=7.6766936
		,000475=6.6766936
		,0000475=5.6766936

must be 9, because $9-9=0$, that is *no Cypher* is to be prefix'd. And thus, a *Decimal* that has 1, 2, 3, 4, &c. *Cyphers*, must have the *Indices*, accordingly 8, 7, 6, 5, &c. because $9-8=1$, $9-7=2$, $9-6=3$, $9-5=4$, &c. denoting the *Cyphers* to be prefix'd to the *Decimal Numbers*. This is evident by the Examples in the Table above; but it is proper to dott these new *Indices* on each Side, to distinguish 'em from those which belong to the Logarithms of *Integers*.

17. From all that has been said hitherto, 'tis evident, that while the *significant Figures* in any Number whatever, *Integral* or *Decimal*, are the same, the *Logarithm* of those Figures will be the same also; the Variation occasion'd by the differing Nature of the Number, being only in the *Indices*, as denoted in the said Table. But something farther will be said of the *Nature* and *Variety of Indices*, when we come to treat of the *Practical Rules of Addition and Subtraction* of Logarithms; I shall now proceed to the next Chapter, wherein the *mysterious Nature*, or *Doctrine of Logarithms* will be farther illustrated, and render'd more obvious to the Senses by *Geometrical Schemes and Demonstrations*.

C H A P. III.

The Doctrine of the Nature and Properties of Logarithms farther explained and illustrated, by means of the LOGARITHMIC CURVE.

1. **I**N explaining the Nature of *Logarithms* from the *Logarithmic Curve*, 'twill be expedient to represent *Proportional Quantities* by *Letters or Species*; as being suited to a more *universal Theory* than *Numbers*, and better applicable to *Right Lines*, by which alone both *Numbers* and their *Logarithms* are represented in the *Geometrical Method* now before us.

2. In a *Series of Proportionals*, then, increasing from Unity, let the first be $=a$; then will the Series be $1, a^1, a^2, a^3, a^4, a^5, \&c.$ where 'tis evident the *Indices* or *Exponents* of the several *Powers* of a , are a *Series of Numbers* in *Arithmetical Progression*, each whereof shews the *Place* or *Distance* of its *Term* from Unity; thus the *Term* a^4 is shewn by its *Index* 4 to be in the *fourth Place* from Unity; and a^5 is in the *fifth Place*; or a^4 and a^5 are *four* and *five times* more distant from Unity than the *first Term* a ; which is here the *common Ratio* also.

3. From hence you observe, that the *Exponents* of the *Powers* of the *Terms* in the *Series* $1, a^1, a^2, a^3, a^4, a^5, a^6, a^7, \&c.$ are the *Logarithms* of those *Terms* respectively. Let the *Exponent* be $=e$; then
 { the *Terms*, $1, a, aa, aaa, aaaa, aaaaa, aaaaaa, \&c.$
 { the *Expon.* $0, e, 2e, 3e, 4e, 5e, 6e, \&c.$
 Therefore

{ As $aaaaa=a^5$; $axa^5=a^6$; and $axaaaa\sqrt{a}=a^5\sqrt{a}$,
 { So $2e+3e=5e$; $e+5e=6e$; and $2e+3\frac{1}{2}e=5\frac{1}{2}e$.
 and so every where, agreeable to the Nature of *Logarithms*

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garithms before describ'd, in the two preceding Chapters.

4. If between 1 and a there be put one *mean Proportional*, viz. $\sqrt{a}$, its *Index* or *Exponent* must be wrote $\frac{1}{2}$; thus $\sqrt{a} = a^{\frac{1}{2}}$; for 0, $\frac{1}{2}$, 1, have an *Arithmetical Ratio*; and so the Terms 1, $a^{\frac{1}{2}}$, a , are *Geometrical Proportionals*. Thus, a *mean Proportional* between a and aa , is $a\sqrt{a} = a^{1\frac{1}{2}} = a^{\frac{3}{2}}$. Also if you conceive two *mean Proportionals* between 1 and a , they shall be $a^{\frac{1}{3}}$, $a^{\frac{2}{3}}$; for $1 : a^{\frac{1}{3}} :: a^{\frac{2}{3}} : a$; and 0, $\frac{1}{3}$, $\frac{2}{3}$, 1, have an *equal Difference*.

5. Moreover the same Series of *Geometrical Proportionals* may be continued both ways, and be made *decreasing* as well as *increasing*; that is, it may as well *descend below Unity* to the *left Hand*, as *ascend* above it to the *Right*. Thus the Terms $a^{\frac{1}{2}}$, $a^{\frac{1}{3}}$, $a^{\frac{1}{4}}$, $a^{\frac{1}{5}}$, $\frac{1}{a}$, a , a^2 , a^3 , a^4 , a^5 , &c. are all in the same *Geometrical Progression*. And since the Distance of a^2 from Unity towards the *right Hand* is *positive*, and denoted by the *positive Exponent* $+2$; so $a^{\frac{1}{2}}$ being equally distant on the *contrary Side*, or *below Unity*, and this Distance being *negative*, therefore the Exponent thereof may be thus *negatively* express'd, $a^{-2} = a^{\frac{1}{2}}$ 1, and so $a^{\frac{1}{3}}$ is the same as a^{-3} ; thus the Series a^{-5} , a^{-4} , a^{-3} , a^{-2} , a^{-1} , 1, a , a^2 , a^3 , a^4 , a^5 , &c. is the same as the last foregoing.

6. Therefore, in those Series, if a represent any *Right Line* (see Fig. 1.) aa , or the *Square* thereof, is not to be taken as a Quantity of *two Dimensions*, or *Surface*, viz. a *Geometrical Square*; but only as a *Line* that is a *third Proportional* to some *Line* taken as *Unity*, and the *Line* a . So likewise aaa , or aaa , is not a Quantity of *three Dimensions*, or a *Geometrical Cube*; but a *Line* that the *fourth Term* or *Proportional* in a *Geometrical Series*, whose *first Term* is 1.

and *second a.* And thus you are to conceive of all the rest; viz. simply as *Proportionals of Length* only.

7. This being premised then, if on any Line, as AN (Fig. 2.) both ways indefinitely extended from A, be taken the equal Distances $AC=CE=EG=GI=IL=LN$, to the *right Hand*; and $Ac=ce=eg$, &c. on the left; and if on the Points $g, e, c, A, C, E, G, I, L, N$, be erected to the Right Line gN, the Perpendiculars $gh, ef, cd, AB, CD, EF, GH, IK, LM, NO$, which let be in a continual *Geometrical Proportion* to each other, and represent *Numbers*, whereof $AB=1$, or Unity. The Tops of these Lines being duely join'd, from what is call'd the *Logarithmic Curve*, viz. hBHO; by which we are farther to explain and illustrate the *Nature and Properties of Logarithms*.

8. But a more compleat Idea of the *Logarithmical Curve*, may be conceiv'd by a twofold Motion of the Line AB; the one *equable*, the other *accelerated* or *retarded* in any given *Geometrical Ratio*: For Example, if the Right Line AB moves uniformly along the AN, so that the End A describes *equal Spaces* $AC=CE=EG$, &c. in *equal Times*; while in the same Time the said Line AB so *increases*, that the *Increments* thereof generated in *equal Times*, be *proportional* to the *whole* increasing Line; that is, if AB in moving forward to ab , be *increased* by the *Increment* ob , and in an *equal Time*, when it comes to CD, the *Increment* thereof is Dp ; and $Dp : ab :: bo : AB$; then the End B of the said Line AB thus *continually increasing* or *decreasing* in the *same Ratio*, will describe the *Logarithmic Curve*. For since $AB : bo :: ab : Dp :: DC : dq$, &c. it shall be, by Composition of Ratio, as $AB : ab :: ab : DC :: DC : dc$; and so on.

9. Since (from what has been said) the Line $AB=1$, and the other Lines CD, EF, GH, &c. proceed from thence to *increase* in a *Geometrical Ratio*,
and

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and their Distances AC, CE, EG, &c. are all equal to each other (by Art. 7.) Therefore it follows that the Distances or Lines AC, AE, AG, &c. are the *Logarithms* of those Numbers represented by the Right Lines CD, EF, GH, &c. according to the *Definition of Logarithms*, Chap. I. Art. 1, and 13. For if $AC=1$, then $AE=2$, $AG=3$, &c. and so in the Series of *proportional Numbers* AB, CD, EF, GH, IK, LM, NO, &c. we have the *Logarithms*

0, AC, AE, AG, AL, AN, } &c.
0, 1AC, 2AC, 3AC, 4AC, 5AC, 6AC, } &c.

10. As *Logarithms*, then, are the *Exponents* of the *Ratio's* of *proportional Numbers* to *Unity* in any Series; or shew the *Place*, *Power*, or *Order* of the *Numbers* with respect thereto; 'tis plain that the *Logarithm* of AB or *Unity* must be $=0$, because *Unity* is not distant from it self. Also if the *Ratio* of CD to AB be $=x$, then shall the *Ratio* of the Number EF to AB $=x \times x = xx$; or *Duplicate* of the former; and the *Ratio* of GH to AB $=x \times x \times x = x^3$, or *TriPLICATE* of the first *Ratio* x . Thus the *Ratio* of NO to AB $=x^6$. And consequently the *Numbers*, their *Ratio's*, and *Logarithms*, will stand in the following Order, viz.

Prop. N°. AB, CD, EF, GH, IK, LM, NO, &c.
Ratio's . . . 1, x , x^2 , x^3 , x^4 , x^5 , x^6 , &c.
Logarithms 0, AC, 2AC, 3AC, 4AC, 5AC, 6AC, &c.

11. If four Numbers be such, that the Distance between the *first* and *second*, be equal to the Distance between the *third* and *fourth*, (be the Distance between the *second* and *third* what it will) then these Numbers will be *proportional*; for let the four Numbers be AB, CD, LM, and NO; then because $AC=LN$, it will be $AB : Dp :: LM : OT$, by Art. 8. Therefore (by Composition of Ratio) $AB : DC :: LM : NO$. And thus of any other four Numbers in the *Geometric Series*. The *Converse* also of this Proposition is as manifestly true.

12. Since $AB=1$, and its Logarithm $=0$; and since *Unity* is to the *Multiplier* as the *Multiplicand* is to the *Product*, in every Multiplication; therefore to every Addition of Logarithms, there corresponds a Multiplication of their Numbers. Thus,

Numbers, $CD \times EF = GH$; and $EF \times IK = NO$.

Logarithms, $AC + AE = AG$; and $AE + AI = AN$.

And in Division, since *Unity* is to the *Divisor* as the *Quotient* to the *Dividend*; then (by Art. 11.) for every Subtraction or Difference of Logarithms, there corresponds a Division of their Numbers. Thus,

Numbers, $GH \div EF = CD$; and $NO \div EF = IK$.

Logarithms, $AG - AE = AC$; so $AN - AE = AI$.

agreeably to what was shewn in Chap. I. Art. 10.

13. Again, since *Unity*, any assum'd Number, the Square thereof, the Cube, the Biquadrate, &c. are all continual Proportionals, their Distances from each other will be equal to one another; and are therefore as the Numbers AB, CD, EF, GH, &c. in the Series. Consequently if the Distance or Logarithm of the Number CD be multiplied by 2, 3, 4, &c. there will answer an Involution of the said Number to the Square, Cube, Biquadrate, &c. Power.

Thus for the Square

Numb. $CD \times CD = EF$; and $EF \times EF = IK$.

Log. $AC \times 2 = AE$; so $AE \times 2 = AI$.

For the Cube

Numb. $CD \times CD \times CD = GH$; and $EF \times EF \times EF = NO$.

Log. $AC \times 3 = AG$; and so $AE \times 3 = AN$.

For the Biquadrate

Numb. $CD \times CD \times CD \times CD (= EF \times EF) = IK$.

Log. $AC \times 4 = (AE \times 2) AI$.

Which is the same as was observed in Chap. I. Art. 12.

14. If the equal Distances AC, CE, EG, GI, IL, be bisected, and in the Points of Bisection *a, c, e, g, i*, there

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there be erected the Perpendiculars *ab, cd, ef, gb, ik*; these, by means of the Curve, will be all *Proportionals*; and the Number LM will be in the *tenth Place* from Unity or AB: if then we put $LM=10$, and suppose its *Ratio* to Unity be 10000000, such *Ratiunculae*, as that of *no* to AB, or Unity is 1. Then will the *Numbers* and their *Logarithms* be as express'd in the following Table.

Numbers.	Logarithms.
AB = 1	0.
<i>ab</i> = 1.259, &c.	<i>Aa</i> = 1000000
CD = 1.585, &c.	AC = 2000000
<i>cd</i> = 1.996, &c.	<i>Ac</i> = 3000000
EF = 2.512, &c.	AE = 4000000
<i>ef</i> = 3.163, &c.	<i>Ae</i> = 5000000
GH = 3.982, &c.	AG = 6000000
<i>gb</i> = 5.072, &c.	<i>Ag</i> = 7000000
IK = 6.310, &c.	AI = 8000000
<i>ik</i> = 7.944, &c.	<i>Ai</i> = 9000000
LM = 10.	AL = 10000000.

Numbers.	Logarithms.
AB = 1	0
CD = 2	AC = 3010300
EF = 3	AE = 4771212
GH = 4	AG = 6020600
IK = 5	AI = 6989700
LM = 6	AL = 7781512
NO = 7	AN = 8450980
PQ = 8	AP = 9030900
RS = 9	AR = 9542425
TV = 10	AT = 10000000

15. If BKV (Fig. III.) be the *Logarithmic Curve*, and $AB=1$, $TV=10$, or $10AB$; and all the intermediate Lines CD, EF, GH, &c. be as *Digits* 2, 3, 4, &c. to 10; these Lines will be situated

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ated at *unequal Distances* from each other ; and so their *Logarithms* AC, AE, AG, &c. not proceeding to increase by equal Differences, shew the Numbers AB, CD, EF, &c. are not *ordinately* (or all of them) in a *Geometrical Ratio* or *Proportion*. If the Ratio of 10 to 1, or TV to AB be supposed to consist of 10000000 *Ratiunculae*, and consequently the whole Logarithm AT of 10000000 *equal Parts*, then the Distances of each of the *nine Digits* from Unity, viz. AC, AE, AG, &c. shall consist of such Numbers of those small *equal Parts*, as are expressed in the Table, opposite to the said Digits.

16. Yet since some of the Digits are in a *Geometric Ratio*, as 1, 2, 4, 8, their Logarithms will be *equidifferent* ; so $AC=CG=GP$; and because $1:3::3:9$, there is $AE=ER$; and thus it appears that tho' the nine Digits are not all in continual *Geometrical Proportion*, yet some of them are so ; and the rest are some of *those Proportionals*, of which there be 10000000 between $AB=1$, and $TV=10$. If the *first Term* from Unity be called x , the second will be x^2 , the third x^3 , &c. and since the Number 10 is the 10000000th Term of the Series, it will be $TV=x^{10000000}=10$. Also $CD=x^{3010300}=2$; and $EF=x^{4771212}=3$; and so on. Whence *every Digit* is *some Power* of that Number which is the *first Proportional* from Unity : The *Exponents* or *Indices* of the Powers being the *Logarithms* of the Numbers, agreeable to Art. 10. of this.





C H A P. IV.

*The Nature of Logarithms and their Indices,
when the Numbers are Fractions, farther ex-
plain'd by the LOGARITHMIC CURVE.*

1. **W**E have hitherto principally considered the Nature and Properties of Logarithms of *whole Numbers* or *Integers*, and have observ'd that in the *Decuple Series* 1, 10, 100, 1000, &c. the Terms have their Ratio's to Unity *affirmative* or *positive*, viz. 1, 2, 3, &c. or thus, +1, +2, +3, &c. the contrary of which happens when the Number of the Logarithm is not *integral*, but *fractional*, or expresses only some *fractionate* Part of Unity. For there the Series being continued on the *other Side*, or *below Unity*, hath the *Indices* of the Powers of the Terms of a *Quality* directly *opposite* to the former; and therefore as *those Indices* were *positive*, so *these* will be of a *negative Nature*, and import the Terms to be *below the State of Unity*, or rather of *Integrity*; and will be affected with the Sign —, as —1, —2, —3, &c. as is evident Chap. III. Art. 5.

2. Wherefore since AB represents *Unity*, (Fig. II.) all the *Numbers* in the *Series* towards the *right Hand*, or *above Unity*, CD, EF, GH, &c. being *integral*, and having the *Ratio* greater than *Unity*, will have the Logarithms thereof *positive*, viz. +AC, +AE, +AG, &c. But those *Numbers* or *Terms* on the *Left*, or *below Unity*, cd, ef, gh, &c. being *fractionate*, and having the *Ratio* less than *Unity* or *decreasing*, will have the Logarithms thereof *Negative*, viz. —Ac, —Ae, —Ag, &c. And so the *Indices*

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of those *Logarithms* will be respectively *affirmative* or *negative*.

3. Since then (as hath been all along shewn) the *Indices* of the *Logarithms* of *Integers*, as being *affirmative*, must be *added*, that so their *Sum* may indicate the *Product* of the *Numbers* multiplied; as $AC + AE = AG$, which shews that its *Number* $GH = CD \times EF$, the *Numbers* multiplied; so if the *Index* of one *Logarithm* be *negative*, and the *Index* of another be *affirmative*, the *Difference* of those two *Indices* must be taken for the *Product* of the *Numbers*. Thus to multiply the *Integer* GH by the *Decimal* cd , their *Indices* being $+AG$, and $-Ac$, their *Difference* $AG - A = AE$, and shew the *Number* EF is the *Product* of the other two. And here, because the greater *Index* GH is *affirmative*, the *Difference* also AE is *affirmative*, and the *Product* EF to be an *Integer* or on the *right Hand* of *Unity*.

4. But if the *Decimal* gh be to be multiplied by the *Integer* CD , whose *affirmative Logarithm* or *Index* AC is the *negative Index* of the *Decimal*, viz. Ag , their *Difference* $Ag - AC = Ae$, is *negative* also, and so shews the *Product* ef will be a *Decimal*, or below *Unity*. Again, if both the *Indices* of *Logarithms*, whose *Numbers* are to be multiplied, are *negative*, their *Sum* shall be a *negative Index* whose *Logarithm* points out the *Product* (in this Case) always a *Decimal*, or in the *Series* below *Unity* AB . For Example, to multiply the two *Decimals* cd , ef , the *Sum* of their *negative Indices* $Ac + Ae = Ag$ is *negative*, and shews the *Decimal Product* gh , ever below *Unity*.

5. The Reason of all which is very plain; for since *Unity* is to the *Multiplier* as the *Multiplicand* to the *Product*, and the *Logarithm* of *Unity* is $= 0$; *Unity*, any two *Numbers* and their *Product*, together with their *Indices*, will stand as follows:

$$1. \begin{cases} \text{Ind. } o, AC, AE, AG, \\ \text{Num. } AB : CD :: EF : GH, \end{cases}$$

therefore $\begin{cases} AC + AE = AG + o = AG, \\ CD \times EF = GH \times 1 = GH. \end{cases}$

$$2. \begin{cases} o, -Ac, +AG, +AE, \\ AB : cd :: GH : EF. \end{cases}$$

therefore $\begin{cases} AG - Ac = AE + o = AE, \\ GH \times cd = EF \times 1 = EF. \end{cases}$

$$3. \begin{cases} o, +AC, -Ag, -Ac, \\ AB : CD :: gh : ef, \end{cases}$$

therefore $\begin{cases} AC - Ag = o - Ac = -Ac, \\ CD \times gh = 1 \times ef = ef. \end{cases}$

$$4. \begin{cases} o, -Ac, -Ac, -Ag, \\ AB : cd :: ef : gh, \end{cases}$$

therefore $\begin{cases} Ae + Ac = o - Ag = -Ag, \\ ef \times cd = 1 \times gh = gh. \end{cases}$

6. From whence 'tis evident, if the Numbers are both Integers, the *Product* will fall in the *Scale* above (or be greater than) either; if they be of different Sorts, the *Product* will fall between them, above Unity if the greater Number be an Integer, or below Unity, if it be a Decimal. Lastly, if both the Numbers be Decimals, the *Product* will fall below Unity and them both, or will be less than either of them. These things well observ'd and understood, make all the *Mystery* of the *Arithmetic* of *Fractions* by *Logarithms* vanish, where it is taught with the Use of negative Indices to Logarithms of fractional Numbers, as it is to be found in some Books. Note, I have all along in this Chapter, said affirmative or negative Indices, and not affirmative and negative Logarithms; because the Indices only shew the Quality of the Numbers, viz. whether above or below Unity, greater or lesser than it, i. e. whether they be Integers, or wholly Fractions; and therefore these Indices must be carefully added or subtracted, as occasion requires,

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and as before directed; but the *Logarithms* shewing the *Distances* of *Numbers* from *Unity* indifferently, or without respect to the *Order* of *above* or *below*, are *always* to be *added*.

7. But since this Method of *negative* or *different Indices* to *Logarithms* is attended with *Addition*, *Subtraction*, and other intricate *Cautions* peculiar to themselves, it can't be recommended so much as another Method *more approv'd*, and therefore *more generally used*; wherein the *Indices* of *all Logarithms*, (both of *fractional* as well as *integral Numbers*) undergo the same common *Management* with their *Logarithms*; and so can't be so difficult to *Learners* as the *other*; tho' it is also attended with *particular Rules*, as you'll find farther on, and is as follows:

	Numbers.	Indices.	Log ^s .
Fractions.	ut = ,0000000001	o — 10	0 90.0000000
	rs = ,000000001	tr = — 9	1. 91.0000000
	pq = ,00000001	tp = — 8	2. 92.0000000
	no = ,00000001	tn = — 7	3. 93.0000000
	lm = ,0000001	tl = — 6	4. 94.0000000
	ik = ,000001	ti = — 5	5. 95.0000000
	gh = ,00001	tg = — 4	6. 96.0000000
	ef = ,001	te = — 3	7. 97.0000000
	cd = ,01	tc = — 2	8. 98.0000000
	ab = ,1	ta = — 1	9. 99.0000000
Integers.	AB = 1	tA = 0	10. 100.0000000
	CD = 10	tC = + 1	11. 101.0000000
	EF = 100	tD = + 2	12. 102.0000000

8. Let AB be = 1, or *Unity* (Fig. IV.) and let tu be a *fractional* Number 10 times as far *below* *Unity* AB, as CD is *above* it; then will At = 10 AC, and supposing the Number CD = 10, 'tis evident tu = ,0000000001 of ,0000000001; if then, instead of

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0000000000 0000000000 00000000001 and 1, will of course be *positive*; and so in this Class, the *Indices of Logarithms* of all *decimal Numbers* (greater by far than *Use*, or even *Curiosity*, can desire) will be *positive* also, as well as those of the *Logarithms of integral Numbers*; and therefore in short, we have obtain'd *positive Indices* for the *Logarithms of all Numbers in general*, and so the *Trouble of Addition and Subtraction of different Parts of Logarithms* at the same time, is avoided, as was proposed.

11. But (as I have before observed, Chap. II. Art. 16.) the *Indices of Logarithms of Decimals* should be distinguished from the *Indices of Logarithms of Integers in Operations*; and this I think is best done as there directed, *viz.* by fixing a *Dott* on *each Side* the said *Indices*. And thus manifestly appears the *Reason* of all that is deliver'd in Chap. II. from Art. 13. to the End.

12. Now in *multiplying, dividing, &c. of Fractions*, the *Factors* may be either *both Decimal*; or *one Decimal*, and the *other an Integer*; if *both the Factors are Decimal*, the *Product* will be *Decimal*, in *Multiplication*. If the *Factors be of different Sorts*, and the *Ratio of Unity to the Decimal* be either *less, equal to, or greater* than the *Ratio of Unity to the Integer*; then shall the *Product* be *greater, equal to, or less than Unity*; i. e. it will be *Integer, Unity, or Decimal*, in *multiplying*; all which will be evident by the following *Examples*, serving as so many *Rules* for the *right ordering of Indices in Operations*.

$$13. \left\{ \begin{array}{l} AB : cd :: ab : ef \\ tA. .tc. .ta. .te. \\ 10. .8. .9. .7. \\ \text{That is} \end{array} \right\} \begin{array}{l} \text{theref.} \\ \\ \\ \end{array} \left\{ \begin{array}{l} cd \times ab = 1 \times ef = ef. \\ .tc. + .ta. = tA + te. \\ .8. + .9. = 10. + 7. \\ .8. + .9. - 10. = 7. = \end{array} \right.$$

Index of ef.

$$2. \left\{ \begin{array}{l} AB : ab :: EF : CD \\ tA. .ta. tE. tC. \\ 10. .9. 12. 11. \end{array} \right\} \text{theref.} \left\{ \begin{array}{l} ab \times EF = 1 \times CD = CD. \\ .ta. + tE = tA. + tC. \\ .9. + 12. = 10. + 11. \end{array} \right.$$

And consequently $.9. + 12. - 10 = 11. =$
Index of CD.

$$3. \left\{ \begin{array}{l} AB : ab :: CD : AB \\ tA. .ta. tC. tA. \\ 10. .9. 11. 10. \end{array} \right\} \text{theref.} \left\{ \begin{array}{l} ab \times CD = 1 \times 1 = 1 \\ .ta. + tC = tA + tA. \\ .9. + 11 = 10 + 10. \end{array} \right.$$

Therefore $.9. + 11. - 10 = 10 =$
Index of I.

$$4. \left\{ \begin{array}{l} Ab. cd :: CD : ab. \\ tA. .tc. tC. .ta. \\ 10. .8. 11. .9. \end{array} \right\} \text{theref.} \left\{ \begin{array}{l} CD \times cd = 1 \times ab = ab. \\ tC. + .tc. = tA. + .ta. \\ 11. + .8. = 10. + .9. \end{array} \right.$$

and therefore again . . . $11. + 8. - 10 = .9. =$
Index of ab.

14. From these Observations well consider'd, 'tis easy to apprehend the Truth of what is deliver'd in Art. 12. above. And since AB, in these *Examples* of the *Products*, hath for the *Index* of its Logarithm 10, 'tis equally obvious what the *Indices* of the *Logarithms* of the *Products* would be, were the said *Index* of the Logarithm of AB made 100. In this *present Case* were $tA = 10$, if we reject 10, the *Indices* of the Logarithms of *integral Products* will be the same as if the *Logarithms* began at AB or Unity; as is plain in the 2d and 3d *Examples*; and accordingly if $tA = 100$, we reject 100, and the Case is the same.

15. But since 'tis most convenient to have the *Indices* of all Logarithms of *integral Numbers* to begin from *Unity* in the simple Order, 0, 1, 2, 3, &c. as if the *Logarithms* did really begin from thence; so 'tis but rejecting 10 from the said *Indices* in their present State, and what we desire is obtain'd.

Thus

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$$\begin{array}{l} \text{Thus the fore-} \\ \text{going Cases} \end{array} \left\{ \begin{array}{l} 1. \ .8. + .9. - 10 = .7. \\ 2. \ .9. + 12. - 10 = 11. \\ 3. \ .9. + 11. - 10 = 10. \\ 4. \ 11. + .8. - 10 = .9. \end{array} \right.$$

by rejecting 10 from the *Indices* of the *Logarithms* of *Integers*, becomes

$$\left\{ \begin{array}{l} .8. + .9. - 0 = .17. \\ .9. + 2 - 0 = 11. \\ .9. + 1 - 0 = 10. \\ 1 + 8 - 0 = .9. \end{array} \right. \begin{array}{l} .7. \\ 1. \\ 0. \\ .9. \end{array}$$

Then from the *Sums* .17. 11. 10. .9. again rejecting 10 (where 'tis found) we have the *Remainders* .7. 1. 0. .9. the *true Indices* of the *Logarithms* of the *Products*, as required.

16. If either *Factor*, or the *Product* of them, exceeds the Limit of $\frac{1}{10000000000}$, or is less than ,0000000001, we shall find it most convenient to use the *Indices* of those *Logarithms* of which the *Index* of the *Logarithm* of *Unity* or AB is = 100, viz. those *Indices* in the 3d Class in Table of Art. 8. hereof. And if, in all the foregoing Cases, instead of *rejecting* 10, we now reject 100, we shall have the *Indices* of the *Logarithms* of the *Products* the same kind as before.

$$\begin{array}{l} \text{Thus} \end{array} \left\{ \begin{array}{l} 1. \ .98. + .99. - 100 = .97. \\ 2. \ .99. + 102 - 100 = 101. \\ 3. \ .99. + 101 - 100 = 100 \\ 4. \ 101. + .98. - 100 = .99. \end{array} \right.$$

by rejecting 100, becomes

$$\left\{ \begin{array}{l} .98. + .99. - 0 = .197. \\ .99. + 2 - 0 = 101. \\ .99. + 1 - 0 = 100. \\ 101. + .98. - 0 = 199. \end{array} \right. \begin{array}{l} .97. \\ 1. \\ 0. \\ .99. \end{array}$$

Thus here you see the Effect the same as above, Art. 15. if .97. .99. be deducted from 100, as .7. .9. were from 10, the *two middle Indices* being the same in both Cases.

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17. In *Division*, the *Divisor* is to *Unity* as the *Dividend* is to the *Quotient*; and so the *Distance* between *Unity* and the *Divisor* is equal to that between the *Dividend* and *Quotient*. If then the *Fraction* ef be divided by ab , because $aA = ec$, therefore cd is the *Quotient*, the *Index* of whose *Logarithm* is tc ; but $tc = tA + te - ta$. Also if the *Integer* CD be divided by the *Fraction* ab , because $aA = CE$, therefore EF is the *integral Quotient* whose *Logarithm* is tE ; but $tE = tA + tC - ta$. Again, let the *Fraction* ab be divided by the *Integer* CD , because $CA = ac$, therefore the *Fraction* cd is the *Quotient*; whose *Logarithm* $tc = tA + ta - tC$; that is, in every Case, the *Logarithm* of the *Divisor* being taken from the *Logarithm* of the *Dividend*, if to the *Remainder*, you add the *Logarithm* of *Unity*, the *Sum* will be the *Logarithm* of the *Quotient*: which is but the *Converse* of the *Rules* for the *Logarithms* in *Multiplication*, as is evident by *Inspection* of *Art. 13.* foregoing. And the *Methods* there mention'd for dually adjusting the *Indices*, are to be equally observed here.

18. In *Involution*, or raising the *Powers* of *Fractions*, 'tis evident that the *Distance* between *Unity* and the *Root*, is equal to the *Distance* between the *Root* and the *first Power*, the *first* and *second Power*, the *second* and *third Power*, and so on. Therefore since $Aa = ab = ce = eg$, &c. $cd = \text{Square of } ab$, $ef = \text{Cube}$, $gh = \text{Biquadrate}$, &c. *Power* of the *Root* ab . But since $AB : ab :: ab : cd :: cd : ef :: ef : gh$, &c. therefore the *Logarithms* $tA + tc = 2ta$, and so $tc = 2ta - tA$. That is, the *Logarithm* of the *Square* of the *Root* is equal to double the *Logarithm* of the *Root* less the *Logarithm* of *Unity*. Again, $\frac{ta + tc}{2} = tc = 2ta - tA$, and so $ta + tc = 4ta - 2tA$; that is, $tc = 3ta - 2tA$; in Words, the *Logarithm* of the *Cube* is equal to triple the *Logarithm* of the *Root*, less double the *Logarithm*

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garithm of *Unity*. Moreover, $\frac{tc+tg}{2} te=3ta-2tA$, therefore $tc+tg=6ta-4tA$; but $tc=2ta-tA$, subduct this from the *last Equation*, there will remain $tg=4ta-3ta$; or the Logarithm of the *Bi-quadrato*.

19. And *universally*, if the *Power* of a *Fraction* be x , and the *Logarithm* $=L$, the *Logarithm* of the *Power* x shall be $=xL-x-1 \times tA$, or $=xL-x tA + tA$. Thus, suppose you would know the *Logarithm* of the *Square* of the *Fraction* dc ; here $x=2$, and $L=tc$, therefore $2tc-tA=tg$, the *Logarithm* of the *Square* gh , as required. If the *Logarithm* of the *Cube* be desired of the *Fraction* ef , we have $x=3$, $L=tc$, and so $3tc-2tA=tr$, the *Logarithm* of the *Cube* rs , as *desir'd*. And thus you proceed for the *Logarithms* of *other Powers*.

N. B. The *Indices* of *Logarithms* of *all Powers* of *Fractional Numbers* (I mean such as are *purely* so) must be *doubly dotted*, since those *Powers* always fall below the *Root*, which is supposed a *pure Fraction*.

20. *Evolution*, or the *Extraction* of the *Roots* of *Powers*, is just the *reverse* of the foregoing *Process*. For suppose the *Fraction* cd were given, whose *square Root* was required. Because $AB : ab :: ab : cd$, therefore $AB \times cd = ab \times ab$; and so $\sqrt{AB \times cd} = ab$; therefore the *Logarithm* $\frac{tA+tc}{2} = ta$, is the *Logarithm* of ab the *square Root* sought. Also if the *Cube Root* of the *Fraction* ef be required; because $AB : ab :: cd : ef$, we have $AB \times ef = ab \times cd$; and so the *Logarithms* $tA + te = ta + tc$. But $tA + te - ta = tc = 2ta - tA$, that is, $3ta = 2tA - te$, and consequently $\frac{2tA+te}{3} = ta$ the *Logarithm* of ab the *Cube Root* required.

21. And universally, if the Logarithm L of the Root of any Power x , of any Fraction r s, be required, we have this Theorem $\frac{tr + xtA - tA}{x} = L$, or $\frac{tr + x - 1xtA}{x} = L = te$, that is, in Words, the Number $x - 1xtA$ added to the *Index* of the Logarithm of the *Fraction*, the Logarithm thus augmented being divided by x , the *Quotient* shall be the Logarithm of the Root sought. Or, since $tA = 10, 100$, the Number $x - 1$ prefix'd to the *Index* of the Logarithm of the Power, and the Logarithm thus divided by x , the *Quotient* shall be the Logarithm of the Root sought.



CHAP. V.

The original Construction of Logarithms by the Lord Neper, and the Alteration thereof to the present Form by himself and Mr. Briggs, explain'd and illustrated by the LOGARITHMIC CURVE.

1. **T**HE noble Inventer of *Logarithms*, the Lord *Neper*, having duely contemplated their wonderful Nature, first constructed and publish'd a *Canon* thereof; but those far different from what we now commonly use. And this was no Wonder, since scarcely any thing receives its *Invention*, and utmost *Perfection* at the same time.

2. In the first kind of Logarithms that *Neper* published, the *first Term* of the *continual Proportionals*, was placed only so far distant from *Unity*, as that *Term* exceeded *Unity*. Thus, for Example, if o n be the *first Term* of the Series from *Unity* AB (see Fig. I.) the Logarithm thereof, or the Distance $A n$, or $B y$, was by him put equal to vy or the *Increment*

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of the Number n y above Unity. If then we suppose $yn=1.0000001$, the Excess of this Number above $AB=1$, is 0.0000001 , which therefore, by him, was made its *Logarithm*; that is, $An=0.0000001$.

3. From hence, by Computation, the Number 10 will be the 23025850th Term of the Series; which Number therefore is the *Logarithm* of 10 in this Form of *Logarithms*; and expresses its Distance from Unity in such Part whereof vy , or An is one. Also the *Logarithm* of 2 (in this Form) is 6931471; of 3, is 10986122; of 4, is 13862943, &c.

4. But this Position of the Ratio of the Terms is entirely at Pleasure; for the Distance of the first Term may have any given Ratio to its Excess above Unity; that is, An may be indifferently less, equal to, or greater than vy ; and according to that various Ratio (which may be supposed at pleasure) between An , or By and vy , i. e. the Increment of the first Term above Unity, and the Distance of the same from Unity, there will be produced different Forms of *Logarithms*.

5. The *Logarithms* of this first Form, were found by the sagacious Inventer not to answer the Design in the best manner as could be wish'd; and therefore he changed 'em into another more convenient Form, wherein he put the Number 10, not as the 23025850th Term of the Series, but the 10000000th Term: And after Neper's Death, the learned and indefatigable Mr. Briggs, with great pains, made and publish'd a Canon of *Logarithms* according to this new Form. Now since in this Canon the *Logarithm* of 10 is 1.0000000, and since 1, 10, 100, 1000, &c. are *Proportionals*, they shall be *equidistant* from each other; wherefore the *Logarithm* of 100 shall be 2.0000000; of 1000, 3.0000000, and the *Logarithm* of 10000 will be 4.0000000, and so on. And this Form of *Logarithms* hath been ever since in use, and are those in present Use; the Nature and Properties

Properties of which we have been hitherto explaining.

6. The *Rationale* of the Method by which Mr. Briggs computed his Logarithms, is best explain'd from the *Logarithmic Curve*, according to Dr. Keil, as follows. In the *Logarithmic Curve* HBD (Fig. V.) let there be three *Proportionals* AB, ab, qs, very nearly equal to one another; that is, let their Differences have a *very small Ratio* to the said *Ordinates*, (for such are those *Proportionals*,) and then the *Differences* of the *Logarithms* will be *proportional* to the *Differences* of the *Ordinates*; that is, it will be $sr : bc :: Br : Bc :: Aq : Aa$. For since the *Ordinates* AB, ab, qs, are *nearly equal* to one another, they will be *very nigh* to one another; and so the *Parts* of the *Curve* Bs, Bb, intercepted between them, will *nearly coincide* with a *Right Line*; for it is possible that the *Ordinates* may be so near to each other, that the *Difference* between the *Part of the Curve* and the *Right Line* subtending it, may have to that *Subtense*, a *Ratio less* than any *given Ratio*. Consequently the *Triangles* Bcb, Brs, may be taken for *Right-lined* ones, and will be *equiangular*: and therefore, since a b is parallel to qs, they will be *similar*, and their *homologous Sides proportional*; viz. $rs : bc :: Br : Bb$, or $Aq : Aa$.

7. From hence, by the way, appears also the *Reason* of the *Correction* of *Numbers* and *Logarithms* by *Differences* and *proportional Parts*. For putting $AB=1$, or *Unity*, 'tis evident, that the *Logarithms* Bc, Br, are *proportional* to the *Differences* cb, rs, of the *Numbers* AB, ab, rs; as we shall hereafter prove by *Facts*, in the *practical Part*.

8. If a *mean Proportional* be found between 1 and 10, and then again another *Mean* between that and *Unity*; and if proceeding thus, you continually find a *mean Proportional* between the *Mean last found* and *Unity*, bisecting the *Logarithms* still as you proceed
(in

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(in the manner of the *Example* in Chap. II. Art. 6.) you will at last get a Number whose *Distance* from Unity shall be less than the $\frac{1}{1000000000000}$ Part of the Logarithm of 10.

9. After Mr. Briggs, in this manner, had made
 54 *Extractions* of the *Square Root*, he arrived to the
 Number 1.00000 00000 00000 127819149320032-
 3442, and its Logarithm was 0.00000 00000 00000-
 05551115123125782702. Suppose this Logarithm
 be equal to Aq or Br ; and let the Number found by
 this Extraction, be $=qs$; and then its *Excess* above U-
 nity will be $=rs$.

That is, $\left\{ \begin{array}{l} Aq = 0.0000000000000000555111512- \\ \quad 3125782702. \\ qs = 1.00000000000000001278191493- \\ \quad 200323442. \\ rs = 0.00000000000000001278191493- \\ \quad 200323442. \end{array} \right.$

10. Now by means of *these Numbers* the Logarithms of all *other Numbers* may be found in the following Manner. Between the *given Number* (whose Logarithm is to be found) and *Unity*, find (by the Extraction of Roots, as above) so many *mean Proportionals* till at last a Number be obtain'd *so little exceeding Unity*, that there be 15 *Cyphers* next after it, and as many *significant Figures* after those. Suppose the small Number thus found be ab , and let the *significant Figures* with the 15 *Cyphers prefix'd* before them, denote the Difference bc ; then say, as the Difference rs is to the Difference bc , so is the *given Logarithm Br*, to Bc the *Logarithm sought* for the Number ab . If now this Logarithm Bc or Aa , be continually doubled the *same number of times* as there were *Extractions of the Square Root*, you'll have at last the Logarithm of the Number propos'd as required.

11. If the *Tangent* TB be drawn to touch the *Curve* in B, then may the *Subtangent* AT be found by

by the Numbers above in *Art. 9.* For since AB, Br, are parallel to rs, AT; therefore the *Right-lin'd Triangles* Brs and BAT, are similar, and so as $sr : rB :: AB : AT$, the Subtangent; but since $AB=1$, therefore $\frac{rB}{sr} = AT$; or thus;

As $rs = 0.00000\ 00000\ 00000\ 127819149320032-$

3442,

Is to Br $= 0.00000\ 00000\ 00000\ 055511151231257-$

8270,

So is $AB=1.00000\ 00000\ 00000\ 00000\ 00000\ 00000$

0000,

to AT $= 0.434294481903251827651128918916-$

6051.

12. If the *proportional Right Lines* GH, EF, AB, CD, (Fig. V.) are Ordinates to the Axis CV of the *Logarithmic Curve*, and if their *Ends* FH, DB, be join'd by *Right Lines*, which produced meet the *Axis* in the Points P and K, then the *Right Lines* GP, KA, will be *always equal*. For since $GH : EF :: AB : CD$; it will be, as $GH : Ft :: AB : DR$ (by Chap. II. Art. 8.) But because of the *similar Triangles* PGH HtF, as also KAB BRD, we have $PG : Ht (:: GH : Ft :: AB : DR) :: KA : BR$. But $Ht=BR$, and therefore $PG=AK$; which was to be demonstrated.

13. If the *Right Lines* CD, EF *equally accede* to AB, GH, so that the Point D at last may coincide with B, and the Point F with H, then the *Right-Lines* DBK, FHP, which did *before* cut the Curve, will *now* only *touch* it in the Points B and H; that is, they will be chang'd into the *Tangents* BT, and HV. And so the *Right-Lines* AT, GV, will *always be equal* to each other; and so the *Subtangent* AT, or GV, in whatever Part of the Axis it be, is *always one constant given Length*; and this is one of the most remarkable and useful Properties of the *Logarithmic*

garithmic Curve: For the *different Species or Forms* of those Curves are determined by the *Subtangents*.

14. If the Excess *cb* of any Number *a b* extremely near Unity, or but a *small matter* exceeding it, be given, the *Logarithm* of its Distance from Unity *Aa* or *Bc*, will be known by means of the constant *Subtangent AT*; for by Art. 11. we have $bc : Bc :: AB : AT$; therefore $AT \times bc = Bc \times AB = Bc$, the *Logarithm* required. Thus also $AT \times rs = Br$, the *Logarithm* of the Number *qr*; and so the *Logarithm* of any *prime Number* 2, 3, 7, 11, 13, &c. may be found independently of the *Logarithm* of any other Number.



CHAP. VI.

A Method of constructing the Logarithms derived and demonstrated from the Nature of Numbers only, by Dr. Edm. Halley.

1. **T**HE *admirable Method* now before us, is one of the many great and wonderful Inventions and Discoveries of the celebrated Dr. *Halley*, the present *Astronomer Royal*, and *Fellow of the Royal Society*; and whose Name amongst the *Literati* will be had in *everlasting Remembrance*. This Method not only comprehends all the Improvements made by others by means of the *Hyperbola* and other *Geometrical Figures*, but shews with great Accuracy from the *pure Properties* of Numbers (as most natural and agreeable in a *Business purely Arithmetical*, as the *Logarithmotechny* is) how the *Logarithms* may be produced to any desired Number of Places, with far greater *Ease* and *Expedition* than by any Method before known. According to him, therefore,

2. Suppose an *infinite Number* of equal *Ratio's* or *Ratiunculae* between any two Terms in a continued *Scale* or *Series* of *Proportionals*; and those *Ratiunculae* express the *Ratio* of those two Terms, as of 1 to $1+x$. If then between *Unity* (1) and any *Number* proposed ($1+x$) there be taken any *Infinity* (n) of *mean Proportionals*, the infinitely little *Augment* or *Decrement* of the *first* of these *Means* from *Unity* will be a *Ratiuncula* or *Fluxion* (x) of the *Ratio* of *Unity* to the said *Number*; and the *Terms* of the *Series* will stand thus; viz. 1. $1+x$. $1+x^2$. $1+x^3$. $1+x^4$. &c. to $1+x^n$; and the

Exponents $\left\{ \begin{array}{l} 0. 1. 2. 3. 4. \text{ \&c. to } n; \text{ or thus,} \\ 0. x. 2x. 3x. 4x. \text{ \&c. to } nx. \end{array} \right.$

From whence 'tis evident that not only the *Number* (n) of the *Proportionals* or *Ratiunculae*, but also their *Sum* (nx) may be put for the *Logarithm* of $1+x$. And thus also Nj may be put = *Logarithm* of $1+y$, and consequently it will be, as $L, 1+x : L, 1+y :: nx : Ny$.

3. If $x=j$, that is, if the *Ratiunculae* composing divers *Ratio's* have the same *Magnitude*, then are those *Ratio's* proportional to the *Numbers* of *Ratiunculae* contain'd between their *Terms*, viz. $L, 1+x : L, 1+y :: n : N$. For if $x=1$, and $y=2$; then $1+x=2$, and $1+y=3$; and if $n=30103$, &c. N will be found = 47712, &c. that is, if there be the *infinite Number* 30103, &c. of *Ratiunculae* in the *Ratio* of 1 to 2; there shall be the *infinite Number* 47712, &c. of the same *Ratiunculae* in the *Ratio* of 1 to 3.

4. On the contrary, if $n=N$, then $L, 1+x : L, 1+y :: x : y$, that is, suppose the *infinite Number* of *Ratiunculae* in one *Ratio* equal to the *infinite Number* of *Ratiunculae* in any other *Ratio*, then are the *Logarithms* of those *Ratio's* directly as their *Fluxions*,

or as the *Magnitudes* of the *Ratiunculæ* respectively. For instance, let the Ratio of 10 to 1, 100 to 1, 1000 to 1, &c. all and every of them be supposed to consist of 5 *Ratiunculæ*; as follows: Thus,

{ Ratio's	0.	1.	2.	3.	4.	5. &c.
{ Terms	1.	10.	100.	1000.	10000.	100000, &c.
{ Ratiunc.	5v.	5w.	5x.	5y.	5z,	&c.

'Tis plain the *Ratiuncula* *v* of the whole Ratio of 10 to 1 is $\frac{1}{5}$; of 100 to 1, is $\frac{2}{5}$, &c. that is, the *Ratiunculæ* are $v=\frac{1}{5}$, $w=\frac{2}{5}$, $x=\frac{3}{5}$, $y=\frac{4}{5}$, $z=\frac{5}{5}$; but those *Fractions* are as the *natural Numbers* 1, 2, 3, 4, 5. Wherefore the Logarithms of the Ratio's of 10 to 1, 100 to 1, 1000 to 1, &c. are directly as the natural Numbers 1, 2, 3, 4, 5, &c. and so may be expressed by them.

5. Since then the *Logarithms* of Ratio's are as their *Fluxions*, therefore the Logarithm of any Number is found by taking the *Difference* of *Unity* and the *infinite Root* of that Number; that is, because $1+x^n$ is the first Term from *Unity*, or *Ratiuncula*, $\overline{1+x^n}$ $= 1+x$, is the *infinite Power* to be resolv'd; and $\sqrt[n]{1+x^n}$, or $\overline{1+x^n}^{\frac{1}{n}} = 1+x$, and so $\overline{1+x^n}^{\frac{1}{n}} - 1 = x$ = Logarithm of $1+x$.

6. In order to extract the Root of the infinite Power $1+x$, (which, to some, may seem *strange* and next to *impossible*) we must make use of Sir *Isaac Newton's* celebrated Theorem for that purpose. Supposing then the Power be $1+x$, according to his Theorem, $\overline{1+x^n}^{\frac{1}{n}} = 1 + \frac{1}{n}x + \frac{1-n}{2nn}xx + \frac{1-3n+2nn}{6n^3}x^3 + \frac{1-6n+11nn-6n^2}{24n^4}x^4$, &c. the Root of the Power $1+x$, when the *Index* (*n*) is *finite*; but (*n*) being in the present Case *infinite*, all the Terms of the *Coefficients*, wherein (*nn*) is found a Divisor (as being *infinitely infinite*) will vanish, as being *infinitely* less than

than nothing. But the Co-efficient $\frac{1-n}{2n} = \frac{1}{2n} - \frac{1}{2n} =$

$-\frac{1}{2n}$; and $\frac{1-3n+2n^2}{6n^2} = -\frac{1}{6n^2} + \frac{1}{3n} = \frac{1}{3n}$; Also the

Co-efficient $\frac{1-6n+11n^2-6n^3}{24n^3} = \frac{1}{12n^3} - \frac{1}{4n} = \frac{1}{4n}$; &c.

Wherefore the foregoing Root will become $1 + \frac{1}{n}x - \frac{1}{2}nx^2 + \frac{1}{3}nx^3 - \frac{1}{4}nx^4 + \frac{1}{5}nx^5$, &c. That is,

$\frac{1}{n}xx - \frac{1}{2}nx + \frac{1}{3}xxx - \frac{1}{4}x^4 - \frac{1}{5}x^5$, &c. $= 1 + x^n - 1 = x^n$
Logarithm of $1+x$.

7. And whereas the *infinite Index* (n) may be taken at pleasure, an *Infinity* of *different Scales* of Logarithms may be produced; and those different Logarithms will be to one another as $\frac{1}{n}$, or *reciprocally* as the *Indices* (n). And as it hath been shewn (in Chap. V.) that in making the *first kind of Logarithms* by *Neper*, the *infinite Index* of the Logarithm of 10 would be 23025850, &c. But in making the second sort after by *Briggs*, the said *infinite Index* was put = 10000000, &c. Consequently in the foregoing Series for Logarithms, if $n=10000000$, &c. the Lord *Neper's* Logarithms will be produced; and the Series will be *simply* $x - \frac{1}{2}x^2 + \frac{1}{3}x^3 - \frac{1}{4}x^4 + \frac{1}{5}x^5$, &c. On the contrary, if $n=23025850$, &c. then *Briggs's Logarithms* will arise from the Series; and because $\frac{1}{n} = \frac{1}{23025850}$, &c. $= 0.434229448$, &c. therefore $\frac{1}{n}$ (=AT) is the *Subtangent* of the *Logarithmic Curve* for the *Briggian Logarithms*, as is plain from Chap. I. V. Art. 11. Whence if a Logarithm of *Neper's Form* be multiplied by 0.434229448, &c. or divided by 2.3025850, &c. it is converted into a Logarithm of *Briggs's Sort*, or those in present Use.

8. If the Logarithm of a decreasing Ratio be sought, as of 1 to $1-x$, the Power being $1-x$, its *infinite Root* will be $\frac{1}{1-x^n} = 1 + \frac{1}{n}x + \frac{1}{2n}x^2 + \frac{1}{3n}x^3 - \frac{1}{4n}x^4 -$

$\frac{1}{2}nx^4 - \frac{1}{2}nx^6$, &c. that is, $\frac{1}{n} \times x + \frac{1}{2}x^3 + \frac{1}{3}x^5 + \frac{1}{4}x^7 + \frac{1}{5}x^9$, &c. $= 1 - 1 - x^2 = x$ the Logarithm of $1 - x$; the first Term next below Unity, or Root of this infinite decreasing Series being $1 - x$. And so in this Case, according as the infinite Index (n) is made $= 10000000$, &c. or 2.3025850 , &c. so Neper's or Briggs's Logarithms of those negative Numbers will be produced.

9. Instead of the Terms $1 : 1 + x$, let $a : b$ express the Terms of any Ratio universally; and make $a + b = s$, and $a - b = d$; and since it is $1 : 1 + x :: a : b$, therefore $a + ax = b$, and so $x = \frac{b-a}{a} = \frac{d}{a}$. Again because (in the decreasing Ratio) it is $1 : 1 - x :: b : a$; therefore $b - bx = a$, and so we have again $x = \frac{b-a}{b} = \frac{d}{b}$. Whence the Logarithm of the same Ratio $a : b$, may be doubly expressed; viz. for the increasing Ratio, the Series will be

$$\frac{1}{n} \times \frac{d}{a} - \frac{d^2}{2a^2} + \frac{d^3}{3a^3} - \frac{d^4}{4a^4} + \frac{d^5}{5a^5}, \text{ \&c. or,}$$

$$\frac{1}{n} \times \frac{d}{b} + \frac{d^2}{2bb} + \frac{d^3}{3b^3} + \frac{d^4}{4b^4} + \frac{d^5}{5b^5}, \text{ \&c. for}$$

the decreasing Ratio; all which is evident from the three last Articles.

10. But if we suppose the Ratio of a to b , viz. $\frac{a}{b}$ composed of two Parts; viz. of the Ratio of a to the Arithmetical Mean between the Terms a and b , and of the Ratio of the said Arithmetical Mean to the other Term b ; that is, suppose $\frac{a}{b} = \frac{a}{\frac{a+b}{2}} \times \frac{\frac{a+b}{2}}{b}$, (for $\frac{a+b}{2}$ is the Arithmetical Mean between a and b ;) then the Sum of the Logarithms of those two Ratio's, $\frac{a}{\frac{a+b}{2}}$, $\frac{\frac{a+b}{2}}{b}$, will be equal to the Logarithm of the Ratio of a to b . Or, $L, \frac{a}{\frac{a+b}{2}} + L, \frac{\frac{a+b}{2}}{b} = L, \frac{a}{b}$.
And

And thus also we have $L, \frac{1}{2}s + L, \frac{b}{\frac{1}{2}s} = L, \frac{b}{a}$.
 Now because the Ratio of $\frac{1}{2}s$ to b is encreasing, therefore $1 : 1 + x :: \frac{1}{2}s : b$; and so $1 + x = \frac{b}{\frac{1}{2}s}$; consequently $x = \frac{b}{\frac{1}{2}s} - 1 = \frac{b - \frac{1}{2}s}{\frac{1}{2}s} = \frac{d}{s}$ a-new; again, because the Ratio of $\frac{1}{2}s$ to a is decreasing; therefore $\frac{1}{2}s : a :: 1 : 1 - x$, and so $1 - x = \frac{a}{\frac{1}{2}s}$; and $x = 1 - \frac{a}{\frac{1}{2}s} = \frac{\frac{1}{2}s - a}{\frac{1}{2}s} = \frac{d}{s}$.

11. Therefore since $x = \frac{d}{s}$ for both Ratio's, viz. of a to $\frac{1}{2}s$, and $\frac{1}{2}s$ to b , we shall have (by the foregoing Rule, Art. 9.) $\frac{1}{s} \times \frac{d}{s} + \frac{d^2}{2s^2} + \frac{d^3}{3s^3} + \frac{d^4}{4s^4} + \frac{d^5}{5s^5}$, &c. = (A) L, a to $\frac{1}{2}s$, and $\frac{1}{s} \times \frac{d}{s} - \frac{d^2}{2s^2} + \frac{d^3}{3s^3} - \frac{d^4}{4s^4} + \frac{d^5}{5s^5}$, &c. = (B) $L, \frac{1}{2}s$ to b . Then $\frac{1}{s} \times \frac{2d}{s} * + \frac{2d^3}{3s^3} * + \frac{2d^5}{5s^5}$, &c. = (A+B) L, a to b .

Thus you have a Series expressing the Logarithm of the Ratio of a to b , whose Sum is $s=a+b$, and Difference $d=a-b$: and *this Series* converges twice as swift as the former in Art. 8. and therefore is more proper for *making* or *examining* of Logarithms, which it performs with great Expedition, when d the Difference is but the 100th Part of s the Sum; the first Step $\frac{2d}{s}$ sufficing for 7 Places of the Logarithm, and the second $\frac{2d^3}{3s^3}$ for 12 Places.

12. Because the Difference of the Logarithms of the Ratio's of a to $\frac{1}{2}s$, and $\frac{1}{2}s$ to b is the Logarithm of the Ratio of a to b ; or thus, because

$$\left(\frac{\frac{1}{2}s}{b} \right)$$

$\frac{\frac{1}{2}s}{b} \left(= \frac{ab}{\frac{1}{4}ss} \right)$, therefore the $L, \frac{a}{\frac{1}{2}s} - L, \frac{\frac{1}{2}s}{b} = L, \frac{ab}{\frac{1}{4}ss}$
 $(A-B) = \frac{1}{n} \times \frac{2d^2}{2s^2} * + \frac{2d^4}{4s^4} * + \frac{2d^6}{6s^6}, \text{ \&c. but}$
 half the Ratio $\frac{ab}{\frac{1}{4}ss}$ is the Ratio $\frac{\sqrt{ab}}{\frac{1}{2}s}$ (for $\frac{\sqrt{ab}}{\frac{1}{2}s} \times \frac{\sqrt{ab}}{\frac{1}{2}s} =$
 $\frac{ab}{\frac{1}{4}ss}$), that is, the Ratio of the *Geometrical Mean*
 $\sqrt{ab}$ to the *Arithmetical Mean* $\frac{1}{2}s$; consequently
 the Logarithm of $\frac{\sqrt{ab}}{\frac{1}{2}s} = \frac{1}{n} \times \frac{d^2}{2s^2} * + \frac{d^4}{4s^4} * + \frac{d^6}{6s^6},$
 \&c. which is a Theorem of good Dispatch for find-
 ing the Logarithm of $\frac{1}{2}s$.

13. But the same Logarithm may yet be much more
 advantageously obtained by a Method like the for-
 mer. For if we make the Terms of the Ratio
 $\frac{ab}{\frac{1}{4}ss} = \frac{A}{B}$, and put $S = ab + \frac{1}{4}ss$, the Sum of the
 Terms, and $D = ab - \frac{1}{4}ss$; 'tis evident the
 Logarithm of $\frac{A}{B} \left(= \frac{ab}{\frac{1}{4}ss} \right) = \frac{1}{n} \times \frac{2D^2}{S} + \frac{2D^3}{3S^3} +$
 $\frac{2D^4}{5S^5} + \frac{2D^7}{7S^7}, \text{ \&c. by Art. 11. But because } \frac{1}{4}ss =$
 $\frac{1}{4}a^2 + \frac{1}{4}ab + \frac{1}{4}b^2$, therefore $D = ab - \frac{1}{4}ss = ab -$
 $\frac{1}{4}a^2 - \frac{1}{4}ab - \frac{1}{4}b^2 = \frac{1}{4}a^2 - \frac{1}{4}ab + \frac{1}{4}bb = \sqrt{\frac{1}{4}a} - \frac{1}{4}b$
 $= \frac{1}{4}d^2 = 1$, in the present Case of finding the Loga-
 rithms of Prime Numbers; for suppose the Loga-
 rithm of 23 be sought, then $a = 22$, $b = 24$, $\frac{1}{4}s = 23$,
 and $d = 2$; also $A = ab = 528$, and $B = \frac{1}{4}ss = 529$,
 and therefore $ab - \frac{1}{4}ss = A - B = D = \frac{1}{4}d = 1$. Where-
 fore, since $D = 1$, the Series above becomes $\frac{1}{n} \times \frac{2}{S} +$
 $\frac{2}{3S^3} + \frac{2}{5S^5} + \frac{2}{7S^7}, \text{ \&c.} = \text{the Logarithm of the}$
 Ratio of $\frac{ab}{\frac{1}{4}ss}$; and so the half of it, *viz.* the Series
 $\frac{1}{n} \times \frac{1}{S} + \frac{1}{3S^3} + \frac{1}{5S^5} + \frac{1}{7S^7}, \text{ \&c.} = \text{the Loga-}$
 rithm

rithm of the Ratio of $\sqrt{ab}$ to is . . And this converges much sooner than any Theorem hitherto invented, and beyond which nothing better can be hoped for, in the *great Author's* Opinion.

14. The *Logarithm* given to find what *Ratio* it expresses, is a Problem solv'd with *like Ease*, and demonstrated by a *like Process* to that foregoing for finding the Logarithm of a *given Ratio*. For as the Logarithm of the Ratio of 1 to $1+x$ was proved to be $1+\overline{x^n}-1$, (by Art. 5.) and that of the Ratio of 1 to $1-x$ to be $1-\overline{1-x^n}$ (by Art. 8.) so the Logarithm, which we will call L , being given, since $L=1+\overline{x^n}-1$, therefore $L+1=1+\overline{x^n}$ in the first Case; and $1-L=1-\overline{x^n}$, in the latter: Consequently $1+L=\overline{1+x}$, and $1-L=\overline{1-x}$. That is, according to Sir *Isaac's Theorem*, $1+nL+\frac{1}{2}n^2L^2+\frac{1}{6}n^3L^3+\frac{1}{24}n^4L^4+\frac{1}{120}n^5L^5$, &c. $=1+x$; and also $1-nL+\frac{1}{2}n^2L^2-\frac{1}{6}n^3L^3+\frac{1}{24}n^4L^4-\frac{1}{120}n^5L^5$, &c. $=1-x$; consequently the Number $1+x$ or $1-x$ is readily known by those Series, be the *Species* of the Logarithm what it will. That is, whether it be *Neper's* Logarithm, where $n=100000$, &c. and so $1+x=1+L+\frac{1}{2}L^2+\frac{1}{6}L^3+\frac{1}{24}L^4+\frac{1}{120}L^5$, &c. or whether $n=23025850$, &c. for *Briggs's* Logarithm.

15. If one Term of the *Ratio*, whereof L is the Logarithm, be given, the other Term will easily be had by the same Rule. Let a = the *least Term* of the *Ratio*, and b = the greatest; then because it is $1 : 1+x :: a : b$; and so $1+x = \frac{b}{a} = 1+L+\frac{1}{2}L^2+\frac{1}{6}L^3+\frac{1}{24}L^4$, &c. if $n=100000$, &c. and therefore $b=a+aL+\frac{1}{2}aL^2+\frac{1}{6}aL^3$, &c. if a were given; but if b were given, because $1 : 1-x :: b : a$,
therefore

therefore $1 - x = \frac{a}{b}$; and so $a = b - bL + \frac{1}{2}bL^2 - \frac{1}{6}bL^3 + \frac{1}{24}bL^4$, &c. Wherefore by the Help of the *Tables*, the Number belonging to any Logarithm will be *exactly had* to the utmost Extent of the *Tables*.

16. Suppose $\frac{b}{a} = N$, the Number belonging to the given Logarithm L of the Ratio $\frac{b}{a}$, then $\frac{b}{N} = a$, let the Logarithm of the Ratio $\frac{b}{N}$ be $\mathfrak{z}$, and let the Term b be known; then (*per* Rule, Art. 15.) we have $1 : 1 - x :: b : N$, and so $1 - x = \frac{N}{b}$, and $N = b - b\mathfrak{z} + \frac{1}{2}b\mathfrak{z}^2 - \frac{1}{6}b\mathfrak{z}^3$, &c. if $\mathfrak{z}$ be *Neper's* Logarithm, but $N = b - b\mathfrak{z} + \frac{1}{2}b\mathfrak{z}^2 - \frac{1}{6}b\mathfrak{z}^3$, &c. if $\mathfrak{z} = 2.3025$, &c. as in *Briggs's* Logarithm. But if the Ratio be $\frac{a}{b} = N$, then $\frac{a}{N} = b$, and so $1 + x = \frac{N}{a}$, therefore $N = a + a\mathfrak{z} + \frac{1}{2}a\mathfrak{z}^2 + \frac{1}{6}a\mathfrak{z}^3$, &c.

Or $N = a + a\mathfrak{z} + \frac{1}{2}a\mathfrak{z}^2 + \frac{1}{6}a\mathfrak{z}^3$, &c. Note, here a and b denote the Number belonging to the nearest *next lesser* or *next greater* Logarithm than the given Logarithm L , and the Logarithm $\mathfrak{z}$ is the *Difference* of those Logarithms; wherefore as $\mathfrak{z}$ is less, the Series converges the swifter; and finds the Number N of the Logarithm L , much sooner and easier than the Rule in Art. 15.

17. In the foregoing Series $a + a\mathfrak{z} + \frac{1}{2}a\mathfrak{z}^2 + \frac{1}{6}a\mathfrak{z}^3$, &c. $= N$; the three first Steps may be abridg'd thus, $a + \frac{a\mathfrak{z}}{1 - \frac{1}{2}\mathfrak{z}} = a + a\mathfrak{z} + \frac{1}{2}a\mathfrak{z}^2$, very nearly, and may serve with *Exactness* enough for Numbers not exceeding 14 Places, which is more than sufficient for *common Use*. Therefore we may take

$a +$

$a + \frac{ax}{1-\frac{1}{2}x} = N$, or $b - \frac{bx}{1+\frac{1}{2}x} = N$; and if the Index n taken for *Brigg's* Logarithms, we shall have

$a + \frac{ax}{1-\frac{1}{2}x} = N$, or $b - \frac{bx}{1+\frac{1}{2}x} = N$; that is

(putting $x = \frac{1}{n} = .43429$, &c.) $\frac{xa + \frac{1}{2}ax}{x - \frac{1}{2}x} = N$, or

$\frac{xb - \frac{1}{2}bx}{x + \frac{1}{2}x} = N$; which Equation may be resolv'd into

the following Analogies;

$$\text{viz. } \begin{cases} x - \frac{1}{2}x : x + \frac{1}{2}x :: a : N; \text{ or;} \\ x + \frac{1}{2}x : x - \frac{1}{2}x :: b : N. \end{cases}$$

18. If more Steps of *this Series* be desir'd, it will be found as follows, viz. $a + \frac{ax}{1-\frac{1}{2}x} - \frac{\frac{1}{2}a^2x}{1-\frac{1}{2}x} + \frac{\frac{1}{2}a^2x}{1-\frac{1}{2}x}$, &c. $= N$; also the Rule $1 + nL + \frac{1}{2}nnLL + \frac{1}{6}n^3L^3$, &c. may be thus contracted, viz. $1 + \frac{2 + nL + \frac{1}{2}nnLL \times \frac{1}{2}nL}{2} = N$. What is said concerning this Method of making either *Logarithms* or *Numbers*, I presume is sufficient to render it very intelligible to any common Capacity, and to shew the admirable *Usefulness* and *Excellency* thereof beyond any other, hitherto invented.



C H A P. VII.

*The *Logarithmic Series aforegoing, demonstrated also by *FLUXIONS, and from the Nature of the *Hyperbola.*

1. **T**HE preceding Series for the Logarithms, which has been demonstrated *purely* from *Arithmetical Principles*, or the *Properties of Numbers*, may also be prov'd from the Doctrine of the *Fluxions of Logarithms*. For the Writers on *Fluxions* variously demonstrate the *Fluxion of the Logarithm of any Number* is equal to the *Fluxion of that Number (whose Logarithm it is)* divided by the said Number it self.

2. Let the Number proposed be $1+x$, the Fluxion of which is $\dot{x}$, therefore the *Fluxion of its Logarithm* will be $= \frac{\dot{x}}{1+x}$; from whence the foregoing infinite Series for the Logarithm of the Number $1+x$ may be derived, as follows. The Logarithm of $1+x$ is equal to the *flowing Quantity* or *Fluent* of the said Fluxion $\frac{\dot{x}}{1+x}$. But $\frac{\dot{x}}{1+x} = \dot{x} \times \frac{1}{1+x}$; and $\frac{1}{1+x} = 1+x) 1 (=1-x+x^2-x^3, \&c.$

$$\begin{array}{r}
 1+x \\
 \hline
 -x \\
 \hline
 xx \\
 \hline
 +xx \\
 \hline
 +xx+x^2 \\
 \hline
 ..-x^3 \\
 \hline
 ..-x^2-x^4 \\
 \hline
 ..+x^4, \&c.
 \end{array}$$

3. The Quotient then $1 - x + x^2 - x^3 + x^4$, &c.
 $= \frac{1}{1+x}$; but $\dot{x} \times 1 - x + x^2 - x^3 + x^4$, &c. $= \dot{x} -$
 $x\dot{x} + x^2\dot{x} - x^3\dot{x} + x^4\dot{x}$, &c. $= \frac{\dot{x}}{1+x}$. The *Fluent* there-

fore of that *infinite fluxionary Series* (by the *inverse Method of Fluxions*) is found to be $x - \frac{1}{2}x^2 + \frac{1}{3}x^3 - \frac{1}{4}x^4 + \frac{1}{5}x^5$, &c. which therefore is the *Logarithm* of the Number $1+x$; and is the very same with that Series in the foregoing Chap. VI. Article 7. for *Neper's Logarithms*.

4. Again, if the Ratio be *decreasing*, or the Number be $1-x$, the Fluxion of this also is $\dot{x}$, and therefore the Fluxion of its *Logarithm* $\frac{\dot{x}}{1-x}$. But

$\frac{\dot{x}}{1-x} = \dot{x} \times \frac{1}{1-x} = 1+x+x^2+x^3$, &c. $\dot{x} \times \dot{x} = \dot{x} + x\dot{x} + x^2\dot{x} + x^3\dot{x}$, &c. The *Fluent* of which is $x + \frac{1}{2}x^2 + \frac{1}{3}x^3 + \frac{1}{4}x^4$, &c. the same Series as that in Article 8. of the *preceding Chapter*, for *Neper's Logarithm* of $1-x$.

5. The *same Series* is likewise deduced from the Nature or Equation of the *Hyperbola*. For let FCH be an *Hyperbola* (Fig. VI.) AE, AI, the *Asymptotes*; draw BC, DC parallel to AI and AE; also draw the Ordinate EF parallel to the Ordinate BC, or *Asymptote* AI. Let AB= a , EF= y , and BE= x . The Equation $aa=ay+xy$ expresses the Nature of the *Hyperbola* between the *Asymptotes*. Now the Fluxion of the Space between the *Abscissa*, *Ordinate*, and *Curve*, is always equal to the *Product* of the Ordinate into the *Fluxion* of the *Abscissa*; that is, in this Case, $=y\dot{x}$. Therefore to determine the Fluxion of the *Asymptotic Space* contain'd between the *Abscissa* BE, the *Ordinates* EF, BC, and the *Curve* of the *Hyperbola* FC, we have $\frac{aa}{a+x} = y$, therefore $\frac{aa\dot{x}}{a+x} =$
 $y\dot{x}$ = Fluxion of the said Space FCBE.

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6. But $\frac{ax}{a+x} = \frac{ax}{a+x} \times 1$, and $\frac{ax}{a+x} = a+x) ax$
 $(= a-x + \frac{xx}{a} - \frac{x^3}{aa} + \frac{x^4}{a^3}, \&c. \text{ Wherefore } \frac{ax}{a+x} =$
 $ax - \frac{xx}{a} + \frac{x^3}{aa} - \frac{x^4}{a^3} + \frac{x^5}{a^4}, \&c. \text{ But the flow-}$
ing Quantity of this fluxionary Series is $ax - \frac{1}{3}x^2 +$
 $\frac{x^3}{3a} - \frac{x^4}{4aa} + \frac{x^5}{5a^3}, \&c. = \text{the Space FCBE. Suppose } a=1, \text{ then } x - \frac{1}{3}x^2 + \frac{1}{6}x^3 - \frac{1}{24}x^4 + \frac{1}{120}x^5, \&c. = \text{the}$
said Asymptotic Space FCBE, as before. But 'tis evi-
dent this is again the very same Series as was invented
by Dr. Halley for the Logarithm of the Number 1+x.

7. The *Asymptotic Spaces*, then, are with respect
to the *Abscissæ*, as *Logarithms* in respect to *Numbers*.
That is, since $AB=1$, and the Logarithm of 1 is $=0$,
the Spaces $B a b C$, $B c d C$, $B e g C$, $B b k C$, &c.
are the *Logarithms* of the Numbers $A a$, $A c$, $A e$,
 $A b$, &c. Again, because the *Abscissæ* are in a *re-*
ciprocal Proportion of the Ordinates, that is,
 $AB : AE :: EF : BC$; therefore the *Asymptotic Spaces*
are in respect of the *Ordinates* as *Logarithms* in re-
spect of *Numbers*: Yet so, that while the *Ordinates*
 BC, ab, cd, eg, bk, EF , decrease in a *Geometrical*
Ratio, the Spaces $B a b C$, $B c d C$, &c. may in-
crease in an *Arithmetical Ratio*. And since in *Ne-*
per's Logarithms $\frac{1}{n} \times x = \frac{1}{n}xx + \frac{1}{n}xxx - \frac{1}{n}x^4, \&c.$
 $n=100000, \&c.$ 'tis plain his Logarithms become
the same with the *Hyperbolic Logarithms* just now
consider'd.





CHAP. VIII.

The Method of constructing Logarithms by the Infinite Series, exemplified and illustrated.

1. **T**HE Manner of raising *Theorems* for the Construction of Logarithms hath been sufficiently explain'd; it therefore remains that we illustrate the same by proper Examples. The *Theorems* for doing this directly are,

Theorem I. $\frac{1}{n} \times x = \frac{1}{n}x + \frac{1}{2n}x^2 + \frac{1}{3n}x^3 + \frac{1}{4n}x^4 + \frac{1}{5n}x^5, \&c.$
 $= L, \overline{1 \pm x}.$

Theorem II. $\frac{1}{n} \times \frac{2d}{s} + \frac{2d^3}{3s^3} + \frac{2d^5}{5s^5} + \frac{2d^7}{7s^7}, \&c.$
 $= L, \frac{a}{b}.$

Theorem III. $\frac{1}{n} \times \frac{d^2}{2s^2} + \frac{d^4}{4s^4} + \frac{d^6}{6s^6} + \frac{d^8}{8s^8},$
 $\&c. = L.$

Theorem IV. $\frac{1}{n} \times \frac{1}{s} + \frac{1}{3s^3} + \frac{1}{5s^5} + \frac{1}{7s^7},$
 $\&c. = L$

Note, in these *Theorems*, $\frac{1}{n}$ is all along applied to adapt them to *all sorts of Logarithms*.

2. Since $n =$ the Logarithm of 10, we must therefore first suppose $n = 10000000, \&c.$ and thence *Neper's Logarithms* will be produced; and so these are the first sort of Logarithms which Nature affords: The others, as *Briggs's Logarithms*, &c. are made from them. In order then to find a *Briggian Logarithm*, 'tis necessary first to find *Neper's Logarithm* of 10. This may be done several Ways, either by the Number 10 it self, or by its component Parts. If we attempt it by the Number 10 it self, then because

$$1+x=$$

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$1+x=10$, we shall have $x=9$, which, because it is greater than 1, will occasion that the first Theorem will not converge; and so the second Theorem must be used. In this $\frac{a}{b} = \frac{1}{10}$, and therefore $s=1+10=11$, and $d=10-1=9$. And thus the second Series for *Neper's* Logarithm of 10 will be $\frac{2d}{s} + \frac{2d^3}{3s^3} + \frac{2d^5}{5s^5}$, &c. $= \frac{18}{11} + \frac{1458}{1331} + \frac{117858}{1677161}$, &c. $= 2.30258$, &c. the Logarithm sought.

3. But this Series converging so *extremely slow*, renders the Business very tedious, and therefore the said Logarithm must be attempted from the *component Parts* of 10. And since $8 \times 1\frac{1}{4} = 10$, and $2 \times 2 \times 2 = 8$, therefore $3L, 2 + L, 1\frac{1}{4} = L, 10$. Consequently by finding the Logarithm of 2 and $1\frac{1}{4}$, we find the Logarithm of 10. Now *Neper's* Logarithm of 2, is found either by Theorem I. which converges very slowly; or by Theorem II. which converges much faster; and therefore to be chosen. Here $\frac{a}{b} = \frac{1}{2}$, and $a+b=s=3, a-b=d=1$, and so

the Theorem $\frac{2d}{s} + \frac{2d^3}{3s^3} + \frac{2d^5}{5s^5}$, &c. $= \frac{2d}{s} + \frac{1}{3} \times \frac{2d^3}{s^3} + \frac{1}{5} \times \frac{2d^5}{s^5}$, &c. $= \frac{2}{3} + \frac{1}{3} \times \frac{2}{3^3} + \frac{1}{5} \times \frac{2}{3^5}$, &c. $= L, 2$; this multiplied by 3, is $= \frac{6}{3} + \frac{1}{3} \times \frac{6}{3^3} + \frac{1}{5} \times \frac{6}{3^5}$, &c. $= 3L, 2$. Now put $\frac{6}{3^3} = A$, and because $\frac{1}{3^4} \times \frac{6}{3^3} = \frac{6}{3^5}$, and $\frac{1}{3^2} = \frac{1}{9}$, therefore also put $\frac{1}{9} A = B$, and so $B = \frac{6}{3^5}$; and thus $\frac{1}{9} B = C, \frac{1}{9} C = D$, and so on. Whence the Theorem will become $2 + \frac{1}{3} A + \frac{1}{5} B + \frac{1}{7} C + \frac{1}{9} D + \frac{1}{11} E + \frac{1}{13} F$, &c. $= 3L, 2$.

4. By the same Theorem II. we obtain *Neper's* Logarithm of $1\frac{1}{2}$; for because $\frac{5}{4} = 1\frac{1}{2}$, therefore $\frac{1}{1\frac{1}{2}} = \frac{5}{4}$ $\left(= \frac{4}{5} = \frac{a}{b} \right)$. Whence $a + b = s = 9$, and $d = a - b = 1$; and so the Theorem $\frac{2d}{s} + \frac{2d^3}{3s^3} + \frac{2d^5}{5s^5}$, $\mathcal{E}c.$ $= \frac{2}{9} + \frac{1}{3} \times \frac{2}{9^3} + \frac{1}{5} \times \frac{2}{9^5}$, $\mathcal{E}c.$ But because $\frac{2}{9} = \frac{6}{3^2} = A$; $\frac{2}{9^3} = \frac{6}{3^7} = \frac{6}{3^5} \times \frac{1}{9} = \frac{1}{3} B = C$; $\frac{2}{9^5} = \frac{6}{3^{11}} = \frac{6}{3^9} \times \frac{1}{9} = \frac{1}{3} D = E$, $\mathcal{E}c.$ therefore the said Theorem will become $A + \frac{1}{3}C + \frac{1}{3}E + \frac{1}{3}G + \frac{1}{3}I + \frac{1}{3}L$, $\mathcal{E}c. = L$, $1\frac{1}{2}$ as required. If now this Series be added to the foregoing (in Art. 3.) we shall have the Theorem $2 + \frac{1}{3}A + \frac{1}{3}B + \frac{1}{3}C + \frac{1}{3}D + \frac{1}{3}E + \frac{1}{3}F + \frac{1}{3}G + \frac{1}{3}H$, $\mathcal{E}c. = 3L$, $2 + L$, $1\frac{1}{2} = L$, 10. See the Operation in the Table below.

A = 0.22222222222222	2. = 2.00000000000000
B = . . 246913580246	$\frac{1}{3}A = .2962962962962$
C = . . . 27434842249	$\frac{1}{3}B = . . . 49382716049$
D = ... 3048315805	$\frac{1}{3}C = . . . 13064210595$
E = ... 338701756	$\frac{1}{3}D = ... 338701756$
F = ... 37633529	$\frac{1}{3}E = ... 98531420$
G = ... 4181503	$\frac{1}{3}F = ... 2894887$
H = ... 464612	$\frac{1}{3}G = ... 876124$
I = ... 51624	$\frac{1}{3}H = ... 27330$
K = ... 5763	$\frac{1}{3}I = ... 8454$
L = ... 637	$\frac{1}{3}K = ... 273$
M = ... 71	$\frac{1}{3}L = ... 85$
	$\frac{1}{3}M = ... 3$

Thus *Neper's* Log. of 10 = 2.3025850929940, $\mathcal{E}c.$

5. The Logarithm thus found (if continued on) will be 2.302585092994045684017991454684364207601101488628772976033328, $\mathcal{E}c. =$; and therefore

fore in making the *Briggian Logarithms*, we shall have $\frac{1}{n} = 0.434294481903251827651128918916605082294397005803666566114454$, &c. the Reciprocal of the former; which henceforth let be call'd z ; that is, let $\frac{1}{n} = z$. And now we are prepar'd to find the Logarithm (of any Form) of any other Number.

6. For Example, let it be required to find *Briggs's Logarithm* of z , to 10 Places of Figures. In order to this, the Index $\frac{1}{n}$ must be assum'd of a Figure or two more than the intended Number of Places in the Logarithm. The *second Theorem* is most proper for this Purpose; for here again $\frac{a}{b} = \frac{1}{3}$, $d = 1$, and $s = 3$, and also $z = 0.434294481903$; and the Theorem $\frac{1}{n} \times \frac{zd}{s} + \frac{zd^3}{3s^3} + \frac{zd^5}{5s^5}$, &c. $= z \times \frac{1}{3} + \frac{1}{3} \times \frac{z}{3^3} + \frac{1}{3} \times \frac{z}{3^5}$, &c. $= L, 2$, and therefore $\frac{z}{3} + \frac{1}{3} \times \frac{z}{3^3} + \frac{1}{3} \times \frac{z}{3^5}$, &c. $= \frac{1}{3} L, 2$. Here put $\frac{z}{3} = A$, and because $\frac{z}{3^3} = \frac{z}{3} \times \frac{1}{3^2} = \frac{1}{9} \times A = B$, and thus $\frac{z}{3^5} = \frac{z}{3^3} \times \frac{1}{3^2} = \frac{1}{9} B = C$, and so on. Therefore the said Theorem will here again become $A + \frac{1}{3}B + \frac{1}{9}C + \frac{1}{27}D + \frac{1}{81}E + \frac{1}{243}F$, &c. $= \frac{1}{3}L, 2$, as is evident from the following Operation.

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$\frac{1}{2}z = A = 0.144764827301$	$A = 0.144764827301$
$B = 16084980811$	$\frac{1}{2}B = 5361660270$
$C = 1787220090$	$\frac{1}{2}C = 357444018$
$D = 198580010$	$\frac{1}{2}D = 28368572$
$E = 22064445$	$\frac{1}{2}E = 2451605$
$F = 2451605$	$\frac{1}{2}F = 222873$
$G = 272400$	$\frac{1}{2}G = 20953$
$H = 30266$	$\frac{1}{2}H = 2017$
$I = 3362$	$\frac{1}{2}I = 197$
$K = 373$	$\frac{1}{2}K = 19$
$L = 41$	$\frac{1}{2}L = 1$
The Sum is $\frac{1}{2}L, 2 = 0.150514997826$	
Multiplied by 2	
The <i>Briggian</i> Logarithm of 2 = 0.301029995652	

7. This Logarithm may yet be much easier and sooner obtain'd by this Consideration, viz. That $\frac{1}{2}^{10} = \frac{1}{1024}$, and $\frac{1000}{1024} \times \frac{1}{1000} = \frac{1}{1024}$; therefore $L_{\frac{1000}{1024}} + L_{\frac{1}{1000}} = L_{\frac{1}{1024}} = L_{\frac{1}{2}} = L_2$. But

$L_{\frac{1000}{1024}} = L_{\frac{125}{128}}$. Wherefore put $\frac{125}{128} = \frac{a}{b}$ a-new, then $a+b=253$, and $a-b=d=3$; thus Theorem II. will converge much faster, and will become, in

Numbers, $2 \times \frac{2}{1} \times \frac{1}{253} + \frac{2}{1} \times \frac{3^3}{253^3} + \frac{2}{1} \times \frac{3^5}{253^5}, \&c.$

Or if $\frac{a}{b} = y$, the said Theorem, in *Species*, is $2zy + \frac{2}{1}zy^2 + \frac{2}{1}zy^3, \&c.$ Suppose $2zy = A$, then $\frac{2}{1}zy^2 = \frac{1}{1} \times 2zyxy^2 = \frac{1}{1}Axy^2 = B$, also $\frac{2}{1}zy^3 = \frac{1}{1} \times 2zyxy^2xy^2 = \frac{1}{1}Bxy^2 = C$, and so on; and thus the Theorem is $A + \frac{1}{1}Ay^2 (=B) + \frac{1}{1}By^2 (=C) + \frac{1}{1}Cy^2 (=D) + \frac{1}{1}Dy^2, \&c.$

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$$8. \text{ Wherefore } \begin{cases} 2zy=A=0.01029947387912 \\ \frac{1}{2}Axy=B=..48271995 \\ \frac{1}{4}Bxy=C=..4072 \end{cases}$$

The Sum is the Logarithm } .. 0.01029995663980
of $\frac{1000}{1024} (=L\frac{12}{128}) =$

Add the Logarithm of $\frac{2}{1000}$.. 3.00000000000000

And $\frac{1}{10}$ of that Sum will } .. 0.30102999566398.0
be $L2 =$

Thus you see 3 Steps of the Series thus ordered, are sufficient for 14 Places of Figures, whereas before (Art. 6.) 11 Steps produced the Logarithm true only to 10 Places.

9. Let the next Example be to find the *Briggian Logarithm* of 3. This may also be done by Theorem II. where $\frac{a}{b} = \frac{1}{2}$, and $d=2$, and $s=4$; also

$$\frac{d}{s} = \frac{1}{2}, z=0,43429, \&c. \text{ as before. Then } z \times \frac{2d}{s} + \frac{2d^3}{3s^3} + \frac{2d^5}{5s^5}, \&c. = \frac{1}{2} \times \frac{z}{2} + \frac{1}{2} \times \frac{z}{8} + \frac{1}{2} \times \frac{z}{32}, \&c.$$

$$= L 3. \text{ The half thereof } \frac{z}{2} + \frac{1}{2} \times \frac{z}{8} + \frac{1}{2} \times \frac{z}{32}$$

$$\&c. = \frac{1}{2} L 3. \text{ Put } \frac{z}{2} = A, \text{ then } \frac{z}{8} = \frac{z}{2} \times \frac{1}{4} =$$

$$\frac{1}{4} A = B, \text{ and } \frac{z}{32} = \frac{z}{8} \times \frac{1}{4} = \frac{1}{4} B = C, \text{ and so on.}$$

Whence $A + \frac{1}{4}B + \frac{1}{16}C + \frac{1}{64}D + \frac{1}{256}E, \&c. = \frac{1}{2}L 3$. But this Series converges so very slow, that as many Steps will be necessary as you intend Places of Figures in the Logarithm, and more; therefore Recourse must be had to Theorem IV. which here comes into play, because the Logarithms on each Side of it are known, *viz.* the Logarithm of 2 and 4.

10. Therefore (according to Theorem IV.) $a=2$, $b=4$, $\frac{a+b}{2} = \frac{1}{2}s = 3$, $ab=8$, consequently $\frac{1}{2}ss=9$,

$$\text{and so } \frac{1}{2}ss + ab = S = 17. \text{ Wherefore } \frac{z}{S} + \frac{1}{2} \times \frac{z}{S^2}$$

+

$+\frac{1}{3} \times \frac{z}{8^3} + \frac{1}{7} \times \frac{z}{8^7}, \&c. = L \sqrt[3]{z}$, put $\frac{z}{8} = A$,
 then $\frac{1}{3} \times \frac{z}{8^3} = \frac{1}{3} \times A \times \frac{1}{8^2} = B$, $\frac{1}{7} \times \frac{z}{8^7} = \frac{1}{7} \times B \times \frac{1}{8^4}$
 $= C, \&c.$ Also $\sqrt[3]{\frac{1}{8}} \times \frac{1}{\sqrt[3]{8}} = \frac{1}{8}$, wherefore $L \sqrt[3]{\frac{1}{8}} +$
 $L \frac{1}{\sqrt[3]{8}} = L 3$. Therefore $L \frac{1}{\sqrt[3]{8}} + A + B + C + D,$
 $\&c. = L 3$. See the Operation following.

$$\begin{array}{lcl} \left\{ \begin{array}{l} L \frac{1}{\sqrt[3]{8}} = \frac{L_3 + L_4}{2} = \dots\dots 0.4515449934959 \\ \frac{1}{3} \times \frac{z}{8^3} = A = \dots\dots\dots 255467342296 \\ \text{Thus } \frac{1}{3} \times \frac{1}{8^2} \times A = B = \dots\dots\dots 294656680 \\ \frac{1}{7} \times \frac{1}{8^4} \times B = C = \dots\dots\dots 611744 \\ \frac{1}{7} \times \frac{1}{8^4} \times C = D = \dots\dots\dots 1511 \end{array} \right. \end{array}$$

The Sum is the Log. of $\dots\dots 3 = 0.477121254719.0$

11. But this Logarithm may yet much sooner
 and with less Trouble be found, by the Artifice used
 in Art. 7. For the Ratio $\frac{2^{21}}{5 \times 3^8} = \frac{12768}{32803} = \frac{4}{6}$, and
 so $a + b = 65573$, and $a - b = 37$; and since
 $\frac{2^{21}}{5 \times 3^8} \times \frac{f}{2^{11}} = \frac{1}{3^3}$, therefore $L \frac{2^{21}}{5 \times 3^8} + L \frac{f}{2^{11}} =$
 $L 3^3$; but $L \frac{f}{2^{11}} = L 2^{11} - L 5$. And $L \frac{2^{21}}{5 \times 3^8}$
 $= \frac{2xd}{8} + \frac{2xd^2}{38^3}, \&c.$

$$\text{Therefore } \left\{ \begin{array}{l} L 2^{11} - L 5 = \dots\dots\dots 3.81647993062 \\ \frac{2xd}{8} = \dots\dots\dots 0.00049010708 \end{array} \right.$$

The Sum is the Logarithm of $3^3 = 3.81697003770$

And $\frac{1}{3}$ thereof is the Log. of $3 = 0.47712125471$;
 thus the *first Step* of Theorem II. gives the Loga-
 rithm true to 11 Places.

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12. The next Example shall be that which Dr. Halley has given for finding the Logarithm of 23, which is done by Theorem IV. In this Case, $a=22$, $b=24$, $\frac{1}{2}a=23$, $\frac{1}{2}ss=529$, $ab=528$, and $\frac{1}{2}ss+\frac{ab}{2}=1057=S$. And $\frac{x}{S} + \frac{x}{3S^2} + \frac{x}{5S^3}$, &c. $= L \sqrt{\frac{ab}{\frac{1}{2}ss}}$. But $\sqrt{\frac{ab}{\frac{1}{2}ss}} \times \frac{1}{\sqrt{ab}} = \frac{1}{\frac{1}{2}S}$; therefore $L \frac{\sqrt{24+22}}{\frac{1}{2}S} + L \frac{1}{\sqrt{24+22}} = L \frac{1}{\frac{1}{2}S}$; and because $2 \times 2 \times 2 \times 3 = 24$, and $2 \times 11 = 22$, therefore also $3L, 2 + L 3 = L 24$; and $L 2 + L 11 = L 22$. And $\frac{L24+L22}{2} = L \sqrt{ab}$; therefore, (proceeding in the Operation according to Art. 10.) we have

$$\left\{ \begin{array}{ll} \frac{L24+L22}{2} = \dots 1.36131696126690612945009172669805 \\ \frac{28}{S} = A = \dots 41087462810146814347315886368 \\ \frac{1}{3} \times \frac{1}{SS} \times A = B = \dots 12258521544181829460074 \\ \frac{1}{3} \times \frac{1}{SS} \times B = C = \dots 6583235184376175 \\ \frac{1}{3} \times \frac{1}{SS} \times C = D = \dots 4208829765 \\ \frac{1}{3} \times \frac{1}{SS} \times D = E = \dots 2930 \end{array} \right.$$

The Sum $= L 23 = \dots 1.36172783601759287886777711225117$ which Logarithm is true to 32 Places of Figures, and thus you may proceed for any other.

13. In making Logarithms for *Prime Numbers*, the *Artifice*, or *greatest Advantage* consists in finding such a *Ratio* or *Fraction*, whose *Terms* are the *greatest* possible and their *Difference* the least; and the *Number* whose Logarithm is sought, or some *Power* thereof, is an *aliquot Part* or *Submultiple* of one of the *Terms* of the said *Ratio* or *Fraction*. For this once obtain'd, the Logarithm is soon acquired by Theorem

rem II. with ease. Thus the Fraction $\frac{211}{2110} = \frac{211 \times 11}{80 \times 29} = \frac{b}{a}$; where $d=1$, and $s=4641$ the Series

will converge very swift for the Logarithm of $\frac{211 \times 11}{80 \times 29}$

but $\frac{211 \times 11}{80 \times 29} \times \frac{80 \times 29}{11} = \frac{211}{1} = 211$, therefore $L80 +$

$L29 - L11$ added to the Series, gives the Logarithm of 211. But the *Ratio* or *Fraction* $\frac{1187041}{121 \times 211^2} = \frac{1187041}{121 \times 211^2}$

will make the Series converge very much

sooner than before; for here $d=1$, and $s=10774081$.

For $L80 + L54 + L29 + L43 - L121$ added to the Series (or Theor. II.) gives the Logarithm of 211^4 , half which is the Logarithm of 211. Lastly, the

Fraction $\frac{1082122441}{1082119440} = \frac{211^4}{60 \times 28 \times 53 \times 113 \times 197} = \frac{b}{a}$, where

$d=1$, and $s=3964238881$, converges to that Degree that the first Step of the Series quotes the Logarithm of the Fraction to 29 Places, to which add the Logarithms of the 5 Numbers in the *Denominator*, and it gives the Logarithm of 211^4 , then $\frac{1}{4}$ of that is the Logarithm of 211, as before.

14. The greatest Difficulty consists in finding out proper Numbers for producing such Fractions as aforesaid; and the best Method of this is by *prudent Tryals*. An Example of which is here subjoin'd. Suppose I would procure a convenient Fraction for the Logarithm of 223, I make tryal thus;

$$\text{First } 223 \begin{cases} \times 7 = 1561 \\ \times 8 = 1784 \end{cases}$$

$$\text{Then } 223 \begin{cases} \times 87 = 19401 \\ \times 2 = 446 \end{cases}$$

$$\text{Therefore } 223 \times 387 = 64001$$

Having thus obtain'd the Term 64001, 'tis easy to observe the other may be 64000, wherefore the Fraction is $\frac{64001}{64000} = \frac{287 \times 223}{64000}$, and finds the Logarithm of

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223 with good Dispatch. Or thus, to find a Fraction of *larger Terms*; suppose I assume the Numerator $159000 = 1000 \times 53 \times 3$, then to find the other Term as near this as may be, I try thus;

$$223 \begin{cases} \times 3 = 669 \\ \times 1 = 223 \end{cases}$$

$$223 \begin{cases} \times 13 = 2899 \\ \times 7 = 1561 \end{cases}$$

Therefore $223 \times 713 = 158999$, which is within Unity as great as the other Term 159000, and consequently the Fraction $\frac{159000}{158999}$ is that required, and thus you proceed to raise the Terms of any other.

15. Let the Terms of any Fraction be represented by a =Least, and b =Greatest. Then if the *Ratio* be *increasing* it will be $\frac{a}{b}$, but if *decreasing*, $\frac{b}{a}$; let that

Term, in which the Number sought is *ingredient*, be express'd by the Product cx , where c =the Number (or Product of Numbers) whose Logarithm is known, and x =the Number whose Logarithm is sought.

If $a=cx$, then $\frac{cx}{b} = \frac{a}{b}$, or $\frac{b}{cx}$; but if $b=cx$, then

$\frac{a}{cx}$, or $\frac{cx}{a}$; also let there be put the 2d Theorem

$\frac{2d}{8} + \frac{2d^3}{3s^3} + \frac{2d^5}{5s^5} + \frac{2d^7}{7s^7}$, &c. $= z$; then if the un-

known Number x be in the *Denominator* of the *increasing Ratio* $\frac{a}{b}$, viz. $\frac{a}{cx}$; or in the *Numerator* of

the *decreasing Ratio* $\frac{b}{a}$, viz. $\frac{bx}{a}$; then it will be

$Z + La - Lc = Lx$. And, *vice versa*, if it be $\frac{cx}{b}$ or

$\frac{b}{cx}$, the Theorem will be $Z + Lb - Lc = Lx$. From

hence the Operations in the foregoing Articles for making Logarithms have their Grounds and Reason; and

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and every thing there asserted is from this Process very evident.

16. To find the natural Number of any Logarithm proposed; this is best done by the Theorem in Art. 17. Chap. VII.

$$\text{viz. } \left\{ \begin{array}{l} a \\ b \end{array} \right\} x^{1-\frac{1}{n}} + \frac{1}{n} x^{2-\frac{1}{n}} + \frac{1}{2n^2} x^{3-\frac{1}{n}} + \frac{1}{6n^3} x^{4-\frac{1}{n}} + \frac{1}{24n^4} x^{5-\frac{1}{n}} + \dots$$

$$\text{Etc.} = N.$$

For Example, let it be required to find the Interest of one Pound for one Day, at the rate of 6*l. per Cent. per Annum, Compound Interest*; which is to extract the Root of 1.06 taken as the 365th Power; thus the Logarithm of 1.06 = 0.0253058652647702408-46731186351, &c.

Which divide by 365, } $L = 0.0000693311377116$
the Quotient is } 99289991044346, &c.

The next nearest } $b = 1.00016 = 0.0000694815587$
Log. & its N^o. } 28037517724712696, &c.

Their Difference is $a = 0.000000150421016338227$
733668350, &c.

Mult. this by $n = 2.3025850929940456840179914$
54684, &c.

The Product is

$$\left\{ \begin{array}{l} n a = 0.000000346357189893416971322305 \\ n^2 a^2 = \dots\dots\dots 119963302990864503 \\ n^3 a^3 = \dots\dots\dots 41550152514 \\ n^4 a^4 = \dots\dots\dots 14391 \end{array} \right.$$

The Powers of $n a$. Then

$$1 + \frac{1}{n} n a = 1.000000000000059981651495432251$$

$$\frac{1}{2} n^2 a^2 = \dots\dots\dots 599$$

$$\text{The Sum } 1 + \frac{1}{n} n a + \frac{1}{2} n^2 a^2 + \dots\dots\dots \left\{ \begin{array}{l} 1.00000000000005998165- \\ 1495432851 = X. \end{array} \right.$$

The odd Powers

$$\left\{ \begin{array}{l} n a = 0.000000346357189893416971322305 \\ \frac{1}{3} n^3 a^3 = \dots\dots\dots 6925025419 \end{array} \right.$$

$$\text{Sum } n a + \frac{1}{3} n^3 a^3 = 0.00000034635718989342389634-7724 = Z.$$

Then

Then the Value of the Series is

$$X - a = 0.999999653642870088227599085126.$$

Which multiply by $b = 1.00016$, produces

$$\sqrt[35]{1.06} = 1.000159653587452947441715500980,$$

&c. = N , the Number sought of the given Logarithm L , and that to 30 Places of Figures. The same Number may be seen produced to 60 Places in Mr. Sberwin's Mathematical Tables.



CHAP. IX.

*Of the *Logarithmic *Spiral; and the Nature and Construction of the Table of *Meridional *Parts, or the Nautical *Meridian *Line, deduced therefrom.*

1. IF any Right-Line pW be moved with an *equable Motion* about the fix'd Point p , and at the same time the Point W be mov'd towards the Point p , with a Velocity such that the Radii $pW, pV, pS, \&c.$ form'd thereby, be in a *Geometrical Ratio* decreasing, then the Curve $WVSQ, \&c.$ is called the *Logarithmic Spiral*; and that for the same Reason as the Logarithmic Curve before describ'd received its Appellation. See Chap. III. Art. 8, 9, 10.

2. For suppose the Arches $AC = CE = EG, \&c.$ and therefore in *Arithmetical Progression*; and since, from the *Generation* of the *Spiral*, the Radius $pB : pD :: pD : pF :: pF : pH, \&c.$ 'tis evident the Arches $AC, AE, AG, \&c.$ are the *Exponents* of the *Ratio's* of the Radii $Dp, Fp, Hp, \&c.$ to the first pB ; and so those *Arches* are in respect of the *Radii*, as *Logarithms* in respect of *Numbers*; as is sufficiently manifest from the preceding *Theory of Logarithms*. Wherefore

fore if Bp be = 1, 10, 100, &c. and PW (the 10th Proportional from pB) be = 10, 100, 1000, &c. then shall the Arch AC =, 1000000, AE =, 2000000, AG =, 3000000, &c. AW = 1,0000000; be the Logarithms of the Numbers pD = 1,259, &c. pF = 1,585, &c. pH = 1,996, &c. pW = 10; of Mr. Briggs's Form.

3. This Spiral is also called the *Equiangular Spiral*; because it intersects all the Radii pW , pQ , pB , at equal Angles. For suppose the Arches NP , TW , infinitely small, and equal to each other, then may the Parts of the Spiral OQ and VW , be esteemed Right-Lines; and so since in the Triangles pOQ , pVW , the Sides are proportional, viz. $Op : pQ :: Vp : pW$, and the Angle $OpQ = VpW$, those Triangles are similar; and consequently the Angle $pOQ = pVW$, or $pQO = pWV$; and thus it will be every where.

4. Now let the whole Scheme be considered as the *Stereographic Projection* of one Quarter of a parallel Hemisphere, then shall p be the Pole; WLA , a Quadrantal Arch of the Equator; the Radii pW , pT , pR , &c. the several Meridians projected on the Plane of the Equator. And since 'tis the Property of every Rumb Line to make equal Angles with every Meridian on the Globe, and the Angles contained between circular Arches on the Globe, are equal to the Angles between the same Arches in this Projection, therefore the *Logarithmic Spiral* WQB is the Projection of a Rumb Line; since it has the same Property on the Projection, as the Rumb on the Globe, as was proved Art. 3. hereof.

5. Moreover, since all Right Circles, such as are the Meridians in this Case, are projected into Right Lines equal to the Tangents of half the Arches, the Lines pB , pD , pF , pH , &c. will here represent the Tangents of half the Complements of the Latitudes AB , CD , EF , GH , &c. And since the Arches in the Equator AC , AE , AG , &c. are the Differences

of Longitude made by sailing from the Latitude B to the Latitudes D, F, H, &c. on the Rumb or Spiral BOW; and it has been shewn that those Arches are the *Logarithms* of the Radii pD, pF, pH, &c. therefore the Difference of Longitude is the *Logarithm* of the *Tangent* of half the *Complement* of Latitude, reckoning from the Meridian A p whence the Logarithms begin.

6. Therefore the Difference of Longitude RT, made by sailing from the Latitude S to the Latitude V, is equal to the Difference of the Logarithms (AT—AR) of the Tangents of the half Complements (Sp, Vp) of the Latitudes TV, RS. And since the Ratio of the *Progression*, or of pW to pV, may be infinitely varied, 'tis plain the infinite Number of Rumbs in a Quadrant of the Compass determine so many Scales of *Logarithms* in the Equator of the Tangents of the half Complements of the Latitudes proper to those Rumbs.

7. Since then every different Rumb is a *Logarithmic Spiral*, or determines a peculiar Scale of *Logarithms* for the Tangents of the Half-Complements of its Latitudes, therefore any Canon or Table of Logarithm-Tangents, whether of *Neper's*, *Briggs's*, or any other Form whatsoever, is the Scale of the Differences of Longitude on some determinate Rumb or other. And consequently if this Rumb be investigated for the Canon of *Briggs's Logarithms* (now in common Use,) the said Canon may be made to answer all the Purposes of the *Nautical Meridian Line*, in Propositions of *Navigation by Mercator's Chart*.

8. In order to this it must be considered, that the *Meridian Line* is a Table or Scale of Longitudes to every Degree of Latitude on the Rumb which makes an Angle of 45 Degrees with the Meridian; since in this Case the Differences of Longitude are always equal to the *Meridional*, or enlarg'd Differences of Latitude. And since there is a certain Rumb on which

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which *Neper's* or *Briggs's* Logarithm-Tangents are the *Differences* of Longitude, and the Differences of Longitude on different Rumbs are to one another as the Tangents of the Angles of those Rumbs with the Meridian ; therefore by having given the Difference of Longitude on the Rumb of 45° , in Logarithms of *Neper's* Form, and the Length of the Arch of one Minute or Degree in Parts of the Radius, we can thence find the Angle of that Rumb which determines that *Species of Logarithms*.

9. Now the *Momentary Augment* or *Fluxion* of the Tangent-Line of 45° , is exactly double to the *Fluxion* of the Arch of the Circle (as is easily proved), and the *Tangent* of 45° being equal to *Radius*, the *Fluxion* also of the *Logarithm-Tangent* will be double to that of the Arch, if the Logarithm be of *Neper's Form* ; but for *Briggs's* Form, it will be as the same double Arch multiplied into $\frac{1}{n} = 0.43429$, &c. or divided by $n = 2.30258$, &c. the Index for *Briggs's Logarithms*. See Chap. VI. Art. 7.

10. Now since the Radius of a Circle being put $= 1$, the *Periphery* thereof will be 6.2831853 , &c. therefore $360)6.2831853$, &c. $(0.01745329$, &c. = the Length of the Arch of *one Degree*. Also $60)0.01745329$, &c. $(0.0002908882$, &c. = the Length of an Arch of *one Minute*, in Parts of the Radius. If one Minute be supposed Unity, then the Proportion for finding the Angle of the Rumb required for *Neper's* Logarithms, will be, as $1 : 2.908882$, &c. $:: \text{Radius} = 1000000$, &c. $: \text{the Tangent} = 2908882$, &c. of the Angle $71^\circ 1' 42''$, whose Logarithm is 10.463726117 , &c. and under that Angle is the *Meridian* intersected by that *Rumb Line*, on which the Differences of *Neper's* Logarithm-Tangents of the Complements of the Latitudes are the true Differences of Longitude, estimated in Minutes and Parts, taking the first 4 Figures for Integers.

11. But since *Neper's* Logarithms are to those of *Mr. Briggs's* Form, as 2.302585, &c. is to 1.000000, &c. therefore to find the Angle of the Rumb for the Logarithms of *Briggs's* Form; this must be the Analogy, As 2302585, &c. : 2908882, &c. :: 1000000 = Radius : 12633114, &c. = the Tangent of the Angle $51^{\circ} 38' 9''$, whose Logarithm is 10.101510428, &c. Wherefore in the *Rumb* Line that makes an Angle of $51^{\circ} 38' 9''$ with the *Meridian*, the *common* (viz. *Briggs's*) Logarithm-Tangents are the true Differences of Longitude.

12. But if a Table or Scale of *Logarithm-Tangents* be made by Extraction of the Root of the *infinitest Power*, whose *Index* is the Length of the Arch you put for *Unity* in the said Scale; then such a Scale of *Logarithm-Tangents* shall be the true *Meridian Line* required. If then the *Radius* or Tangent of 45° , be put = 1; and the Difference between Radius and any other Tangent *T*, be called *t*; so that it be $R \pm t = T$; the Logarithm of the *Ratio* of *Radius* to such *Tangent* will be

$\frac{1}{n} \times t - \frac{t^2}{2} + \frac{t^3}{3} - \frac{t^4}{4} + \frac{t^5}{5}, \&c. = t =$ the Logarithm of the Tangent *T*, when it is $R + t = T$.

Or $\frac{1}{n} \times t + \frac{t^2}{2} + \frac{t^3}{3} + \frac{t^4}{4} + \frac{t^5}{5}, \&c.$ when it is $R - t = T$. All which is evident from Chap. VI. Art. 6.

13. According to the same Doctrine (Art. 9. of the same Chap.) if *T* be any given Tangent, and *t* the Difference thereof from another Tangent; then the Logarithm of their Ratio will be $\frac{1}{n} \times \frac{t}{T} - \frac{t^2}{2T^2} + \frac{t^3}{3T^3} - \frac{t^4}{4T^4} + \frac{t^5}{5T^5}, \&c.$ when *T* is the lesser Term. But

$\frac{1}{n} \times \frac{t}{T} + \frac{t^2}{2T^2} + \frac{t^3}{3T^3} + \frac{t^4}{4T^4} + \frac{t^5}{5T^5}, \&c.$ when *T* is the greater Term,

14. Again, it was shewn in the same Chap. VI. Art. 10, and 11. that this Series may be made to converge twice as swift, omitting all the even Powers, by putting T = the Sum of the Tangents, and t = the Difference, as above. For thus the Logarithm will be $\frac{2}{n} \times \frac{t}{T} + \frac{t^3}{3T^3} + \frac{t^5}{5T^5} + \frac{t^7}{7T^7}$, &c. = the Logarithm of the Ratio of those two Tangents.

15. But the Ratio of T to t , or of the Sum of two Tangents to their Difference is the same as that of the Sine of the Sum of those Arches to the Sine of their Difference; that is, again, as the Ratio of the Co-Sine of middle Latitude (or half Sum of the Arches) to the Sine of half the Difference. Therefore putting S = Sine-Complement of middle Latitude; and s for the Sine of half the Difference of Latitudes; then

$\frac{t}{T} = \frac{s}{S}$; and so the Series will become $\frac{2}{n} \times \frac{s}{S} + \frac{s^3}{3S^3} + \frac{s^5}{5S^5} + \frac{s^7}{7S^7}$, &c. wherein as the Differences of Latitude are smaller, fewer Steps will suffice.

16. So that, if the Equator be put for Middle-Latitude, then shall S = Radius, and s = Sine of the Latitude; then the Meridional Parts reckon'd from the Equator will be $\frac{2}{n} \times \frac{s}{r} + \frac{s^3}{3r^3} + \frac{s^5}{5r^5} + \frac{s^7}{7r^7}$, &c.

Here because $r=1$, therefore $\frac{2}{n} \times s + \frac{s^3}{3} + \frac{s^5}{5} + \frac{s^7}{7}$, &c. the half of which is $\frac{s}{n} + \frac{s^3}{3n} + \frac{s^5}{5n} + \frac{s^7}{7n}$, &c. = half the Logarithm of the Ratio of $r + s$ to $r - s$; that is, of the versed Sines of the Distances from both Poles. See Chap. VI. Art. 11.

17. I shall exemplify this Series by shewing how the Meridional Parts answering to 30° Latitude, are to be found thereby, and that by the Logarithms, as follows.

The

The Logarithm of the }
Sine of 30 Degrees is } $s = 0,5000000 = 9.6989700$

Multiply by 3

The Logarithm of $s^3 = 9.0969100$

Subtract the Logarithm of 3 = 0.4771213

There remains the }
Logarithm of } $\frac{s^3}{3} = 0.0416667 = 8.6197887$

And proceeding thus, $\left\{ \begin{array}{l} \frac{s^5}{5} = 0.0062500 = 7.7958800 \\ \frac{s^7}{7} = 0.0011160 = 7.0476920 \\ \frac{s^9}{9} = 0.0002171 = 6.3364875 \\ \frac{s^{11}}{11} = 0.0000444 = 5.6472773 \end{array} \right.$
you'll find the o-
ther Steps of the
Series by their Lo-
garithms, as here
set down.

The Sum of $s + \frac{s^3}{3} + \frac{s^5}{5} + \frac{s^7}{7} + \frac{s^9}{9} + \frac{s^{11}}{11}$, &c. is } $= 0.5492542 = 9.7398051$

To which add }
the Log. of $\pi = 3437.74677078, \&c. = 3.5362739$

The Sum is the
Log. of the Me-
ridional Parts } $\text{viz. } 1888,334, \&c. = 3.2760790$
for the Arch
of 30°

18. And thus you may proceed to find the Length of any other Arch, or the Distance from the Meridian of its Parallel of Latitude; and so the *Meridian Line* may be constructed *de novo*, if any one thinks it worth while. But tho' it may be done with greater Accuracy and Exactness by these *infinite Series* than what we have by the *common Method*; yet the Table of *Meridional Parts*, or *Nautical Line* made from thence, now in Use, is abundantly sufficient for all the Purposes of Sailing; and consequently

quently renders a new Calculation thereof unnecessary, and a Matter of mere Curiosity. And indeed, since it has been shewn above (Art. 9, 10, 11.) the *Meridian Line* is no other than a Scale of the Logarithm-Tangents of the Half-Complements of the Latitude on the Rumb of $51^{\circ} 38' 9''$, the Propositions of Sailing by this Method are resolvable by only the Canon of Logarithm-Tangents at the End of this Treatise; so that where this Canon is at hand, neither Meridional Table or Line can be necessary, as will appear by a Chapter particularly on this Subject, in the *Practical Part*. They who would see the Theory of this Branch of the Art, may peruse N°. 219. of the *Philosophical Transactions*, where they will find a most learned Tract on this Subject, wrote by Dr. *Halley*; from whence the Substance of this Chapter is taken.



C H A P. X.

*Of the Construction of a Large *Logarithmic Scale, exhibiting by Inspection a distinct * Idea of the Nature and Agreement of * Numbers and their * Logarithms.*

1. **I**T is an Observation of the earliest Antiquity, that we have no *Ideas* in the *Mind* which were not *first* in the *Senses*; or that the *Senses* of the *Body* are the only *Inlets* or *Entries* by which the *Idea's* of *Objects* present themselves to the *Mind*. It follows then, that the *Idea's* must needs be so much the more *clear* and *distinct* in the *Mind*, and consequently be the better understood by it, by how much the more *fully*, *completely*, and *obviously* they first of all affect our *Senses*. Single uniform Objects easily in-

sinuate

sinuate themselves, and make strong and clear Impressions on the *Mind*, while those which involve *Multiplicity* and *Variety* in their Nature, are proportionally more difficultly apprehended by the Senses, and consequently affect the Mind with *imperfect, confused, and slight Impressions*, which therefore must produce a more *perplex'd, obscure, and uncertain Nation or Conception* of the Things themselves.

2. From this Consideration we may easily learn the Reason why, of all the vast number of Persons who understand the *practical Use* of Logarithms, so very few of them know any thing of the *Nature and Construction* of them.. The Use of Logarithms is very obvious to the Senses by easy Examples, but their Nature and Construction lead the Mind too much upon the Contemplation of *Infinites* both of *Quantity* and *Variety*, which are Subjects too vastly *abstruse* and *remote* from *Sense*, ever to be very commonly understood; unless some *Expedients* be contrived, which may help to facilitate so difficult an Affair.

3. And as there are principally but three Ways, whereby the Nature of Logarithms are explained, *viz.* by *Numbers, Species, and Lines*, the *Expedient* aforesaid must be sought in *one* of these *three Methods*. But *Numbers*, of all things else, exhibit the most *complex* and *various Idea*, therefore it cannot be hoped for from them. *Species*, on the other hand, are too *simple and concise a Representation* of so *vast and various Idea's* as are those of *Logarithms*, and have nothing of their *Resemblance* in their *Form*; this *Expedient* therefore is not to be expected from this Head. It remains then, that *Right* or *Curve Lines* be used for the Purpose of explaining the Nature of Logarithms, by making the whole Matter obvious to the Senses. And here indeed we shall find all that can be desired, or is necessary to the Purpose.

4. For Example, let the complex *Idea* of a *thousand Units* be to be express'd most advantageously to a Mind unexercis'd about such *complex'd Notions*; if you do it by *Numbers* or *Figures*, it must be by this Expression 1000; but there are but *four Characters* to form an *Idea* of a *thousand* separate Objects in the Mind. In Species, this *great* and *complex Idea* is often represented by one Character alone, as x ; or two, as $1+x$; which are still more obscure and absolutely unintelligible without some *Pre-Notions* of the Matter. But a Line may be taken of a Length sufficient, that by proper Divisions, all the thousand Units may be render'd distinct and obvious to the Sense, in any variety of Magnitude almost, but especially if they are equal to each other, as in the Case of Logarithms before us. Wherefore, since by *Lines* such *great* and almost *inconceivably complex Ideas* are capable of being represented to the Senses, *distinct* and separate in their *proper Parts*, and the *Doctrine of Logarithms* depending entirely on such *Ideas*, 'tis evident that by Means of Lines of a sufficient Length, the *Nature* and *Properties* of Logarithms, and the *Operations* thereby, may be render'd more *apparent* and *compleat* to the Senses, and so be better understood in the Mind, than by any other Means whatsoever.

5. The Consequence of all which, is, that the young Tyro, and all who would have a *true Notion* and most *clear Understanding* of this *abstruse* and *mysterious* Doctrine, should be assisted with such a *large lineal Construction of the Logarithms*, as hath been hinted at. And this, I hope, I have effected in the *large Diagram* on the Copper-Plate before you, with considerable Exactness, which I call the *Logarithmic Scale*. Wherein all that has been said in the *General Theory* foregoing, or may follow in the *practical Operations of Logarithms*, is evident even to *Sense it self*, to a very wonderful degree, by a bare

Inspection, or a Glance of the *Eye* only ; and therefore cannot but conduce to form a very distinct and agreeable *Idea* or *Notion* both of the *Theory* and *Praxis* of this admirable Art.

6. The Scale consists of *three principal Lines* which bound it ; the *first* is AB on the Side, which is 22 *Inches* in Length, and is divided into 1000 equal Parts which represent the *natural Numbers* from 1 to 1000, all which are visible and distinct to the *naked Eye* ; which Numbers therefore are affixed to every 10th Division. The second is AC at the Bottom, divided into 300 *equal Parts*, (as being but 16 $\frac{2}{3}$ *Inches* long.) These represent the *Logarithms* ; if each of these *equal Parts* be supposed to represent 10, or 100, the *Logarithms*, then, for all Numbers under 1000, will be exhibited by Lines only to 4 or 5 Places of Figures, including the *Indices*. The *third* principal Line is the *Logarithmic Curve* eDEFB, in which all the Lines of Numbers and *Logarithms* terminate, and whose *Genesis* and *Properties* have been before described. See Chap. III, IV, V, &c.

7. The Scale consists (or is made up) of Lines of Numbers, and others which are the *Complements* of the *Logarithms* to 3.0000. The first are *perpendicular* to the *Logarithmic Line* AC in its several Divisions, and increase in Length in a *Geometrical Ratio* ; as hath been observed : thus dividing AC into 3 equal Parts $CG=GH,=HA$, if GD be the 10th *proportional* Term from $Ce=1$, or *Unity*, then shall $GD=10Ce=10$, and $HE=10GD=100Ce=100$. Lastly, $AB=10HE=100GD=1000Ce=1000$; as is evident from the *Nature* of the *Curve*, and by *Inspection*. The *Complements* of the *Logarithms* are the *Lines* which run across the *Diagram*, parallel to the *Line* of *Logarithms* AC ; these at the *Curve* refer the *Numbers* to their proper *Logarithms*, and by means of those Lines thus crossing each other in

in every Part of the Scheme, the *Logarithms* of *Number*, and the *Numbers* of *Logarithms* are most easily and obviously found; for the Extent of the Scale, by *Inspection* only.

8. It is not pretended that this (or any other) *Instrument* is capable of any great *Exactness* in *practical Operations*; 'tis sufficient for my Design, if it only *illustrates* and *proves the Truth* of every part of the *Doctrine* of *Logarithms* to the Sense, and thereby renders it easier to the *Intellects* of young *Learners*. If the *Theory* before deliver'd be examined by this *Scale*, it will be found to agree with it to a *sensible Exactness*; it being as it were but the same thing at large. In the following Part, I shall shew the *Correspondence* and *mutual Agreement* between the *fundamental Operations* by *Logarithms* wrote by *Numbers*, and the same performed on this *Scale*; than which nothing more, that I know of, can be said or expected.



CHAP. XI.

Of the Construction of the Artificial LINES of NUMBERS, SINES, and TANGENTS, by means of the LOGARITHMS.

1. **T**HE *Canon* of *Logarithms* being compleated and orderly *digested* in *Books*, tho' this was a greater Advantage than the *Mathematicians* of any former Age enjoy'd, yet not content to have a *bulky Book* of *Logarithms*, fit to be used in *Studies* and with the *Pen* only, the restless and unsatisfy'd Faculty of *Invention* in Men put them upon Contrivances to new-model and reduce the *voluminous Art*

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to *Miniature*, that so it might be render'd more easily manageable, and more *universally* useful.

2. In the Pursuit of this Design they also very well succeeded; for since Numbers of any kind are capable of being represented by *Right-Lines*, they were not long unappriz'd that the *whole Body of the Canon of Logarithms* might be laid down and express'd in the Divisions of one *strait Line*. Mr. *Gunter*, Professor of Geometry at *Gresham-College*, was the first who took this matter in hand, and constructed such an *artificial Line* of Logarithms; which therefore from him was called (ever since) *Gunter's Line*, or simply, the *Gunter*. The same Person also constructed *artificial Lines* of *Sines* and *Tangents*; and all those Lines, with some others laid down on a Scale, make what we commonly call *Gunter's-Scale*.

3. The *Construction* of those *artificial Lines* is easy to be understood, and is as follows. Draw the *Right-Line* AB (Fig. VII.) which divide into 10 great equal Parts, as is there denoted by 1, 2, 3, 4, &c. and each of these into 10 others, and so on. Conceive these *several Divisions*, or *equal Parts*, to represent the Logarithms in the Canon for the natural Numbers. Now suppose the whole Length $AB=10$, then the first grand Divisions will be 1, 2, 3, 4, &c. But if $AB=100$, then the *first Divisions* will be 10, 20, 30, &c. and the *second Divisions* 1, 2, 3, &c. Again if $AB=1000$, the *prime Divisions* will be 100, 200, 300, &c. and the *secondary Divisions*, 10, 20, 30, &c. Suppose the latter Case, viz. $AB=1000$; then draw another *Right-Line* CD equal and parallel to AB, the *natural Line* of Logarithms.

4. Now in the Line CD, such Divisions are to be made as may represent the Places of the natural Numbers 1, 2, 3, &c. or 10, 20, 30, &c. or 100, 200, 300, &c. But neglecting the *Indices* of Logarithms,

arithms, 'tis plain, since the Logarithms of the Numbers 1, 10, 100; 2, 20, 200; 3, 30, 300; &c. are the same, the Distances of those Numbers will be the same on the Line or Scale CD. And therefore since the Logarithm of 1, is $= 0$, the Number 1 must be placed at the very Beginning of the Line CD, from whence the Logarithms begin in the Line AB. Then because the whole Line AB $= 1000 =$ Logarithm of 10 $=$ CD, therefore against the Logarithm of 2, which is $= 301$ in the Line AB, make a Division in the Line CD, and by it place the Number 2. Again, because the Logarithm of 3 is $= 477$ in AB, therefore correspondent to the Point in AB, make another Division in CD, and by it place the Number 3. The Logarithm of 4 is 602, therefore from 602 in AB make a Division in CD, by which you must place the Number 4; and thus you proceed to find the Divisions for the other Numbers to 10 in the Line CD, by the Logarithms of those Numbers in the Line AB.

5. If the Divisions in the Line CD now found for the Numbers 1, 2, 3, 4, &c. be suppos'd, instead of them, to be for the Numbers 10, 20, 30, 40, &c. then each of those Divisions may be subdivided into 10 others, by the *Logarithmic Parts* in the Line AB. Thus, because the Logarithms of 11, 12, 13, 14, &c. are 41, 79, 113, 146, &c. therefore against these latter Numbers in the Line AB, make Divisions in the Line CD, so shall the first grand Division from 1 to 2 be divided again into 10 others. Again, because the Logarithms of 21, 22, 23, 24, &c. are 322, 342, 361, 380, &c. therefore Divisions made in CD against these Numbers in AB will finish the Subdivisions of the Space from 2 to 3, in the said Line CD. And thus proceeding, you may subdivide all the other *grand* or *prime Divisions*, to the End of the Line.

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6. If your Lines be of so great Length; that these last Subdivisions in CD, are still of a Length capable of another *tenfold Division*; then the first grand Divisions must be reputed 100, 200, 300, &c. and so since the Logarithms of the Numbers 101, 102, &c. 201, 202, &c. 301, 302, &c. are 4, 8, &c. 303, 305, &c. 478, 480, &c. therefore if against these Parts in AB, you make Divisions in CD, there will ensue a triple Division of the said Line CD, which is more than is necessary for *Instrumental Uses*, and indeed cannot be done but only for the two or three first Divisions.

7. Thus have you seen the Construction of the *Artificial, Logarithmic, or Gunter's Line*, so famous in all Parts of the Mathematics. A Line which performs the Business of the whole Logarithmic Canon; since the Divisions of this Line have all the same Properties with regard to the natural Numbers on it, as the Logarithms of the Table have to the Numbers corresponding to them. 'Tis plain the Divisions and Relation of these two Lines AB, CD, are the same as At, and AT, in Fig. IV. and III. Consequently what has been said of those Lines heretofore will help to illustrate the *Theory* and Construction of the Lines now under Consideration. But since in Use the *Gunter CD* is supposed to be divided into an 100 Parts at least, therefore you always (or mostly) observe it of a double Length of that which is expressed in Fig. VII. which Length is commonly called *Radius*; and so the *Gunter* in common Use is said to be of a *double Radius*; because else the Divisions for the *nine Digits* would be wanting, since the Distance from 1 to 10 is equal to that from 10 to 100, as is evident from the foregoing Construction, and from the *Theory* of Logarithms.

8. Having thus shewn the *Construction* of the *Line of Numbers*, the *Construction* of the Lines of artificial *Sines* and *Tangents* easily follows; since, as before observed,

observed, the Logarithms of *Sines* and *Tangents* are nothing more than *common Logarithms* of such Numbers as express the *Sines* and *Tangents* of each Minute of the *Quadrant*.

Deg.	N. Sine.	Log.	N. Tang.	Log.
1	17	2418	17	2419
2	34	5428	35	5430
3	52	7188	52	7194
4	69	8345	70	8446
5	87	9402	87	9419
6	104	1.0192	105	1.0216
7	121	1.0858	122	1.0891
8	139	1.1435	140	1.1478
9	156	1.1943	158	1.1997
10	173	1.2396	176	1.2463
20	342	1.5340	363	1.5610
30	500	1.6989	577	1.7614
40	642	1.8080	839	1.9238
50	766	1.8842	1191	2.0761
60	866	1.9375	1732	2.2385
70	939	1.9729	2747	2.4389
80	984	1.9933	5671	2.7536
90	1000	2.0000	Infin.	Infin.

In the little Table above, the first Column contains the *Degrees*, the 2d and 4th the *Natural Sines* and *Tangents*, and the 3d and 5th Columns contain the *Logarithms* of those natural *Sines* and *Tangents*, the *Indices* being omitted, and the *Radius* supposed = 10000.

9. Let three Lines be drawn, and let L. = Line of *Logarithms*, or *double Radius* of 20000 equal Parts; S = a Line for *Sines*; and T = Line for *Tangents*; the two latter must be drawn equal and parallel to the first; as in the Construction of the Line of *Numbers*. Then having graduated the Line L. into 20000 equal Parts,

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Parts, if against such of those Parts as are express'd by the Numbers in the 3d Column, you make Divisions in the Line S, and by those Divisions you place the Numbers in the first Column, you will then have the *artificial Line of Sines S* graduated for the first great Divisions of 1, 2, 3, 4, 5, 6, &c. 10, 20, 30, &c. Degrees: After the same manner by the Table of *Logarithmic Sines* you find Numbers, from whence in the Line L you find Divisions in the Line S for *Minutes*, and *Parts of Minutes*. And thus the Line of *artificial* or *Logarithmic Sines* is finished.

10. Again, from the same Parts of the graduated Line L, as are found in the fifth Column of the *Tablet*, you make Divisions in the Line T, and by them place the Numbers of the first Column, the Line T shall be the *artificial Line of Tangents* graduated for the first great Divisions of 1, 2, 3, 4, 5, &c. 10, 20, 30, &c. Degrees. And the Subdivisions for *Minutes* will be found as before directed. But tho' the *double Radius* on the Line L suffices for graduating the Line of Sines S, to the whole Length of 90 Degrees, because all *Sines* are less than the *Radius* of a Circle, which is the greatest Sine; yet because the *Radius of a Circle* and the *Tangent of 45 Degrees* are equal; therefore 'tis evident the *Logarithms* of all Tangents greater than 45 Degrees, will exceed the Length of the Line L, as is plain from the fifth Column of the foregoing Tablet.

11. But since *Radius* is a *mean Proportional* between the *Tangent* of any *Arch*, and the *Tangent* of that *Arch's* Complement, it follows, that the *natural Tangents* in the *Geometric Ratio* or *Scale* are equally distant on each Side from the *Radius* or *Tangent* of 45 Degrees: and therefore the *Logarithms* of those *natural Tangents*, which are *equidistant* on each Side the *Radius* or *Logarithm* of 45°, are also *equidifferent*; that is, their *Differences* are equal. Thus the Difference of 44° and 46°, from *Radius* is the same; and the

the Differences of 40° and 50° , 30° and 60° , 20° and 70° , &c. are respectively equal to each other; and consequently the first great Divisions from 1 to 45° on the Line T of *artificial Tangents*, will likewise serve for the *Co-Tangents* of those Degrees, that is, for all the *Tangents* from 45° to 90° , reckon'd back again to the beginning of the Line T. And this is the Reason why on those Lines of *Tangents*, you see the Numbers placed at each 10th Division, thus 10 | 10, 20 | 20, 30 | 30, 40 | 40, 45, at the End. For otherwise the said Line of *Tangents*, must be continued out to *double the Length* it now is, which would not be near so convenient.

12. The Numbers in the 2d and 4th Columns are the Divisions on the *Gunter*, which correspond to the *similar Divisions* on the *Lines of Sines and Tangents*; wherefore the *former* being already made, the *two latter* may also easily be constructed by means of that. And these things are all I judge necessary to be said here concerning the Construction of those excellent *Lines of artificial Numbers, Sines, and Tangents*; and as to their Uses, that will be a Subject to be treated of after the *practical Use* of the *Logarithms* themselves is first explain'd and inculcated; for then the Use of these Instruments will be much better conceived and understood.





C H A P. XII.

Of the MANNER of using the TABLES of LOGARITHMS in PRACTICE ; and of the Pre-requisites thereto.

1. **T**HE *Logarithms* being made for *natural Numbers* by some of the Methods foregoing, the next thing necessary was to dispose them into some convenient *Order* or *Form* for *practical Uses*. And such a *Digest* or Collection of *Logarithms*, we call the *Logarithmic Canon* or *Tables*.

2. These *Tables* are of *two Sorts* ; the first contains the *Logarithms* of all *natural Numbers* from *Unity* or 1 to 10000, or 101000 (as those *large Tables* of Mr. *Sherwin*.) In these, the general Manner or Form is such as here express'd in the Margin, which consists of *two Columns* ; in the first are placed the *Numbers*, in the second the *Logarithms* corresponding thereto, with their *Indices*. And three of these *double Columns* fill a Page in common Books of this Form ; and this, of all others, is the most obvious

Numb.	Logarithms.
997	2.9986951
998	2.9991305
999	2.9995655
1000	3.0000000
1001	3.0004341
1002	3.0008677
1003	3.0013009

and easy as to its Use, which therefore can need no Explanation. For by Inspection only is seen what *Logarithm* belongs to *any Number* within the Compass of the Table.

3. But tho' the *aforesaid Form* be the most natural and obvious, yet it is not the most *artful* and *comprehensive* ; therefore another Form or Disposition of the *Tables* for *natural Numbers* has been contrived
more

more concise, or which takes up less room, and is yet as perfect and useful as the other; a *Specimen* of this Form I have here annexed.

N ^o .	Logarithms.				
	0	1	2	3	4
173	238046	238297	238548	238799	239049
174	240549	240799	241048	241297	241546
175	243038	243286	243534	243782	244030

In which the natural Numbers are placed in the *Side-Column* to the *left hand*, all but the *Units Place*, or *first Figure* of the Numbers, which is found in a parallel Column on the top of the Page, in the Order 0, 1, 2, 3 4, &c. as you see in the Specimen. By this means *one Column of Numbers* suffices for *one Page*, whereas in the other Form there are three such Columns, the whole Page itself consisting entirely of the Logarithms, which in this Case admits of five Columns; but the *Indices* are here omitted as not being necessary, since they are known by the Numbers. The manner of using this Form is yet very easy. For Example, to find the Logarithm of the Number 1742; against 174 in the *Side-Column*, and under the Units Place 2 at the top, I find the Logarithm 241048, and since the Number has four Places, the *Index* must be 3; where 3.241048 is the Logarithm compleat for the Number 1742. Thus the Logarithm of 1753 = 3.243782; and so for others.

4. But the Tables of Logarithms are yet capable of a further, and much more curious Improvement with regard to their Contraction or Conciseness; for since the Differences of Logarithms decrease as the Numbers increase, 'tis plain those will grow very small as these become very large; and consequently the *two* or *three first Figures* of the Logarithms to the left will be the *same* for *divers large Numbers together* in

the Canon. Thus for Instance, the Logarithms of all the Numbers between 4168 and 4266, have the two first Figures to the left the same in every one, viz. 62. So likewise all the Logarithms between the Numbers 9954 and 9977 have their first three Figures the same, viz. 998, the Difference of the Logarithms being only in the remaining Figures. The Figures of the Logarithms therefore may be reckoned of two sorts, viz. such as are *permanent* or the same, for certain Intervals; and such as are *variable* or always altering. In this Form of the Canon now under Consideration, these permanent Figures are printed but once for their respective Intervals, and that in the first Column of Logarithms next the Numbers the *variable* Figures only pertaining to each Logarithm make the Substance, or fill the whole Face of each Page, as in the Specimen here subjoin'd.

N ^o .	Logarithms.				
	0	1	2	3	4
132	12.0574	0903	1231	1560	1888
133	3852	4178	4504	4830	5156
134	7105	7429	7752	8076	8399
135	13.0334	0655	0977	1298	1619
136	3539	3858	4177	4496	4814
137	6721	7037	7354	7670	7987
138	9879	*0194	0508	0822	1136
139	14.3015	3327	3639	3951	4263
945	975.432	478	524	570	616
946	.891	937	983	*029	075
947	976.350	396	442	487	533
948	808	854	900	946	991
949	977.266	312	358	403	449

5. The Numbers here, as in the last *Form*, are, for the three first Figures to the left, found in the Side-

Side-Column, the other Figure at top. The Logarithms in this Specimen are 65, yet but six of them need be expressed at length, viz. those for the Numbers 1320, 1350, 1390, 9450, 9470, 9490. The Logarithm for 1320 is thus wrote, 12.0574, to denote the two first Figures 12 (separated by a Dot) are permanent thro' the Interval between 1320 and 1350, that is, they belong to the Logarithms of all the intermediate Numbers between those two, and therefore need be express'd only for the first; the other Part of the said Logarithm which is variable, is express'd alone in all the rest. The *upper Part* of this *Specimen* or *Tariff* consists of Logarithms having two Figures in the *permanent Part*; the lower part is an Example of Logarithms having three Figures in the *permanent Part*. To find a Logarithm therefore to any given Number, will also be very easy in this Form. Thus, suppose the Logarithm be sought for in the Number 1323, 'tis found in this Manner. Take the permanent Part either *against* or *next above* the three first Figures 132, which here is 12, then against 132 and under 3 at the top, you find the *variable Part* 1560, to which prefix the *permanent Part* 12, and you have the Logarithm 121560, which with the *Index*, is 3.121560. Again, to find the Logarithm of 1374, take 13 the *permanent Part* next above the three first Figures 137, then against 137, and under 4 at top, you find the *variable Part* 7987, which annexed to the former Part 13, make 137987, and with the *Index*, 3.137987, the Logarithm sought.

6. Thus also you proceed when the permanent Part consisteth of *three Figures*, where the *Intervals* are much shorter. One caution only is necessary, and that is, that you observe in those Lines where there is found an *Asterism* *, to join all the *variable Parts* in that Line *before* the *, to the *permanent Part* next above; and all *after* it, to that *next below*.

Thus

Thus in the 7th Line, and 2d Column, you see an *Asterism* *, therefore the *variable Part* 9879 before it must be join'd to 13 the *permanent Part* next above it, to form the Logarithm 3.139879 for the Number 1380: But the *variable Parts* 0194, 0508, &c. following it, are to be annexed to 14 the *permanent Part* next below, to form the Logarithms 3.140194, 3.140508, &c. for the Numbers 1381, 1382, &c. The Reason of which I presume must be *self-evident* to every Reader.

7. I have been the more prolix on this *last Form*, lest any *Obscurity* or *Uncertainty* should remain to discourage or prejudice the *young Tyro* against so rare and so *advantageous* a Contrivance. I say, *so rare*; because I have never seen (amongst many) above one, viz. *Sherwin's Canon*, in this *Form*; and that, by reason of its *great Price*, is not very common. The that Gentleman says in his Preface, he has in this *excellent Method* followed Dr. *John Newton* in his *Trigonometria Britannica*, a Book which I have not seen. The Advantage also of this Abbreviation is next to that of the Invention it self; for hereby the *prolix* and *unwieldy Tables* (in their original Form) are reduced or abridg'd to one half the Bulk nearly; all the *superfluous Part* being omitted, and nothing but what was necessary retained in this Canon.

8. According to this most excellent Abridgment therefore, I have first of all, that I know of, published the common Canon of Logarithms for Numbers from 1 to 10000; having taken the no small Pains of transcribing the whole with my own hand from the aforementioned large Work of Mr. *Hen. Sherwin*, which is the most correct of any extant.

9. In this Form or Disposition of the Canon, I have also published the Logarithms of Sines and Tangents; which thing hath not been done before in any Work great or small, that I have ever seen or heard of. This makes the 2d Part of the Logarithmic
Tables,

Tables, as mentioned Art. 2. And since they are here in a different Form from all others, it may not be unnecessary to hint to the young Learner, that the Numbers express'd in the Side-Column are the Degrees, and every 10th Minute, and the Numbers in the parallel Column at top are the Minutes between the 10ths; see the following Tariff of the Logarithms of Sines in this *Form* for the Minutes from 72 Degrees to 73.

D.	0	1	2	3	4
72. 0	978.206	247	288	329	370
10	615	655	696	736	777
20	979.019	059	100	140	180
30	420	460	500	539	579
40	816	855	895	934	973
50	980.208	247	286	325	364

To give an Example, let the Logarithm be sought for the Sine of $72^{\circ} 43'$. Seek in the Side-Column $72^{\circ} 40'$, next above which is the *permanent Part* of the Logarithm 979 in the first Column of Logarithms; then against $72^{\circ} 40$, and under $3'$ at top you find the variable Part 934, which annexed to the other makes 979934; to which prefix the Index (which is set at the top of each Page) 9, and the Logarithm is compleat, viz. 9.979934 for the Sine of $72^{\circ} 43'$; and thus you proceed for any other.

10. I have contracted the Logarithms to six Places of Figures only, as being sufficient in common Use; the *natural Sines* and *Tangents* are not here inserted, since when their Logarithms can be used, they themselves are useless. Besides, whenever they are required, they may be immediately had from their Logarithms. For Example, suppose I would know, the natural Sine and Tangent for $38^{\circ} 47'$, the Logarithm, Sine and Tangent of this Arch, are 9.796836, and

and 9.965009: To these Logarithms (neglecting their *Indices*) find the natural Numbers, by the first Part of the Canon, they will be 626377 and 803542, the natural Sine and Tangent sought. (See the Method below, Art. 15. for finding the Number of a given Logarithm.)

11. The Logarithm of the *Secant* of any Arch, as of $38^{\circ} 47'$, is thus easily obtain'd :

From the double Radius 20.000000

Subtract the Co-Sine of $38^{\circ} 47' = \dots 9.891827$

There remains the Log. Secant of $38^{\circ} 47' = 10.108173$

And thus the Logarithm of any other Secant may be found, and consequently the *natural Secant*, or *natural Number* belonging thereto.

12. The Reason why the *Indices*, on the top of the Pages, of the Logarithms of Sines and Tangents, are so large, viz. 7, 8, 9, 10, 11, &c. is because the *Radius* of the Circle was supposed to consist of 10000000000 equal Parts, whose Logarithm therefore is 10.000000; wherefore a Number of such *equal Parts* expressing the Sine of one Minute $1'$, will consist of 7 Places, whose Logarithm then will have its *Index* 6. The other Sines will consist of 8; 9, and 10 Places, and so the *Indices* of their Logarithms will be 7, 8, 9, as in the Tables; thus also the Numbers expressing the *Tangents* in such *equal Parts* will consist of 7, 8, 9, 10, 11, 12, 13, and 14 Places of Figures, whence the *Indices* of their Logarithms must be 6, 7, 8, 9, 10, 11, 12, 13, according to Art. 11th and 12th of Chap. I. But since, as before said, the first six Places of the Logarithms to the left are sufficient, the rest are rejected as superfluous.

13. In future Operations there will be frequent Occasion for what is called the *Arithmetical Complement* of a Logarithm, which is nothing but the *Difference*

ference between that Logarithm and Logarithm-Rad-
 ius 10.000000.

Thus if from 10.000000
 You subduct the Logarithm 4.877026
 There will remain the *Arithmet. Comp.* = 5.122974
 And this is done *mentally* in an Instant, by taking
 every Figure from 9, except the first; which you take
 from 10.

Note. If the Logarithm be of any Sine or Tan-
 gent; add 10 to the *Index* of the Arithmetical Com-
 plement, and it will be the Logarithm of the Co-
 Secant of the same Arch. For Example,

Suppose the Log: Sine of $3^{\circ} 48'$ = . . . 8.821342
 The Arithmetical Comp. thereof is . . . 1.178658
 to which add 10,
 The Sum is the Log. Secant of $86^{\circ} 12'$ = 11.178658
 Which is evidently the *same Operation* as that in
 Art. 11. hereof.

14. From the *Theory of Logarithms* we learn, *that*
the Differences of great Numbers are proportional to
the Differences of their Logarithms. (See Chap. V.
 Art. 6, 7.) Therefore tho' the Canon of Logarithms
 goes no farther than the Number 10000, it may by
 this means be extended to the Number 10000000,
 or the Logarithm of any Number under 10000000
 may be found by the present Canon, according to
 the following Rule:

First; find the Logarithm of the *four first Figures*
 of the given Number, by the *first Part* of the Canon.

Secondly; subtract this Logarithm from the Lo-
 garithm *next greater or next following* in the Table;
 and reserve the *Difference*.

Thirdly; multiply the *Difference*, by the *remain-*
ing Figures of the given Number; and from the *Pro-*
duct cut off to the *Right hand*, *so many Figures* as
 there were *remaining* in the given Number.

N

Fourthly;

Fourthly ; add the *Remainder* of the *Product* to the Logarithm first found, the *Sum* shall be the Logarithm sought.

For Example, let the Logarithm of the Number 127053 be sought.

The Logarithm of the }
first 4 Figures } .. 127000 = 5.103804

The next greater Log. is of .. 127100 = 5.104146

The *Differences* 100 342

Wherefore say, as 100 : 342 :: 53 : 181,26

$$\begin{array}{r} 53 \\ 1026 \\ 1710 \\ \hline 181 \overline{)26} \end{array}$$

Add the first Logarithm 5.103804.

The Sum is 5.103985 = the Logarithm sought for the Number 127053.

Example 2. Required the Logarithm of the Number 3567894?

The Logarithm of the }
first 4 Figures } .. 3567000 = 6.552303

The Log. next following is of 3568000 = 6.552425

The Diff. of Numb. and Log. 1000 122

Then say, as 1000 : 122 :: 894 : 109,068

Add the first Logarithm . . 6.552303

the Sum is the Log. 3567894 = 6.552412, as was required. And thus you proceed for the Logarithm of any other greater Number than those in the Canon.

15. By a Method *reverse* to the foregoing, you find the *Number* corresponding to a *given Logarithm*; thus, suppose the given Logarithm be 3.567026, and you would know the Number thereof. Seek this Logarithm in the Table, and because you there find it *exactly*, the Number 3690 corresponding thereto, is the Number sought. But if the given Logarithm

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Logarithm be *not exactly contain'd* in the Table, and more than 4 Figures be required, proceed as follows.

First; seek in the Table a Logarithm the *next less* to the given one, for the four first Figures of the Number sought.

Secondly; subtract this Logarithm from the *given one*, and annex to the Remainder, *so many Cyphers* as you seek Figures more than four.

Thirdly; take the *Difference* between the Logarithm just found and the *next greater*, by which divide the said *augmented Remainder*, the *Quotient* annexed to the four first Figures shall compleat the Number sought.

Example 1. Let there be sought the Number to the Logarithm 5.103985, to six Places of Figures.

The Log. next less is of . . . 127000 = 5.103804

The given Logarithm 5.103985

The Difference or Remainder 181

The next greater Logarithm 127100 = 5.104146

Diff. between the *greatest* and *least* Log. 342

Since the Places in the Number sought are 6, augment the first Remainder 181 with *two Cyphers*, and it will be 18100; then $342 \overline{)18100} (=53$, which annex'd to the four Figures 1270 before found, make the Number 127053 required.

Thus also to the given Logarithm 6.552412, you may find its proper Number 3567894, and so for others.

16. In the same manner you proceed to find any Decimal Number to a Logarithm given, only in this Case the *Indices* of the Logarithms are neglected till the Operation is finished, and then so many Figures are to be cut off from the Number found for Decimals, as the Index of the given Logarithm shall indicate; what is here said relates to *plain* or *terminate Decimals* only; but there are other sorts of Decimals which *circulate* or *perpetually repeat* one or

more Figures *ad infinitum*: And those Figures which thus circulate are *Repetends*, as in these Numbers, viz. 235,2222, &c. 27,83333, &c. 2.383838, &c. 702,6026026, &c. 0,2672326723, &c. Now these *Repetends* need be wrote but once if we slur the first and last Figures in each, to denote them such, as thus 235,2 ; 27,83 ; 2,38 ; 702,60 ; 0,26723, &c. See more in my *Universal System, or Body of Decimal Arithmetic*, printed for Mr. Noon.

17. The Logarithms for the Repeating 9 Digits are made by adding the *Arithmetical Complement* of the Logarithm of 9, to the Logarithms of the said Digits, and are such as here annexed. The Logarithms of pure compound *Repetends* are made by adding the *Arithmetical Complement* of so many 9's, as there are Figures in the *Repetend*, to the Logarithms of those Numbers considered as *terminate*. Thus the Logarithm of the *Repetend* 36,5 is found as follows:

To the Logarithm of 36.5 = 1.562293

Add the *Arith. Complement* of . . 999 = 0.000434

The Sum is the Log. of the *Repetend* 36,5 = 1.562727 and thus proceed for others.

18. If the *Repetend* have any prefix'd terminate Part; then from such a mixed *Repetend* subtract its *terminate Part*, and to the Logarithm of the *Remainder* add the *Arithmetical Complement* of the Logarithm of as many 9's as there are Figures in the *Repetend*. For Example, suppose you would find the Logarithm of the mix'd *Repetend* 26892,7, proceed thus;

From

From the *Repetend* 26892,7
 Subtract the *terminate Part* 26
 To the Log of the Rem. . . 26890,1=4.429592
 add the *Aritbm. Complem.* of 9999=0.000043
 The Sum is the Logarithm of 26892,7=4.429635

Note, the *Indices* of the *Aritbmetical Complements* are here (as in these Cases they always must be) omitted.



C H A P. XIII.

Of the Origin and Construction of SHAKERLY'S and STREET'S LOGISTICAL LOGARITHMS.

1. **T**HE Use of *Logistical Logarithms* is in *Astronomical Calculations*, or *Sexagesimal Arithmetical*; but this sort of Arithmetic, which taught the Rules of *Addition*, *Subtraction*, *Multiplication*, *Division*, &c. of *Sexagesimal Fractions*, viz. *Degrees*, *Minutes*, and *Seconds* of *Motion* or *Time*, was, in former times, called *Logistical Arithmetic*. And since the Invention of the common Canon of *Logarithms*, Mr. *Jeremiah Shakerly*, in his *Tabulæ Britannicæ*, first contrived from them a sort of *Logarithms* adapted to the Rules of *Logistical Arithmetic*; and therefore gave them the Name of *Logistical Logarithms*. And since him Mr. *Thomas Street*, in his *Astronomia Carolina*, has invented another and more convenient Form of *Logistical Logarithms*, than *Shakerly's*. And since Tables of both these sorts of *Logistical Logarithms* are extant, 'tis proper to acquaint the Reader with the *Construction* of both, which is as follows.

2. Since *Logistical Logarithms* are altogether concerned in working Proportions of *Degrees, Minutes, and Seconds*, and more especially of *Minutes and Seconds*, together with *Integers*, 'tis evident, if those Sexagesimal Fractions were reduced into the *lowest Denomination*, viz. of *Seconds*, &c. they might then be work'd with the *Logarithms* of *common Numbers*. Thus suppose the Proportion be $60' 00'' : 3' 47'' :: 51^{\circ} 29' : 3' 15''$; if these *fractional Numbers* be reduced to *Seconds*, they will stand thus, $3600'' : 227'' :: 3089'' : 195''$; wherefore 'tis plain, the Proportion in this Case may be wrought by the *common Canon of Logarithms*, as will be hereafter shewn. But then as here is no *Radius*, there will arise *double Trouble* in the Work by *Logarithms* in common Use, in first *adding* the *Logarithms* of the two *middle Terms*, and then *subtracting* the *Logarithm* of the *first* from that *Sum*, in order to have the *Logarithm* of the *fourth Term sought*; or else the *Complement Arithmetical* of the *Logarithm* of the *first Term* must be taken to perform all by *Addition only*. To avoid therefore the *Trouble* attending perpetually either of these *Methods*,

3. Mr. *Shakerly* makes this Proportion, as $3600'' : 227'' :: 100000, \&c. : 0.06305, \&c.$ or with the *Logarithms*, thus;

As the <i>Logarithm</i> of $3600''$	=	3.556302
to the <i>Logarithm</i> of $227''$	=	2.356025
so the <i>Logarithm</i> of <i>Radius</i> 1.00000, &c.	=	10.000000
to the <i>Logarithm</i> of 0.06305, &c.	=	.8.799723

Now 'tis plain, the *two last Logarithms* perform the same as the *two first*, their Properties being the same; but the first of the two latter *Logarithms* is *Radius* 10.000000, which therefore he called the *Logistical Logarithm* of $3600''$ or one Degree, and consequently the *Logarithm* .8.799723 is the *Logistical Logarithm* of $227'' = 3' 47''$. And thus the *Logistical Logarithm*

of

of any other *Degrees, Minutes and Seconds* may be found; *viz.* by reducing them to *Seconds*, and then by taking from the *Logarithm* of those *Seconds* the *constant Logarithm* 3.556302 of *one Degree* in *Seconds*.

4. Thus from the Log. of . . . $1' = 60'' = 1.778151$
 Take the Log. of *one Degree* $= 3600'' = 3.556302$
 there remains the Logist. Log. of $1' = 60'' = 8.221849$

Again, from the }
 Log. of } . . . $1^{\circ} 3' 00'' = 3780'' = 3.577492$
 take the *constant Log.* $1^{\circ} = 3600'' = 3.556302$
 there remains the }
Logist. Log. of } $1^{\circ} 3' 00'' = 3780'' = 10.021190$

By these Examples you easily perceive how the *Logistical Logarithm* may be found for any Number of *Degrees, Minutes and Seconds*, in *Shakerly's Form*. And since there are 60' or 3600'' in *one Hour*, as well as in *one Degree*, therefore a Table of these *Logistical Logarithms* serves equally as well in the *Computation of Time as Motion*.

5. But if the Time of a whole Day or 24 Hours be the *Integer*, since there are but 1440' therein; and $3600'' : 1440' :: 2^{\frac{1}{2}} : 1'$; also since $60' : 24H^{\circ} :: 2^{\frac{1}{2}} : 1H^{\circ}$; therefore if, thro' the course of these Tables of *Logistical Logarithms*, against every $2^{\frac{1}{2}}$ you place the *Hours*, and against every $2^{\frac{1}{2}}$ (or rather every 3d and 5th Second) you place the *Minutes* of an *Hour*; the Table of *Logistical Logarithms* for *Motion and Time* will be compleated; a *Specimen* of which, in this Form of Mr. *Sba-*

°	Motion Time.	
		H
7	4	
"	2700	XVIII.
0	987506	0
1	987522	
2	987538	
3	987554	1
4	987570	
5	987586	2
6	987602	
7	987618	
8	987634	3
9	987651	
10	987667	4

kerley's,

kerley's, I have before annexed. The first Column contains the Degrees, Minutes or Seconds; the second Column the *Logistical Logarithms* thereof; and the third Column contains the Minutes of Time, the Hour being express'd at the top, viz. XVIII. answering to the Motion of $45'$, or $2700''$.

6. But because these *Logistical Logarithms* of Mr. *Shakerley*, consist of many *Figures* throughout the Table, it minister'd occasion to Mr. *Thomas Street* to contrive a more compendious and convenient Form of these Logarithms; and such he invented, which tho' large at the beginning of the Table, yet immediately lessen very fast, and so continue to the End of the Table, or $60'$ or $3600''$; whose *Logistical Logarithm* is $=0$.

7. The Reason of which is manifest from the Manner of their Construction, which is as follows: Suppose any Proportion of *Sexagesimal Numbers*, as that before made use of, Art. 2. viz. $60' : 3' 47'' :: 51' 29'' : 3' 15''$, which reduced to Seconds, stands thus, $3600'' : 227'' :: 3089'' : 195''$. Now in order to obtain a *vacant Term* in this Analogy, Mr. *Street* (instead of *Shakerly's* Analogy $3600'' : 227'' :: 100000 : 83c.$) inverts the *first Ratio*, as thus; $227'' : 3600'' :: \text{Unity} ;$ to a fourth Number whose Logarithm is reputed the *Logistical Logarithm* of the first Term $227''$, as the Logarithm of Unity is of the second $3600''$. See the Work.

The Logarithm of . . . $3' 47'' = 227'' = 2.356026$

The Logarithm of . . $1^0 = 60' = 3600'' = 3.556302$

The Logarithm of Unity 0.000000

The Logist. Log. of . . . $3' 47'' = 227'' = 1200276$

8. Whence it evidently appears, that to find the *Logistical Logarithm* of any Number of Seconds, you need only subtract the common Logarithm of the Number of Seconds from the constant common *Logarithm* of $3600''$, for the *Remainder* or *Difference* will

will be the *Logistical Logarithm* required, in *Street's* Form.

Thus from the Logarithm of . . $3600'' = 3.556302$
 Subtraſt the Log. of $51' 29'' = 3.089 = 3.489818$
 There remains the Logiſt. Log. of $51' 29'' = .66484$
 Again from the ſaid Log. of . . $3600'' = 3.556302$
 Subduct the Logarithm of $3' 15'' = 195 = 2.290035$
 There remains the Logiſt. Log. of $3' 15'' = 1266267$

9. From whence 'tis evident, that the greater the Number of Seconds is, the leſs will be the Logiſtical Logarithm thereof; till you come to the Number $3600''$, whoſe Logiſtical Logarithm is nothing at all, as before ſaid. And thus it appears that the Logarithms of this kind of Mr. *Street's* Form, have in them fewer Places of Figures, and are therefore more convenient for Uſe, by much, than thoſe of *Shakerly's* Form, before deſcrib'd. And for that Reaſon I have choſe to give the Reader a Table of *Street's* Logarithms rather than the other; and tho' Mr. *Leadbetter* has given us Tables of both ſorts, yet I think it intirely needleſs; ſince all the principal Uſes of *Shakerly's*, are much better perform'd in *Street's* Logiſtical Logarithms.

10. In ſhewing the Manner of making theſe Logarithms from the common ones, I have expreſſed them at large, viz. 1200276, 66484, 1266267, the Logiſtical Logarithms of $3' 47''$, $51' 29''$, and $3' 15''$, as per Art. 7, 8. Yet here two things are to be obſerv'd: Firſt, that the Index is not diſtinguiſhed from Logarithm itſelf, with a *Point*, as in the common ſort; but the remaining Figures, both of Logarithms and *Indices*, be they more or leſs, are reputed together, the Logiſtical Logarithm. Secondly, that two Places of Figures to the Right Hand in the Examples, are ſtruck off in the Table; the other being fully ſufficient for all the Purpoſes thereof. So

that in the Table you will find the said *Logistical Logarithms* wrote 12003, 665, 12663, &c.

11. Mr. *Leadbetter* has taken the pains to continue the Table of *Logistical Logarithms* in *Street's* Form, to 120', or 2 Degrees; but as there is little occasion for any more than the *Logistical Logarithms* of 60' or 1 Degree; and when there is, the same *Logarithms* are capable of answering it, I have therefore continued them no farther than the Inventor did, viz. to 60' or 3600".

12. As to the Form of the Table, 'tis very easy to be understood, especially if those of the common *Logarithms* of *Sines* and *Tangents* before described are: The first Column of the Tables contains the *Degrees* or *Minutes*, or *Minutes* and *Seconds* in *Sexagesimals*, as in the common Tables, in the Order of *Denaries* or 10's, the nine Digits running along on the top of the Table, under which, in the several Columns, are the *Logistical Logarithms* abbreviated in the same manner as those of *Sines* and *Tangents*; and are to be taken out according to the Directions there given, which see. In the last Column are contained the Numbers of the first reduced to Minutes or Seconds; and are to be compleated likewise with the *Digits* on the top of the Table. These are referred to when the *Logistical Logarithm* of any integral Number is sought. An Example or two will render all easy.

13. Let it be required to find the *Logistical Logarithm* of 2° 48' or 2' 48". First seek 2 40 in the first Column, and against it and under 8 at top, you find 310, which annexed to its proper permanent Part 13 in the first Column of *Logarithms* makes 13310, the *Logistical Logarithm* of 2 48, as required. Thus the *Logistical Logarithm* of 37' 59" is found to be 1986; and of 54' 40" to be 404; and of 58' 37" to be 101; and of 59' 53" to be 8. And thus the *Logistical*

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gistical Logarithm of $60'$, is $=0$; and so will be a vacant Term in all Analogies for Operation.

14. Let it be required to find the *Logistical Logarithm* of the Number 584. First seek 580 in the last or right-hand Column, and against it and under 4 at top you see *99, which shews it must be join'd to the following permanent Part 78 in the first Column of *Logarithms*, and therewith makes 7899, the *Logarithm* sought. Thus the *Logistical Logarithm* of 1000, is found 5563; and of 3359, to be 301; and of 3596, to be 5; and that of 3600 is nothing.

15. This Table also equally serves for Time, whether for a *Day* and *Minutes*, or *Minutes* and *Seconds*; by help of the little Table at the End of the *Logistical Logarithms*, which shews what Parts of Motion in *Degrees* and *Minutes* correspond to Time in *Hours* and *Minutes*. Thus against $13'$ in Time, you see $33'$ of Motion; against 1 Hour is $2^{\circ} 30'$; and the Motion answering to VII^h $43'$, is $17^{\circ} 30'$, $+ 1^{\circ} 48' = 19^{\circ} 18'$; and that answering XXI^h $21'$ is $52^{\circ} 30'$; $+ 0^{\circ} 53' = 53^{\circ} 23'$; and so for other Parts of Time; consequently

H^o. Time.

The Logistical Logarithm of	{	0	13'	} is {	20378	} as being the same for the Motion of	0 ^o 33'
		1	00		13802		2 30
		7	43		4926		19 18
		21	21		507		53 23
		24	00		0		60 00

The Reason of all which is evident, from Art. 5. hereof.

16. These *Logistical Logarithms* may in like manner be render'd applicable to Computations of *Money*, *Weights*, *Measures*, &c. Thus since $60^{\circ} : 20^{\circ} :: 3 : 1$, therefore the Shillings in a Pound correspond to each 3d Degree or Minute of Motion. And again, since $960^{\text{grs}} : 3600^{\text{''}} :: 1 : 3\frac{1}{2}$; therefore to 1, 2, 3, 4, &c. Farthings there answers $3'$, $7'$, $11'$, $15'$, &c. Minutes

100 *Of Shakerly's and Street's Tables, &c.*

of *Motion*; whence a Table may be form'd to shew the *Logistical Logarithm* for any Number of Farthings under 960, or one Pound. And thus you may proceed to frame a Table to render these *Logistical Logarithms* for *Motion* useful for the *Ounces* and *Pounds* in an *hundred Weight*; and that with great *Ease*, since 1792, the Number of *Ounces* in an *hundred Weight Averdupoise*, is nearly half the Number 3600", the Seconds of the Table. Consequently, the *Logistical Logarithm* of the *Double* of any Number of *Ounces*, is that required for the *Pounds* and *Ounces* equal thereto.

17. Or, lastly, 'tis easy for any one who shall think it worth while, to calculate Tables of *Logistical Logarithms*, for any Species of Computation, peculiar to it self. Thus with respect to Money, if the common *Logarithms* of all the Farthings under 960, be subducted from the constant *Logarithm* of 960, the *Remainders* will be the *Logistical Logarithms* of the Farthings in a Pound. The common *Logarithms* of all the Numbers of *Ounces* under 1792 substracted from the constant *Logarithm* of the said Number 1792, will leave the *Logistical Logarithms* of all the *Ounces* in an hundred Weight. And thus you proceed to make *Logistical Logarithms* of any Kind you please, or may have occasion for. Which is too easy a matter to require an Example, besides those above, Art. 7, and 8.





LOGARITHMOLOGY.

PART II.

The PRAXIS of LOGARITHMS, Common and Logistical:

With its APPLICATION to Vulgar and Duodecimal Arithmetic, Plain and Spherical Trigonometry, Navigation, Mensuration of Surfaces and Solids, Gauging, Timber-Measure, Astronomy, perform'd Numerically and Instrumentally.



CHAP. I.

Of the Rules of ADDITION, SUBTRACTION, MULTIPLICATION, and DIVISION of the Indices of LOGARITHMS.

1. **A**S the *Theory of Logarithms* has been largely premised, the *Rules for a Practical Use and Management* of those artificial Numbers will from thence be easily understood, and the *Rationale of every Operation* be apparent to the intelligent Reader. Before we proceed to the Use of Logarithms as applied to *common Arithmetic*, &c. we must

must first consider some *previous Rules and Methods* which regard the *due ordering and working* those Numbers themselves, on account of the *Indices*, which admit of divers particular Cases in the *Rules of Addition, Subtraction, Multiplication, and Division*: which therefore must be *exemplified and illustrated* as in the *Sequel* of this Chapter.

ADDITION OF LOGARITHMS.

2. As in all *Species of Arithmetic* the first *fundamental Rule* is *Addition*, so in this of *Logarithms*, the Rules which require this *primary Operation* come first to be consider'd, and they are as follow.

(Note, I shall in this Place call the *Indices* of *Logarithms* of *whole Numbers*, *Integral Indices*; and those of the *Logarithms* of *Decimal or Fractional Numbers*, *Decimal Indices*.)

Rule I. If the *Indices* be *both Integral*, add them together for the *Sum required*.

Rule II. If the *Indices* are *both Decimal*, add them as before; and observe, (1.) if the *Sum* be *above* or *just* 10 or 100 ($=tA$; see Chap. IV. Theory) cast away 10 or 100. (2.) If the *Sum* be *under* 10 or 100 ($=tA$) add 10 or 100 thereto; and both the *Sum* in the *latter Case* and the *Remainder* in the *former*, will be *Decimal*.

Rule III. If the *Indices* be of *different Kinds*, *viz. Integral* with *Decimal*, the *Sum*, if *under* 10 or 100, is *Decimal*; if *just* 10 or 100, or *above*, cast away 10 or 100, the *Remainder* is *Integral*.

Rule IV. In case the *Sum* of *two or more decimal Indices* be less than 10 $=tA$, the best way will be to use the larger *decimal Indices*, where $tA=100$; and then their *Sum* will be greater than 100, and so the *Reason* of the *Operation* will be more evident.

3. All these *Rules* relate to the *Indices* of the *Logarithms* only, and are *exemplified* as follow:

Ex-

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Exam. I. To 3.513217 Exam. II. 2.317227
 add 2.303196 0.850891

Rule I. Sum = 5.816413 Sum = 3.168118

Exam. III. To .9.849235 Exam. IV. .97.237406
 add .7.786822 .95.072607

Rule II. }
 Part I. } Sum = .7.636057 Sum = .92.310013

Exam. V. To .4.273760 Exam. VI. .62.346174
 add .3.067247 .21.300725

Rule II. }
 Part 2. } S. = .17.341007 Sum = .183.646899

Exam. VII. To 6.372458 Exam. VIII. .88.426703
 add 2.673842 5.268402

Rule III. Sum = 9.046300 Sum = .93.695105

Exam. IX. To 5.206737 Exam. X. 8.426735
 add .8.312046 .92.105374

Rule III. Sum = 3.518783 Sum = 0.532109

Exam. XI. } .4.203106
 add } .1.312729
 .2.070346
 Rule II. }
 Part 2. } S. = .27.586181

Or rather thus, per Rule IV.
 Ex. XII. .94.203106
 .91.312729
 .92.070346
 .77.586181

*Tis possible these two last Examples may seem somewhat *obscure*; but the *Reason* and *Truth* of each will appear, if they are wrought at *twice*, as follows:

To	.4.203106	And	.94.203106
add	.1.312729		.91.312729
Rule II. } Part 2. }	S. = .15.515835	Rule II. } Part 1. }	S. = .85.515835
to which add	.2.070346		.92.070346
Rule II. } Part 2. }	S. = .27.586181	Rule II. } Part 1. }	S. = .77.586181

From whence appears the Reason why, in these Cases, the *larger Indices* are preferable to the *smaller ones*.

SUBTRACTION of LOGARITHMS.

4. In the *Subtraction of Indices* the following *Rules* are to be observ'd, viz.

Rule I. If they are both *Integral*, and the *higher one the greater*, the *Remainder* will be *Integral*. But if the *lower one be the greater*, then add 10 to the *higher one*, and *subtract*; the *Remainder* will be *Decimal*.

Rule II. If both *Indices* are *Decimal*; and the *higher be the greater*; the *Remainder* is *Integral*; if not, add 10 or 100 to the *higher one*, and *subtract* the *lower*, the *Remainder* will be *Decimal*.

Rule III. If the *Indices* are of *different sorts*, viz. one *Integral* the other *Decimal*; then if the *higher be Integral*, add 10 or 100 to it, and *subtract*; the *Remainder* will be *Integral*. But if the *higher be Decimal* and the *greater*, the *Remainder* will be *Decimal*; if *lesser*, the *larger Indices* must be used.

5. The *Rules* also are easily deriv'd from the foregoing *Theory*, and are illustrated by the following *Examples*.

Exam. I. From	5.816413	Ex. II.	3.168118
Subtract	3.513217		2.317227
Rule I. Rem. =	2.303196	Rem. =	0.850891

Ex.

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Ex. III. From 3.513217 Ex. IV. 0.850891
sub. 5.816413 3.168118

Rule I. Rem. = 7.696804 Rem. = 7.682773

Ex. V. From .7.503617 Ex. VI. .94.420345
sub. .3.467306 .92.673457

Rule II. Rem. = 4.026311 Rem. = 1.746888

Ex. VII. From .3.467306 Ex. VIII. .92.673457
sub. .7.503617 .94.420345

Rule II. Rem. = 5.963689 Rem. = .98.253112

Ex. IX. From 5.816413 Ex. X. 6.2067347
sub. .8.132700 .94.1535012

Rule III. Re. = 7.683713 Rem. = 12.0532335

Ex. XI. From .8.132700 Ex. XII. .94.278769
sub. 5.816413 6.165348

Rule III. } Rule III. }
Part 2. } Re. = 2.316287 Part 2. } R. = 88.113421

MULTIPLICATION of LOGARITHMS.

6. In Chap. IV. Art. 19th and 20th of the *Theory*, the Rules for multiplying the *Indices of Logarithms* (of *Pure Fractions* especially) are demonstrated; and are,

Rule I. If the *Index* be *Integral*, multiply as usual; the Product shall be *Integral*.

Rule II. If the *Index* be *Decimal*, make the Logarithm of *Unity*, or $tA=100$, then shall the *Index* be

P

be

be of the *larger Sort*, which in this Case, will be more convenient for Use ; and then according as you multiply by 2, 3, 4, 5, 6, &c. you must reject 100, 200, 300, 400, 500, &c. from the *Product*, the *Remainder* thereof will be *Decimal*.

$$\begin{array}{rcl} \text{Ex. I. Mult.} & 3.420673 & \text{Ex. II.} & 5.700672 \\ & \text{by} & & 4 \\ & \underline{\hspace{1.5cm}} & & \underline{\hspace{1.5cm}} \end{array}$$

$$\begin{array}{rcl} \text{Rule I. Prod.} & = 6.841346 & \text{Product} = 22.802688 \\ & \underline{\hspace{1.5cm}} & & \underline{\hspace{1.5cm}} \end{array}$$

$$\begin{array}{rcl} \text{Ex. III. Mult.} & .96.130126 & \text{Ex. IV.} & .91.034106 \\ & \text{by} & & 3 \\ & \underline{\hspace{1.5cm}} & & \underline{\hspace{1.5cm}} \end{array}$$

$$\begin{array}{rcl} \text{Rule II. Pr.} & = .92.260252 & \text{Prod.} = .73.102318 \\ & \underline{\hspace{1.5cm}} & & \underline{\hspace{1.5cm}} \end{array}$$

$$\begin{array}{rcl} \text{Ex. V. Mult.} & .84.034121 & \text{Ex. VI.} & .70.061050 \\ & \text{by} & & 5 \\ & \underline{\hspace{1.5cm}} & & \underline{\hspace{1.5cm}} \end{array}$$

$$\begin{array}{rcl} \text{Rule II. Pr.} & = .36.136484 & \text{Prod.} = .50.305250 \\ & \underline{\hspace{1.5cm}} & & \underline{\hspace{1.5cm}} \end{array}$$

In this last Example, the *Index* $.70. \times 5 = 350$; but since from 350 you cannot reject 400, as *per* Rule II ; therefore it must be $400 - 350 = .50$. which subtracted from 99, leaves 49 ; which shews that 49 *Cyphers* above 100, that is, 149 *Cyphers* are to be prefix'd ; and this you are to understand in all Cases where xL is less than $x-1 \times tA$. See Chap. IV. Art. 20th, of *Theory*. Or thus, in general let $tA = 10$ or 100 ; then when xL is *greater* than $x-1 \times tA$, then it will be $xL - x-1 \times tA - 9$ or 99, is equal to the Number of *Cyphers* to be prefix'd. But if $x-1 \times tA$ is *greater* than xL , then it will be $x-1 \times tA - xL - 9$ or 99, = to the said Number of *Cyphers*.

DIVISION of LOGARITHMS.

7. This is but the *Reverse* of the foregoing Operation; and the Rule for decimal *Indices* the reverse to that; which is also derived from Chap. IV. Art. 21st, and 22d of the *Theory*, which see.

Rule I. If the *Index* be *Integral*, divide as usual; and the *Quotient-Index* will be *Integral*.

Rule II. If the *Index* be *Decimal*, use the *larger Sort*; and then adding to the said *Index* 100, 200, 300, &c. divide by 2, 3, 4, &c. the *Quotient-Index* will be *Decimal*.

Ex. I. Divide 6.841348	Ex. II.	22.802688
by 2		4

Rule I. Quot. = <u>3.420673</u>	Quotient = <u>5.700672</u>
---------------------------------	----------------------------

Ex. III. Div. .92.269252	Ex. IV.	.83.102300
by 2		3

Rule II. Qu. = <u>.96.130126</u>	Quot. = <u>.94.367433</u>
----------------------------------	---------------------------

Ex. V. Divide .36.136484	Ex. VI.	.30.305250
by 4		5

Rule II. Qu. = <u>.84.034121</u>	Quot. = <u>.86.061050</u>
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C H A P. II.

Of MULTIPLICATION and DIVISION of
WHOLE NUMBERS and DECIMALS by LO-
GARITHMS.

1. **H**AVING before shewn the Method of finding the *Logarithms* of all kinds of *Numbers*, both *Integers* and *Decimals*, and also of fitting and adjusting proper *Indices* thereto; and in the foregoing Chapter, the *Arithmetical Management* thereof in all *Varieties*: I shall now apply the *Use* of those excellent *Numbers* in the *Rules of Arithmetic*; and first in the *Multiplication* and *Divison* of *Whole Numbers* and *Decimals*.

2. From the foregoing Theory (see Chap. I. Art. 10; and Chap. III. Art. 12.) we obtain this easy and obvious Rule for the *Multiplication* of all kind of *Numbers* by *Logarithms*,

viz. { To the *Logarithm* of the *Multiplicand*,
Add the *Logarithm* of the *Multiplier*;
The *Sum* is the *Logarithm* of the *Product*.

Examples of INTEGERS.

		Logarithms.
3. Example I. Multiply	12 =	1.079181
	by 8 =	0.903090
Product	96 =	1.982271

Ex-

	Logarithms.	
Example II. Multiply 127	= 2.103804	
by 12	= 1.079181	
Product 1524	= 3.182985	

Example III. Multiply 526	= 2.720986	
by 100	= 2.000000	
Product 52600	= 4.720986	

Example IV. Multiply 9876	= 3.994581	
by 517	= 2.713490	
Product 5105892	= 6.708071	

Example V. Multiply 987600	= 5.994581	
by 517000	= 5.713590	
Product 510589200000	= 11.708071	

4. *Examples of MIX'D NUMBERS.*

Example VI. Multiply 7,5	= 0.875061	
by 10	= 1,000000	
Product 75	= 1.875061	

Example VII. Multiply 12,4	= 1.093422	
by 3,6	= 0.556302	
Product 44,64	= 1.649724	

Ex-

		Logarithms.
Example VIII. Multiply . . . 0,762	=	.9.881955
by 570	=	2.755875
Product . . 434,34	=	2.637839

Example IX. Multiply 36,5	=	1.562293
by , 0,00019	=	.6.278754
Product 0,006935	=	.7.841047

Example X. Multiply 473	=	.9.674861
by 62	=	1.803705
Product 30.1	=	1.478566

Example XI. Multiply 6	=	0.823909
by ,8	=	.9.744727
Product 3,76	=	0.568636

Example XII. Multiply . . . 27,23	=	1.326541
by 47,6	=	1.623458
Product 600,718	=	2.949999

5. *Examples of PURE DECIMALS.*

Example XIII. Multiply ,12	=	.9.079181
by ,8	=	.9.903090
Product ,096	=	.8.982271

Ex-

	Logarithms.
Example XIV. Multiply . . . ,0097	= .97.986772
by ,00021	= .96.322219
Product 000002037	= .94.308991

Example XV. Multiply ,00470	= .97.623458
by . . . ,00008	= .95.948847
Product ,0000003735	= .93.572305

Example XVI. Mult. ,00000085	= .93.929419
by ,0000012	= .94.079181
Product ,00000000000102	= .88.008600

Ex. XVII. Mult. ,000000000075	= .90.875061
by ,000006	= .94.778151
Product ,0000000000000045	= .85.653212

6. DIVISION *by* LOGARITHMS.

In the same Part of the *Theory* referr'd to (Art. 2.) for the Rule by which *Multiplication* is performed by *Logarithms*, you will likewise find the Demonstration of the following Rule of *Division* of Numbers by *Logarithms*;

viz. { From the Logarithm of the *Dividend*,
 Subtract the Logarithm of the *Divisor*;
 The Remainder is the Logarithm of the
 Quotient.

Ex-

Examples of INTEGERS.

Logarithms.

$$\begin{array}{rcl}
 7. \text{ Example I. Divide } & \dots\dots 96 & = 1.982271 \\
 & \text{by } \dots\dots 12 & = 1.079181 \\
 & \hline
 \text{Quotient } & \dots\dots 8 & = 0.903090 \\
 & \hline
 \end{array}$$

$$\begin{array}{rcl}
 \text{Example II. Divide } & \dots\dots 1524 & = 3.182985 \\
 & \text{by } \dots\dots 172 & = 2.103804 \\
 & \hline
 \text{Quotient } & \dots\dots 12 & = 1.079181 \\
 & \hline
 \end{array}$$

$$\begin{array}{rcl}
 \text{Example III. Divide } & \dots\dots 52600 & = 4.720986 \\
 & \text{by } \dots\dots 526 & = 2.720986 \\
 & \hline
 \text{Quotient } & \dots\dots 100 & = 2.000000 \\
 & \hline
 \end{array}$$

$$\begin{array}{rcl}
 \text{Example IV. Divide } & \dots\dots 5105892 & = 6.708071 \\
 & \text{by } \dots\dots 517 & = 2.713490 \\
 & \hline
 \text{Quotient } & \dots\dots 9876 & = 3.994581 \\
 & \hline
 \end{array}$$

$$\begin{array}{rcl}
 \text{Exam. V. Divide } & 510589200000 & = 11.708071 \\
 & \text{by } \dots\dots 987600 & = 5.994581 \\
 & \hline
 \text{Quotient } & 517000 & = 5.713490 \\
 & \hline
 \end{array}$$

8. *Examples in MIX'D NUMBERS.*

$$\begin{array}{rcl}
 \text{Example VI. Divide } & \dots\dots 75 & = 1.875061 \\
 & \text{by } \dots\dots 7.5 & = 0.875061 \\
 & \hline
 \text{Quotient } & \dots\dots 10 & = 1.000000 \\
 & \hline
 \end{array}$$

Example VII. Divide 44,64 = 1.649724
 by 12,4 = 1.093422
 Quotient . . . 3,6 = 0.556302

Example VIII. Divide 434,34 = 2.637830
 by 570 = 2.755875
 Quotient . . . ,762 = 9.881955

Example IX. Divide . . . ,006935 = .7841046
 by 36,5 = 1.562293
 Quotient ,00019 = .6.278753

Example X. Divide 30,1 = 1.478566
 by 63 = 1.803705
 Quotient ,473 = 9.674861

Example XI. Divide 3,76 = 0.568636
 by 6 = 0.823909
 Quotient 3 = 9.744727

Example XII. Divide . . . 900,718 = 2.949999
 by 47,6 = 1.623458
 Quotient . . . 21,23 = 1.326541

9. Examples in PURE DECIMALS.

$$\begin{array}{rcl}
 \text{Example XIII. Divide} & \dots & ,096 = .8982271 \\
 \text{by} & \dots & ,12 = .9079181 \\
 \hline
 \text{Quotient} & \dots & ,8 = .903090
 \end{array}$$

$$\begin{array}{rcl}
 \text{Exam XIV. Divide} & ,000002037 & = .94308991 \\
 \text{by} & \dots & ,0097 = .97986772 \\
 \hline
 \text{Quotient} & ,00021 & = .96322219
 \end{array}$$

$$\begin{array}{rcl}
 \text{Ex.XV. Divide} & \dots & ,0000003735 = .93572305 \\
 \text{by} & \dots & 00008 = .95948847 \\
 \hline
 \text{Quotient} & ,00478 & = .97623458
 \end{array}$$

$$\begin{array}{rcl}
 \text{Ex.XVI. Div.} & ,000000000000102 & = .88008600 \\
 \text{by} & \dots & ,00000085 = .93929419 \\
 \hline
 \text{Quotient} & ,0000012 & = .94079181
 \end{array}$$

$$\begin{array}{rcl}
 \text{Ex.XVII. Div.} & ,0000000000000045 & = .85653212 \\
 \text{by} & \dots & ,000006 = .94778151 \\
 \hline
 \text{Quotient} & ,00000000075 & = .90875061
 \end{array}$$

10. I think these Examples in the *Multiplication* and *Division* of Numbers by *Logarithms*, are sufficient to instruct any *docible Genius* in his Practice herein; and as the latter are but the *Converse* of the former, so they mutually illustrate and prove the Truth of each other respectively.



C H A P. III.

Of raising POWERS, and the EXTRACTION of ROOTS by LOGARITHMS.

1. **F**ROM the Theory of Logarithms (Chap. I. Art. 12. and Chap. III. Art. 13.) we have an evident Rule for the *Involvement of Numbers*, or *raising* them to any proposed *Power* by means of Logarithms; which is this,

viz. { Multiply the *Logarithm* of the *given Number*
by the *Index* of the *Power*, viz. 2, 3, 4, 5,
&c. the *Product* shall be the *Logarithm* of
the *Power*, viz. the *Square*, *Cube*, *Biqua-*
drate, *Sur-solid*, &c. *Power* of the said *given*
Number.

Examples in INVOLUTION.

2. Example I. What is the Square of the Number 32?

Multiply the Logarithm of 32 = 1.505150
by the Index of the Power 2

The Prod. is the Log. of the Square 1024 = 3.010300

Ex. II. Required the Square of 3.2 = 0.505150
Multiply by 2

The Prod. is the Log. of the Square 10.24 = 1.010300

Ex. III. Required the Square of 32 = 9.505150
Multiply by 2

The Product is the Answer 1024 = 9.010300

Example IV. Required the several Powers of the Number 1.05 to the Surfolid?

1. The Logarithm of 1.05=0.021189
Multiply by 2

The Product is the *Square* . . . 1,1025=0.042378

2. Multiply the Logarithm of 1.05=0.021189
by 3

The Product is the *Cube* 1,157625=0.063567

3. Multiply the Logarithm of 1.05=0.021189
by 4

Product is the *Biquadrate* 1.21650625=0.084756

4. Lastly, Multiply the same . . 1.05=0.021189
by 5

Product is the *Surfolid* 1.2773315625=0.105945

Example V. Required the *Surfolid* Power of the Number ,0006 ?

Multiply the Logarithm of ,0006=.96.778151
By the Index of the Power . . . 5

The *Surfol.* ,0000000000000007776=.83.890755

Example VI. What is the *Cubo-Cube* Power of ,08 ?

Multiply the Logarithm of . . ,08=.98.903090
by the Index of the Power . . . 6

The *Cubo-Cube* Power ,000000262144=.93.418540

Example VII. What is the 57th Power of the Number ,99 ?

Multiply

Multiply the Logarithm of . . . ,99=.9.995635
by the Index of the Power . . . 57

69969445
49978175

The 57th Power is . . . ,56389, &c.=.9.751195

3. There is another way of raising the Powers of *Decimal Numbers* by Logarithms, and it is thus ;

viz. { Multiply the *Arithmetical Complement* of the
Logarithm of the given Fraction by the
Index of the Power, the *Arithmetical Com-
plement* of the Product is the Logarithm of
the Power sought.

And this in many Cases, (as when the *Index* of the
Power is a *mix'd Number* or *pure Decimal*) will be
found most certain and ready. Thus in the last Ex-
ample this way ;

Example VIII. What is the 57th Power of the
Number ,99 ?

The Logarithm of ,99=.9.995635
The *Arithmetical Complement* 0.004365
which multiply by . . . 57

30555
21825

The Product . . . 0.248805
The Arithmet. Comp. is }
the Log. of the Power } ,56389, &c.=.9.751195

Example IX. What is the 4th or 25th Power of ,2 ?

The

The Logarithm of . . .	$,2 = .9.301030$
The <i>Arithmetical Comp.</i> thereof	$.0.698970$
which multiply by the Index	$,25$

3494850
1397940

The Product . . .	$0,17474250$
The Arithmet. Comp. is } the Log. of the Power }	$,66874, \&c. = .9.825258$

Example X. What is the 6,25th Power of ,0032 ?

The Logarithm of . . .	$,0032 = .7.505150$
The Arithmetical Complement	2.494850
which multiply by . . .	$.6.25$

The Product is . . .	15.5928125
The Arithmetical Complement. of which is	$.84.4071875$
And the Number answering thereto, viz.	
,0000000000000025538 is the 6.25th Power of	
,0032.	

4. EVOLUTION or EXTRACTION of ROOTS by LOGARITHMS.

This is done by a Rule, the *converse* of that for *Invol-
ution*, in Art. 1st ;

viz. { Divide the Logarithm of the Power by the
Index of the Root, the Quotient shall be
the Logarithm of the Root sought.

Examples in EVOLUTION.

Example I. What is the *Square Root* of 1024 ?

Divide the Logarithm of . . .	$1024 = 3.010300$
By the Index of the Root	2

The Quot. is the Log. of the square Root $32 = 1.505150$

Ex-

Example II. Required the *Cube Root* of 1,157626?

Divide the Logarithm of 1,157625 = 0.063567
By the Index of the Root 3

The Qu. is the Log. of the *Cube R.* 1,05 = 0.021189

Example III. What is the *surfsolid Root* of the Power
,0000000000000007776?

The Logarithm thereof is 83.890755
which divide by the Index 5

The Quot. is the Log. of the }
Root sought } ,0006 = .96.778151

Example IV. What is the *Cubo-Cube Root* of the
Power ,000000262144?

The Log. of . . . ,000000262144 = .93.418540
Which divide by the Index 6

The Root sought is . . . ,08 = .98.903090

Example V. What is the 57th Root of the Power
,56389, &c.?

The Logarithm of . . . ,56389, &c. .9.751195
which divide by . . . 57

The Root required is . . . ,99 = .9.995635

5. Another different way to extract the Root of
Decimal Numbers is the converse of that in Art. 3d,
hereof.

viz. { Divide the *Arithmetical Complement* of the Lo-
garithm of the *Decimal* given by the Index
of the Root required, the Arithmetical Com-
plement of the Quotient is the Logarithm
of the Root sought.

Example VI. What is the ,25th Root of the Power ,66874, &c.?

The Logarithm thereof is . . .	.9.825258
The Arithmetical Complement is	0.174742
which divide by . . .	,25

The Quotient . . .	0.698970
--------------------	----------

The Arith. Comp. is the Log. of the Root	} . . ,2=.9.301030

Example VII. Required the 6.25th Root of the Power ,00000000000000025538 ?

The Logarithm thereof is . . .	.84.407187
The Arith. Comp. of which is	15.592813
which divide by . . .	6.25

The Quotient is . . .	2.494850
-----------------------	----------

the Arith. Complem. thereof is . . .	.7.505150
--------------------------------------	-----------

the Logarithm of ,0032 the Root sought.

Example VIII. What is the *Cubo-Cube* or 6th Root of the Power ,1 ?

The Logarithm of . . .	,1=.9.000000
the Arith. Comp. thereof . .	.0.999999
which divide by the Index	6

The Quotient . . .	0.166666
--------------------	----------

The Arith. Comp. is the Log. of the Root	} ,68129, &c. =.9.833333

6. Thus you see the great Use of Logarithms in extracting the Roots of a given Power, which tho' a thing so very difficult by the Rules of common Arithmetic, is yet render'd most easily practicable by this excellent Art; yea 'tis easy to make it appear, that the *Extraction of Roots* is not only most expeditiously performed, but hath a more universal Perfection in this Method, than in any other.



C H A P. IV.

Of the various RULES of PROPORTION, and of finding MEAN PROPORTIONALS by LOGARITHMS.

1. **F**ROM the *Theory* 'tis evident, that the *golden Rule*, or *Rule of Proportionals*, is wrought in Logarithms by only the *Addition* and *Substraction* of the Logarithms of the Terms of the Proportion. And if the Proportion be direct, the Rule is thus ;

viz. { Add the Logarithms of the *second* and *third* Terms, from that *Sum* subtract the Logarithm of the *first* ; the Remainder is the Logarithm of the *fourth* sought.

Examples in the GOLDEN RULE Direct.

Example I. If 12 Pounds cost 1*l.* 15*s.* 9*d.* what will 173 Pounds cost ?

The Logarithm of 12 = 1.079181

To the Log. of 1*l.* 15*s.* 9*d.* = 1.7875 = 0.252246

Add the Logarithm of 173 = 2.238046

The Sum 2.490292

Subtract the first, there remains 25,77*l.* = 1.411111

Wherefore the Answer is 25,77*l.* = 25*l.* 15*s.* 4*d.* 1.

2. But since if you *divide* by any Number, or *multiply* by its *Reciprocal*, the Effect is the same ; and also since the Arithmetical Complement of any Number, is but the Logarithm of the *Reciprocal* of that

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that Number; therefore it follows, that where the *Subtraction* of a Logarithm is required in any Operation, if you take the *Arithmetical Complement* of that Logarithm, the whole may be performed by *Addition* only. Thus in the foregoing Example.

$$\text{Add } \left\{ \begin{array}{l} \text{the Ar. Comp. of the Log. of } 12 = 8.920819 \\ \text{the Log. of } \dots\dots\dots 1,78751 = 0.252246 \\ \text{the Log. of } \dots\dots\dots 173 = 2.238046 \end{array} \right.$$

The Sum is the Answer $\dots\dots 25,771 = 1411111$
the same as before.

Example II. If 2 C. 1 q. 21 l. 14 oz. cost 5 l. 17 s. 8 d. $\frac{1}{2}$.
what will 31 C. 2 q. 26 l. 15 oz. cost?

Then 2 C. 1 q. 21 l. 14 oz. = 2.4453 A. Com. .9.611668

The Log. of 5 l. 17 s. 8 d. $\frac{1}{2}$ = 5.8844 = 0.769702

The Log. of 31 C. 2 q. 26 l. 15 oz. } $\dots\dots = 1.501614$
= 31,7405

The Sum is the Logarithm of 76,381 l. = 1.882984
Therefore 76,381 l. = 76 l. 7 s. 7 d. $\frac{1}{2}$ is the Answer.

3. *Of the Rule of Three Inverse.*

In this Case you must take the *Arithmetical Complement* of the *third* Term, and add it with the Logarithms of the other two as before; so shall the Sum be the Logarithm of the Answer.

Example. Suppose a Field feeds 18 Horses for 7 Weeks, how long will it feed 42, at that rate?

$$\text{Add } \left\{ \begin{array}{l} \text{the Logarithm of } \dots\dots\dots 18 = 1.255272 \\ \text{the Logarithm of } \dots\dots\dots 7 = 0.845098 \\ \text{the Arith. Comp. of the Log. of } 42 = 8.376751 \end{array} \right.$$

The Sum is the Logarithm of Answer 3 = 0.477121

4. Of the double Rule of Three, or Rule of Five Numbers.

As in Questions of this sort, there are always *three conditional or supposed Terms*; the first of which is the *principal Cause of Gain, Loss, Action, &c.* the *second* denotes the *Time, Distance, &c.* and the third is the *Gain, Loss, or Action, &c.* So let these three Terms be denoted by the Capitals P, T, G. Also there are *three* other Terms (similar to the three former) which make the Question to be resolv'd; and let these be represented by the small Letters p, t, g. Two of which are always given, and the other is sought. But since

$$P : G :: p : \frac{Gp}{P}; \text{ and again, since } T : \frac{Gp}{P} :: t : g;$$

$$\text{therefore } Tg = \frac{tGp}{P}, \text{ and consequently } PTg = tpG;$$

from which general Theorem we can easily find p, t,

$$\text{or g. Thus, I. } \frac{PTg}{Gt} = p; \text{ and II. } \frac{PTg}{Gp} = t; \text{ and III.}$$

$$\frac{tpG}{PT} = g. \text{ The Contrivance of these excellent The-}$$

orems we owe to the late Mr. Ward, of Chester.

5. I shall exemplify Questions in this Rule by Examples, as follow.

Example I. If 100 l. in 12 Months gain 6 l. what will 350 l. gain in 9 Months?

Here $P=100$, $T=12$, $G=6$; also $p=350$, $t=9$, to find g.

$$\begin{array}{l} \left\{ \begin{array}{l} \text{the Logarithm of } \dots\dots\dots G=6=0.778151 \\ \text{Add } \left\{ \begin{array}{l} \text{the Logarithm of } \dots\dots p=350=2.544068 \\ \text{the Logarithm of } \dots\dots\dots t=9=0.954242 \end{array} \right. \end{array} \right. \end{array}$$

$$\text{The Sum is the Logarithm of } \dots\dots Gpt=4.276461$$

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Add { the Logarithm of $P=100=2.000000$
 { the Logarithm of $T=12=1.079181$

The Sum is the Logarithm of $PT=3.079181$

Then from the Logarithm of $Gp=4.276461$

Subduct the Logarithm of $PT=3.079181$

There rem. the Log. of $\frac{Gp}{PT}=g=15.75=1.197280$

Wherefore the Answer is $15.75l.=15l. 15s.$

6. Example II. If 36 Bushels will serve 24 Horses
 48 Days, how long will 126 Bushels serve
 96 Horses?

Here $P=24$, $T=48$, $G=36$; also $p=96$, and
 $g=126$; to find t .

Add { the Logarithm of $P=24=1.380211$
 { the Logarithm of $T=48=1.681241$
 { the Logarithm of $g=126=2.100370$

The Sum is the Log. of . . $PTg=5.161822$ from }

Add { the Logarithm of $G=36=1.556302$
 { the Logarithm of $p=96=1.982271$

The Sum is the Log. of $Gp=3.538573$ subf. }

Therefore the Log. of $\frac{PTg}{Gp}=t=42=1.623249$

The Answer, viz. 42 Weeks.

7. Example III. At the Rate of $4\frac{1}{2}l.$ per Cent. per
 Ann. what Principal will produce 35l. 15s.
 in 7.5 Months?

Here $P=100$, $T=12$, $G=4.5$; also $t=7.5$, and
 $g=35.75$; to find p .

Add

Add $\left\{ \begin{array}{l} \text{the Logarithm of } P=100=2.000000 \\ \text{the Logarithm of } T=12=1.079181 \\ \text{the Logarithm of } g=35.75=1.553276 \end{array} \right.$

The Sum is the Log. of . . . $PTg=4.632457$ from

Add $\left\{ \begin{array}{l} \text{the Logarithm of } G=4.5=0.653212 \\ \text{the Logarithm of } t=7.5=0.875061 \end{array} \right.$

The Sum is the Log. of . . . $Gt=1.528273$ sub.

Whence the Log. of $\frac{PTg}{Gt}=p=1271,114=3.104184$

Consequently, $1271,114l.=1271l.2s.3d.\frac{1}{2}$ is the Answer.

Note, These Theorems give the Answer absolutely, without regarding whether the Proportion be Direct or Inverse, or both together, as in Art. 6. Exam. II.

8. Of the Method of finding Mean Proportionals.

In order that a clear Notion of this most useful Problem may be had, I shall premise the following things.

1. Between two Square Numbers AA and BB , there will fall but one Mean Proportional AB ; that is, $A^2 : AB :: AB : B^2$. See *Eucl.* 8. 11.

2. Between two Cubic Numbers A^3 and B^3 , there will fall two Mean Proportionals A^2B and AB^2 ; that is, $A^3 : A^2B :: AB^2 : B^3$. See *Eucl.* 8. 12.

3. Between two Biquadrate Numbers A^4 and B^4 , there will fall three Mean Proportionals A^3B , A^2B^2 , and AB^3 ; that is, $A^4 : A^3B :: A^2B^2 : AB^3 :: AB^3 : B^4$.

4. Again, between two sursolid Numbers A^5 and B^5 , there will fall four Means, viz. A^4B , A^3B^2 , A^2B^3 , and AB^4 ; that is, $A^5 : A^4B :: A^3B^2 : A^2B^3 :: A^2B^3 : AB^4 :: AB^4 : B^5$, will be Proportionals. And thus the

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Number of Mean Proportionals will be always less by one than the *Index* of the Power of the given Extremes.

9. But the *common Ratio* of all such Series is $\frac{B}{A}$

$$\text{For } AA \times \frac{B}{A} = AB; \text{ and } AB \times \frac{B}{A} = BB.$$

$$\text{And } A^2 \times \frac{B}{A} = A^2B; \text{ and } A^2B \times \frac{B}{A} = AB^2; \text{ and}$$

$$AB^2 \times \frac{B}{A} = B^3, \text{ and so in the others. Here I have}$$

supposed A to be the least Number, and B the greatest, and the Series to begin from $A^2, A^3, \&c.$ But if B be less than A , and the Series begin from $B^2, B^3, \&c.$

then the *Ratio* of the Series will be $\frac{A}{B}$.

Now $\frac{B}{A}$ is Root of $\frac{B^2}{A^2}, \frac{B^3}{A^3}, \frac{B^4}{A^4}, \&c.$ and there-

fore from these Premises well understood, 'tis easy to conceive the Reason of the following Rule for finding Mean Proportionals, *viz.*

Rule { Subtract the Logarithm of the *least Term* from the Logarithm of the *greatest*, and divide the Remainder by a Number greater by one than the Number of Means desired; then add the Quotient to the Logarithm of the least Term (or subtract it from the Logarithm of the greatest) continually, and it will give the Logarithms of all the *Mean Proportionals* desired.

10. Example I. Let two Mean Proportionals be sought between 8 ($=A^2$) and 28 ($=B^2$).

The

Chap. IV. *by* LOGARITHMS.

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The Logarithm of $B^2=28=1.447158$

The Logarithm of $A^2=8=0.903090$

The Difference is $\frac{B^2}{A^2}=0.544068$

which divide by 3

The Quotient is $\frac{B}{A}=0.181356$

To which add the Log. of . . . $A^2=8=0.903090$

The Sum is the Log. of $\left\{ \begin{array}{l} A^2B=12,14=1.084446 \\ \text{1st Mean} \end{array} \right.$

To which add again $\frac{B}{A}=0.181356$

The Log. of the 2d Mean $AB^2=18,44=1.265802$

Wherefore the Proportional-
tionals are $\left\{ \begin{array}{l} 8 : 12,14 :: 18,44 : 28 \\ A^2 : A^2B :: AB^2 : B^2. \end{array} \right.$

11. Example II. Between 16 and 64 find five Mean Proportionals.

The Logarithm of $B^6=64=1.806180$

Subduct the Logarithm of . . . $A^6=16=1.204120$

There remains $\frac{B^6}{A^6}=0.602060$

$\frac{1}{6}$ of which is $\frac{B}{A}=0.100343$

To which add the Log. of $A^6=16=1.204120$

The Sum is the Log. of $\left\{ \begin{array}{l} A^6B=20,158=1.304463 \\ \text{the 1st Mean} \end{array} \right.$

To which add again $\frac{B}{A}=0.100343$

The Log. of the 2d Mean $A^4B^2=25,398=1.404806$

And

And thus you produce the Logarithms of

$$\left\{ \begin{array}{ll} 3^{\text{d}} \text{ Mean} & \dots\dots\dots A^3B^3=32=1.505150 \\ 4^{\text{th}} \text{ Mean} & \dots\dots\dots A^4B^4=40.317=1.605493 \\ 5^{\text{th}} \text{ Mean} & \dots\dots\dots AB^5=50.796=1.705836 \end{array} \right.$$

The Series therefore is this,

$$\{ 16. \quad 20,158. \quad 25,398. \quad 32. \quad 40,317. \quad 50,796. \quad 64. \\ \{ A^5. \quad A^4B. \quad A^3B^2. \quad A^2B^3. \quad AB^4. \quad A^5B. \quad B^5.$$

12. If it were required to find 364 *Mean Proportionals* between 0 and 1.06; or 0 and 1.05; or 0 and 1.04, &c. then $A^{365}=0$, and $B^{365}=1.06$; 1.05, &c. and so $\frac{B}{A} = B = \sqrt[365]{1.06}$, or $\sqrt[365]{1.05}$, &c.

Wherefore if you multiply the Logarithm of B, by 2, 3, 4, 5, &c. to 365, you will thereby obtain the *Mean Proportionals* required. And these will be the several *Amounts* of 1 l. and its *Interest* for each *Day* of the Year, at the Rates of 6, 5, 4 l. &c. *per Cent. per Annum, Compound Interest*. But more will be said of this hereafter.



CH A P. V.

SIMPLE INTEREST by LOGARITHMS.

1. **M**Y Design being only to acquaint the Reader with the *Theory* and *practical Uses* of *Logarithms*, and not to treat of the *Theory* of any other Art or Branch of Mathematical Science; it will be sufficient for me barely to mention the *Theorems* or *Rules*, on which the divers Parts of Learning (I shall treat of, depend, and shew how they are most conveniently wrote by *Logarithms*.

Of

Of SIMPLE INTEREST.

2. I have more than once serv'd myself with those excellent Theorems of Interest contrived by the late ingenious Mr. *Ward*; and shall once again make them subservient to my Design in this Place. In order to which

Let $\begin{cases} P = \text{any Principal or Sum put to Interest.} \\ R = \text{the Ratio of the Rate, per Cent. per Ann.} \\ T = \text{the Time the Principal continues at Interest.} \\ A = \text{the Amount of the Principal and Interest.} \end{cases}$

3. Then any three of these being given, the other may be found by the following Theorems.

Theor. I. $TRP + P = A$. Theor. II. $\frac{A}{TR+1} = P$.

Theor. III. $\frac{A-P}{TP} = R$. Theor. IV. $\frac{A-P}{RP} = T$.

Quest. I. What will 275*l.* 15*s.* amount to in 3½ Years, at 4½*l.* per Cent. per Annum?

Here $P=275,75$, $T=3,5$, $R=0,045$, to find A .
Theor. I.

Add $\begin{cases} \text{the Logarithm of } \dots P=275,75=2,440515 \\ \text{the Logarithm of } \dots T=3,5=0,544068 \\ \text{the Logarithm of } \dots R=0,045=8,653212 \end{cases}$

The Sum is the Log. of $PTR=43,431=1,637795$

To which add $\dots P=275,75$

The Sum is $\left. \begin{matrix} \text{the Amount} \end{matrix} \right\} PTR + P = A = 319,181 = 319*l.* 3*s.* 7*d.*$

Quest. II. What Principal or Sum being put to Interest, will amount to, or raise a Stock of 319*l.* 3*s.* 7*d.* in 3½ Years, at the rate of 4½*l.* per Cent. per Annum?

S

The

The Log. of the *Amount* $A = l. 319,181 = 2.504036$

Add { the Log. of the Time ... $T = 3,5 = 0.544068$
 { the Log. of the Rate $R = 0.045 = 8.653212$

The Sum is the Log. of ... $TR = 0.1575 = 9.197280$

Then the Log. of ... $TR + 1 = 1.1575 = 0.063521$
 Subtracted from the Log. } $P = 275,75 = 2.440515$
 of the Amount leaves }

Wherefore the *Principal* } $l. 275,75 = 275l. 15s.$
 required is }

Quest. III. At what Rate *per Cent.* &c. will 275l. 15s.
 amount to 319l. 3s. 7d. in $3\frac{1}{2}$ Years?

Here $A - P = 319,181 - 275,75 = 43.431$.

The Logarithm ... $A - P = 43.431 = 1.637800$

Add { the Logarithm of ... $P = 275,75 = 2.440515$
 { the Logarithm of ... $T = 3,5 = 0.544068$

The Sum is the Logarithm of PT ... $= 2.984583$

Which subd. from the Log. } $R = 0,045 = 8.653212$
 $A - P$, there will remain }

Quest. IV. In what Time will 275l. 15s. raise a
 Stock of 319l. 3s. 7d. at the Rate of $4\frac{1}{2}l.$
per Cent. per Annum?

Add { the Log. of the Prin. $P = 275,75 = 2.440515$
 { the Log. of the Ratio $R = 0,045 = 8.653212$

The Sum is the Log. of $PR = 1.093727$

Which subtract from } $A - P = 43,431 = 1.637800$
 the Log. of }

There

There will remain the } $\frac{A-P}{RP} = T = 3.5 = 0.544073$
 Log. of }
 Therefore the Answer is $3\frac{1}{2}$ Years.

4. Of ANNUITIES, &c. in ARREARS.

Put $\left\{ \begin{array}{l} U = \text{the Annuity, Pension, or yearly Rent.} \\ T = \text{the Time of its Continuance unpaid.} \\ R = \text{the Ratio of the Rate of Interest.} \\ A = \text{the Amount of the Annuity and its Interest.} \end{array} \right.$
 $\frac{2}{R} - 1 = x$

Then the Theorems for finding each of those Particulars, are as follows.

Theor. I. $\frac{TTU-TU}{2} \times R : + TU = A$,

Theor. II. $\frac{2A}{TTR-TR+2T} = U$.

Theor. III. $\frac{2A-2TU}{TTU-TU} = R$.

Theor. IV. $\sqrt{\frac{2A}{RU}} + \frac{x}{4} : -\frac{1}{2}x = T$.

Question I. If 250*l.* yearly Rent (Pension, &c.) be forborn or unpaid 7 Years, what will it amount to in that time at 6*l.* per Cent. for each Payment as it becomes due?

Here $U=250$, $R=0.06$, $T=7$; to find A , per Theor. I.

Add $\left\{ \begin{array}{l} \text{the Logarithm of } \dots U=250=2.397940 \\ \text{the Logarithm of } \dots T=7=0.845098 \end{array} \right.$

The Sum is the Log. of $\dots TU=1750=3.243030$
 add the Logarithm of $\dots T=7=0.845098$

The Sum is the Log. of $TTU=12250=4.088128$

Then the Log. of $\frac{TU-TU}{2} = 5250 = 3.720159$
 to which add the Log. of $R = 0.06 = .8.778151$

The Sum is the Log. of $\frac{TU-TU}{2} \times R = 315 = 2.498310$

To which add $TU = 1750$

The Sum is $\frac{TU-TU}{2} \times R : + TU = A =$ } 2065*l.* the Anf.

Note, if these Payments be made

Quarterly, } $\frac{1}{4}R, \frac{1}{4}U$, and $4T$.
 Half-yearly, } $\frac{1}{2}R, \frac{1}{2}U$, and $2T$.
 Three-quarterly, } $\frac{3}{4}R, \frac{3}{4}U$, and $\frac{3}{4}T$.

Quest. II. What *Year-Rent, Penson, &c.* being forborn or unpaid seven Years, will raise a Stock of 2065*l.* allowing 6 *per Cent. per Annum*, for each Payment as it becomes due?

Here $A = 2065$, $T = 7$, $R = 0.06$; to find U , *per Theor. II.*

Add { the Logarithm of $T = 7 = 0.845098$
 { the Logarithm of . . . $R = 0.06 = .8.778151$

The Sum is the Log. of . . . $TR = 0.42 = .9.623249$
 to which add the Log. of $T = 7 = 0.845098$

The Sum is the Log. of . . . $TTR = 2.94 = 0.468337$
 From which subduct . . . $TR = 0.42$

There remains . . . $TTR - TR = 2.52$
 to which add $2T = 14$

the Sum is $TTR - TR + 2T = 16.52 = 1.218010$
 the Logarithm of $2A = 4130 = 3.615950$

The

The Difference of these Logarithms is $U=250l. = 2.397940$, which is the Annuity sought.

Quest. III. If 250*l.* Yearly Rent, &c. being forborn 7 years, will amount to 2065*l.* allowing simple Interest for each Payment as it becomes due, what must the Rate of Interest be *per Cent. per Annum*?

Add { the Logarithm of $U=250=2.397940$
 { the Logarithm of $T=7=0.845098$

The Sum is the Log. of . . . $UT=1750=3.243030$
 to which add the Log. of . . . $T=7=0.845098$

The Sum is the Log. of . $TTU=12250=4.088128$
 from which subduct $TU=1750$

there remains $TTU-TU=10500=4.021189$
 the Logarithm of $2A-2TU=630=2.799340$

Subduct the former from the }
 latter, there will remain } $R=0.06=.8.778151$

Wherefore, as $1l. : 0.06l. :: 100l. : 6l.$ the Rate required.

Quest. IV. In what time will 250*l.* Yearly-Rent, raise a Stock of 2065*l.* allowing 6*l. per Cent.* &c. for the Forbearance of each Payment as it becomes due?

Here $U=250$, $A=2065$, $R=0.06$, $\frac{2}{R}-1=\frac{2}{0.06}$
 $-1=32, x=x$; to find T , *per* Theor. IV.

Add { the Logarithm of $U=250=2.397940$
 { the Logarithm of $R=0.06=8.778151$

The

The Sum is the Logarithm of $RU=15=1.176091$
 which subf. from the Log. of $2A=4130=3.615950$

there remains $\frac{2A}{RU} = 275,3 = 2.439859$

Add { the Logarithm of $x=32,3=1.509658$
 { the Logarithm of $\frac{1}{x}=8.083=0.907594$

The Sum is the Log. of $\frac{xx}{4} = 261,3605 = 2.417252$

to which add $\frac{2A}{RU} = 275,3333$

the Sum is . . $\frac{2A}{RU} + \frac{xx}{4} = 536,6938 = 2.729724$

Half that Log. } $\sqrt{\frac{2A}{RU} + \frac{xx}{4}} = 23,166\beta = 1.364862$
 is the Log. }

from which deduct $\frac{1}{x} = 16,166\beta$

there will remain $\sqrt{\frac{2A}{RU} + \frac{xx}{4}} : - \frac{1}{x} = T = 7$, the
 Number of Years required.

5. Of the PRESENT WORTH of ANNUITIES, PENSIONS, &c.

Here U, P, R, T, are used to denote the *Annuity*,
present Worth, *Ratio of the Rate of Interest*, and
Time, as in the former Articles ; here also let $\frac{2}{R} -$
 $\frac{2P}{U} - 1 = x$; then the Theorems for Operation are
 as follows ;

Theor. I. $\frac{TTR - TR + 2T}{2TR + 2} \times U = P = \text{Present Worth.}$

Theor. II. $\frac{TR + 1}{TTR - TR + 2T} \times 2P = U, = \text{Annuity, \&c.}$

Theor.

Theor. III. $\frac{2P-2TU}{TU-TU-2PT} = R =$, Ratio of the Rate.

Theor. IV. $\sqrt{\frac{2P}{RU} + \frac{xx}{4}} \pm \frac{1}{2}x = T$, = the Time.

Quest. I. What is the *present Worth* of 75*l.* Yearly Rent, to continue 9 Years, at 6 *per Cent.* &c.

Here $U=75$, $T=9$, $R=0.06$, to find P , *per* Theor. I.

Add { the Logarithm of $T=9=0.954242$
 { the Logarithm of $R=0.06=.8.778151$

The Sum is the Log. of $TR=,54=.9.732393$

Also the Logarithm of . . $TTR=4,86=0.686635$
 subtract $TR=,54$

there remains $TTR-TR=4.32$
 to which add $2T=18.$

The Sum is $TTR-TR+2T=22,32=1.348694$
 the Logarithm of . . . $2TR+2=3.08=0.488551$

the Difference is $=0.860143$
 To which add the Log. of . . . $U=75=1.875061$

The Sum is the Logarithm of

$\frac{TTR-TR+2T}{2TR+2} \times U = P = 543,506 = 2.735204$

Wherefore the present Worth is 543*l.* 10*s.* 1*d.* $\frac{1}{2}$.

Quest. II. What Annuity, Pension, &c. may be purchased for 543*l.* 10*s.* 1*d.* $\frac{1}{2}$, to continue 9 Years, allowing to the Purchaser 6 *per Cent.* *per An.* simple Interest?

Here $P=543,506$; $T=9$; $R=0,06$; to find U , *per* Theor. II,

From

From the Log. of $TR + 1 = 1.54 = 0.187521$
 subduct the }
 Log. of } $TTR - TR + 2T = 22.32 = 1.348694$

to the Difference $.8.838827$
 add the Logarithm of $2P = 1087,012 = 3.036234$

The Sum is the Logarithm of

$$\frac{TR+1}{TTR-TR+2T} \times 2P = U = 75 = 1.875061$$

That is, $75l.$ *per Annum*, is the Annuity, $\text{£}c.$ sought.

Quest. III. If $543l.$ $10s.$ $1d.$ $\frac{1}{2}$ ready Money, will purchase an Annuity, Lease, $\text{£}c.$ of $75l.$ *per Annum*, to continue 9 Years; *Quere* the Rate of Interest *per Cent.* &c.?

Here $P = 543,506$; $U = 75$; $T = 9$; to find R , *per Theor.* III.

From the Log. of $2P - 2TU = 262,988 = 2.419936$
 subd. the }
 Log. of } $TTU - TU - 2PT = 4383,108 = 3.641782$

the Difference is the Log. of $R = 0.06 = .8.778154$

Wherefore, as $1l. : 0,06l. :: 100l. : 6l.$ the Rate required.

Quest. IV. If for $543l.$ $10s.$ $1d.$ $\frac{1}{2}$ I purchase an Annuity, Pension, $\text{£}c.$ of $75l.$ *per Annum*; *Quere*, how long I may enjoy it, at the Rate of 6 *per Cent.* &c. Interest?

Here $P = 543,506$, $U = 75$, $R = 0,06$; to find T , *per Theor.* IV.

From the Log. of . . . $2P = 1087,012 = 3.036234$
 subduct the Log. of $U = 75 = 1.875061$

the Diff. is the Log. of . . $\frac{2P}{U} = 14,493 = 1.161173$

Which

which subtract from $\dots \frac{2}{R} - 1 = 32,33X$

there remains $\frac{2}{R} - \frac{2P}{U} - 1 = x = 17,840 = 1.251395$

add the Log. of $\dots \frac{1}{2}x = 4.46 = 0.649335$

the Sum is the Log. of $\dots \frac{1}{2}xx = 79,5664 = 1.900730$

to which add $\dots \frac{2P}{RU} = 241,558$

the Sum is $\frac{2P}{RU} + \frac{xx}{4} = 321,1244 = 2.506659$

half which is } the Log. of $\sqrt{\frac{2P}{RU} + \frac{xx}{4}} = 17,919 = 1.253329$

subtract $\dots \frac{1}{2}x = 8,92$

there remains $\sqrt{\frac{2P}{RU} + \frac{xx}{4}} - \frac{1}{2}x = T = 8,999 = 9,$

the Years sought.

6. If the Question be of Annuities, &c. in Reversion, you must find the *Amount* of the *Purchase-Money* to the time of Commencement (together with its Interest) by Quest. I. Art. 3. and make that *Amount* the Sum for the Purchase; and then proceed as in the Questions of this last Article. These are all the *fundamental* or *original* Cases of *simple Interest*.



C H A P. VI.

COMPOUND INTEREST by LOGARITHMS.

1. **T**HE former Questions of *simple Interest* might be resolved by the Rules of *vulgar Arithmetic*; and I have only there shewn, they may also be (and that in many Cases, *most conveniently*) wrought by *Logarithms*. But in the present Affair of *compound Interest*, the Use of *Logarithms* is *absolutely necessary*; no other Method of the Solution of Questions in compound Interest being equal to it in Extent and Perfection. And consequently the young Student in Arithmetic is under an indispensable Obligation to be acquainted with this most excellent and useful Branch of the Science.

2. I shall here also proceed according to the Theorems of Mr. *Ward*; and therefore

Put $\left\{ \begin{array}{l} P = \text{the Principal put to Interest.} \\ t = \text{the Time of its Continuance.} \\ A = \text{the Amount of the Principal and Interest.} \\ R = \text{the Amount of 1 l. and its Interest for one Year, at any given Rate.} \end{array} \right.$

Note, you find $100 : 106 :: 1 : 1,06 = R$, at 6 per Ct.

R thus, $100 : 105 :: 1 : 1,05 = R$, at 5 per Ct.

The Amounts of 1 l. $\left\{ \begin{array}{l} 1. \ 2. \ 3. \ 4. \ 5, \ \&c. \text{ Years.} \\ \text{in several Years} \quad \{ R, R^2, R^3, R^4, R^5, \ \&c. \text{ Am. of 1 l.} \end{array} \right.$

Therefore $R^t =$ the Amount of 1 l. in the Time t for the Rate agreed on; this being premised, the Theorems for *Compound Interest* are as follow.

3. Theor. I. $PR^t = A =$ the Amount.

Theor.

Theor. II. $\frac{A}{R} = P$ = the Principal.

Theor. III. $\frac{A}{P} = R$ $\left\{ \begin{array}{l} = \text{the Time.} \\ = \text{the Amount of 1 l.} \end{array} \right.$

By these Theorems the several Questions of Compound Interest are answered most expeditiously by the *Logarithms* in the Manner following.

4. Quest. I. What will 275 l. 15 s. amount to in $3\frac{1}{2}$ Years, at $4\frac{1}{2}$ l. per Cent. per Annum; Compound Interest?

Here $P=275,75$; $R=1,045$; $t=3,5$; to find A, per Theor. I.

The Logarithm of $R=1,045=0.019116$
multiply by the Time $t=3,5$

The Prod. is the Log. of $R^t=1,1665=0.066906$
To which add the Log. of $P=275,75=2.440515$

The Sum is the Log. of $PR^t=A=321,68=2.507421$

So the Amount sought is 321 l. 13 s. 7 d. which is more than the Amount by simple Interest by 2 l. 10 s. See Quest. I. Art. 3. of the foregoing Chapter.

Quest. II. What Principal or Sum being put to Use at $4\frac{1}{2}$ l. per Cent. Compound Interest, will amount to 321 l. 13 s. 7 d. in $3\frac{1}{2}$ Years?

Here $A=321,68$; $R=1,045$; $t=3,5$; to find P, per Theor. II.

From the Logarithm of $A=321,68=2.507421$
Subduct the Logarithm of $R^t=1,1665=0.066906$

The Diff. is the Log. of $\frac{A}{R^t}=P=275,75=2.440515$
therefore the Principal sought is 275 l. 15 s.

Quest. III. At what rate *per Cent.* &c. Compound Interest, will 275*l.* 15*s.* raise a Stock, or amount to 321*l.* 13*s.* 7*d.* in 3½ Years?

Here $A=321,68$; $P=275,75$; $t=3,5$; to find R ,
per Theor. III.

From the Logarithm of $A=321,68=2.507421$
Subtract the Log. of $P=275,75=2.440515$

The Difference is the Log. of $R=1.045=0.066906$
which divide by the Time $t=3,5$

The Product is the Log. of $R=1.045=0.019116$

Then as $1 : 0,045 :: 100 : 4,5=4\frac{1}{2}\%$ *per Cent.* the
Rate required.

Quest. IV. In what Time will 275*l.* 15*s.* raise a Stock of 321*l.* 13*s.* 7*d.* at the rate of 4½*l.* *per Cent.* Compound Interest?

Here $P=275,75$; $A=321,68$; $R=1.045$; to find t ,
per Theor. IV.

From the Logarithm of $A=321,68=2.507421$
Subduct the Logarithm of $P=275,75=2.440515$

The Diff. is the Log. of $R=1.045=0.066906$

Then the Log. of $1.045=0.019116$ $0.066906(3,5=t.$

57348

95580

95580

.....

Thus the Answer is 3 Years and 6 Months.

Note. As the *Amount* of any *Sum* and its *Interest* is greater at *Compound Interest* than at *Simple Interest* for any *Time* above a Year; so it is less at *Compound*

pound than *Simple Interest* for any *Time* less than a Year, as the Learner may easily prove by the Theorems foregoing.

5. Of ANNUITIES, &c. in ARREARS.

Theor. I. $\frac{UR^t - U}{R - 1} = A =$ the *Amount* of any Sum.

Theor. II. $\frac{RA - A}{R^t - 1} = U =$ the *Annuity, Pension, &c.*

Theor. III. $\frac{RA + U - A}{U} = R^t =$ the *Time*.

Theor. IV. $\frac{A - U}{U} = \frac{A}{U} R - R^t =$ the *Amount of 1 l.*

6. Quest. I. If 30 l. yearly Rent be forborn 9 Years, what will it amount to at 5 l. per Cent. &c. Compound Interest?

Here $U=30$, $t=9$, $R=1.05$; to find A ; per Theor. I.

The Logarithm of $R=1.05=0.021189$
multiply by the Index 9

The Product is $R^t=1.516=0.180701$
To which add the Log. of . . . $U=30=1.477121$

The Sum is the Log. of $UR^t=45.48=1.657822$

Subduct $U=30$

There remains . . . $UR^t - U=15.48=1.189771$
Subtract the Log. of $R-1=0.05=8.698970$

The Diff. is the } $\frac{UR^t - U}{R - 1} = A = 309.6=2.490801$
Log. of }

The Amount therefore is 309 l. 12 s. for Answer.

Quest.

Quest. II. What Annuity, &c. will raise a Stock (or amount to) 309*l.* 12*s.* being forborn or unpaid 9 Years, at 5*l.* per Cent. &c. Compound Interest?

Here $A=309,6$; $t=9$; $R=1,05$; to find U , per Theor. II.

Add $\left\{ \begin{array}{l} \text{the Logarithm of } \dots A=309,6=2.490801 \\ \text{the Logarithm of } \dots R=1,05=0.021189 \end{array} \right.$

The Sum is the Log. of $\dots AR=325,08=2.511990$

Therefore the Log. of $AR-A=15,48=1.189771$

Subduct the Log. of $\dots R-1=0.516=9.712650$

The Diff. is the $\left\{ \begin{array}{l} \text{Log. of } \dots \frac{AR-A}{R-1} = U = 30=1.477121 \end{array} \right.$

That is 30*l.* per Annum is the Annuity sought.

Quest. III. In what time will 30*l.* per Annum amount to 309*l.* 12*s.* at 5*l.* per Cent. Compound Interest?

Here $U=30$; $A=309,6$; $R=1,05$; to find t , per Theor. III.

Add $\left\{ \begin{array}{l} \text{the Logarithm of } \dots A=309,6=2.490801 \\ \text{the Logarithm of } \dots R=1,05=0.021189 \end{array} \right.$

The Sum is the Log. of $\dots AR=325,08=2.511990$

Subtract $\dots A-U=279,6$

There remains $AR-U-A=45,48=1.657822$

Subtract the Log. of $\dots U=30=1.477121$

The Diff. is the Log. of $\dots R=1,05=0.180701$

Then the Log. of $1,050.02=1.1890.180701(9=t.180701$

Answer, 9 Years.

Quest.

Quest. IV. At what Rate *per Cent.* Compound Interest will 30*l.* Yearly Rent amount to 309*l.* 12*s.* in 9 Years?

Here $U=30$; $A=309,6$; $t=9$; to find R , *per* Theor. IV.

From the Log. of . . . $A-U=279,6=2.446537$
Subduct the Logarithm of . . . $U=30=1.477121$

The Diff. is the Log. of $\frac{A-U}{U}=9,32=0.969416$

Again from the Log. of . . . $A=309,6=2.490801$
Subtract the Logarithm of . . . $U=30=1.477121$

The Diff. is the Log. of $\frac{A}{U}=10,32=1.013680$

Therefore the said Theorem IV. is reduced to this Equation, *viz.* $9,32=10,32R-R^9$. Now this Equation may be easily resolv'd by a *converging Series*; or still easier by the Tables of the Amount of 1*l.* for several Years successively; for therein R^9 , or the Amount of 1*l.* in 9 Years may be tried for the several Rates *per Cent.* in the Tables. For instance, suppose I pitch upon the Table at 5*l. per Cent.* Compound Interest, then I find $R^9=1.551328$, &c. Wherefore $9,32+1.551328=10.871328=10.32R$; therefore $10,32)10.871328 (=1,05$. Now since 1*l.* : 0,05*l.* :: 100*l.* : 5*l.* the very Rate *per Cent.* assumed in the Tables, it will follow, that is the true Rate sought.

Note, This Question may be resolv'd by the Rule of *false Position*; which I leave to the Learner's Exercise.

7. Of the PRESENT WORTH of ANNUITIES,
&c. at Compound Interest.

Here P denotes the *present Worth* of any Annuity,
Lease, &c. and the rest of the Letters as before,
Art. 6. Then the Theorems are as follows.

Theor. I. $\frac{U - \frac{U}{R^t}}{R - 1} = P = \text{the present Worth.}$

Theor. II. $\frac{PRR^t - PR^t}{R^t - 1} = U = \text{the Annuity, \&c.}$

Theor. III. $\frac{U}{P + U - PR} = R^t = \text{the Time.}$

Theor. IV. $\frac{U}{P} = \frac{U}{P} R^t + R^t - R^{t+1} = \text{the Value of R.}$

Quest. I. What is 30*l.* *per Annum* to continue 9 Years,
worth in ready Money, abating 5*l.* *per Cent.*
&c. Compound Interest to the Purchaser?

Here $U=30$, $t=9$; $R=1,05$; to find P, *per*
Theor. I.

The Logarithm of $R=1,05=0.021189$
Multiply by the Time 9

The Prod. is the Log. of $R^9=1.516=0.180701$
which subtract from the Log. of $U=30=1.477121$

The Diff. is the Log. of $\frac{U}{R^t}=19,788=1.296420$
which take from . . . $U=30$

there remains . . . $U - \frac{U}{R^t}=10.212=1.009111$
Subduct the Log. of . . . $R-1=0.05=.8.698970$

The Diff. is the Log. of $P=204,24=2.310141$
The present Worth is 204*l.* 4*s.* 9*d.* $\frac{1}{2}$, the Answer.

Quest.

Quest. II. What Annuity, &c. to continue 9 Years
may one purchase for 204*l.* 4*s.* 9*d.* $\frac{1}{2}$. abating
5*l.* *per Cent.* &c. to the Purchaser ?

Here $P=204,24$; $t=9$; $R=1,05$; to find U , *per*
Theor. II.

Add $\left\{ \begin{array}{l} \text{the Logarithm of } \dots R=1,516=0,180701 \\ \text{the Logarithm of } \dots R=1,05=0,021189 \\ \text{the Logarithm of } \dots P=204,24=2,310141 \end{array} \right.$

The Sum is the Log. of $PRR^t=325,11=2,512031$

Then from $\dots PRR^t=325,11$

Subtract $\dots PR=309,628$

there remains $PRR^t-PR=15,482=1,189799$

Subduct the Log. of $R-1=0,516 = .9,712650$

The Diff. is the Log. of $\dots U=30=1,477149$

Therefore the Annuity sought is 30*l.* *per Annum.*

Quest. III. What time may one enjoy an Annuity,
&c. of 30*l.* *per Annum*, for 204*l.* 4*s.* 9*d.* $\frac{1}{2}$.
ready Money, abating to the Purchaser 5*l.*
per Cent. &c. ?

Here $P=204,24$; $U=30$; $R=1,05$; to find t ,
per Theor. III.

Add $\left\{ \begin{array}{l} \text{the Logarithm of } \dots P=204,24=2,310141 \\ \text{the Logarithm of } \dots R=1,05=0,021189 \end{array} \right.$

The Sum is the Log. of $PR=214,452=2,331330$

Then $\dots P \pm U=234,24$

The Diff. $\dots P \pm U-PR=19,788=1,296402$
which subtract from the Log. of $U=30=1,477121$

The Diff. is the Log. of $R-1=0,516=0,180719$
 U Then

146 . Compound Interest

Then the Log. of $1.05 = 0.021189$ $0.180719 (= 9 = t.$
 180701

Answer, 9 Years.

..... 18

Quest. IV. Suppose an Annuity, &c. of 30*l.* *per Annum*, to continue 9 Years, be sold for 204*l.* 4*s.* 9*d.* ready Money, what rate of Interest hath the Purchaser allow'd *per Cent.* &c. for his Money?

Here $P = 204.24$; $U = 30$; $t = 9$; to find R , *per Theor. IV.*

From the Log. of the Annuity $U = 30 = 1.477121$
 Subduct the Log. of $P = 204.24 = 2.310141$

The Diff. is the Log. of $\frac{U}{P} = 0.14679 = .9.166980$

Then the said Theor. IV. is reduced to this Equation, *viz.* $0.14679 = 0.14679R^9 + R^9 - R^{10}$; which is to be solv'd by the Method of Infinite Series. But for those who understand not that Method, this Question is much better answer'd by the *Rule of False* ; or yet easier by the Tables of Compound Interest. See a compleat Set, with their Uses, in my *System of Decimal Arithmetic*, published for Mr. Noon, at the *White Hart*, in *Cheapside*, *London*.

8. Of Purchasing FREEHOLD ESTATES.

Estates in Fee Simple (which are such as we commonly call *Freehold* or *Real Estates*) being purchased *for ever*, or without *Reversion* ; 'tis plain, that in the foregoing Theorems for finding the present Worth of Annuities, &c. if those Terms wherein $t (= \text{the Time})$ is found, be made to vanish, as being *Infinite* ; the said Theorems will be reduced

reduced to such as suit the *present Case*, and are as follows.

Theor. I. $\frac{U}{R-1} = P$, the *Present Worth*, or *Purchase-Money*.

Theor. II. $PR - P = U$, the *Annuity*, or *Estate per Annum*.

Theor. III. $\frac{P+U}{P} = R$, the *Amount of 1l. at the given Rate*.

Question. What must be given for a *Freehold Estate* of *50l. per Annum*, allowing the Buyer *6l. per Cent. Compound Interest* for his Money?

Here $U=50$; $R=1,06$; to find P , per Theor. I. †

The Logarithm of . . . $U=50=1.698970$
Subduct the Log. of $R+1=0,06=.8778151$

The Diff. is the Log. of $\frac{U}{R-1} = P = 833,3 = 2.920819$

The *Purchase-Money*, then, is *833l. 6s. 8d.* If it had been at the *Rate of 5l. per Cent. Compound Interest*, the said *Estate* would be worth *1000l. present Money*; which is equal to twenty times the *Yearly Rent*: and therefore the general Method of buying *Freehold Estates*, is by paying 20 or 25 times the *Yearly Rent*; which is commonly called 20 or 25 *Years Purchase*.

The other two Theorems are wrought in the same Manner for U and R ; which, being so very easy, need no Example.





C H A P. VII.

Of VULGAR FRACTIONS by LOGARITHMS.

1. **T**O find the *Logarithm* of a *Vulgar Fraction*, this is the

Rule { From the Logarithm of the *Numerator*;
Subtract the Logarithm of the *Denominator* ;
the Remainder is the Log. of the Fraction.

Example I. What is the Log. of the Fraction $\frac{51}{73}$?

From the Logarithm of . . . 51 = 1.707570
Subtract the Logarithm of . . . 73 = 1.863323

there remains the Log. of $\frac{51}{73} = 0.69862 = 9.844247$

Exam. II. What is the Logarithm of $\frac{1}{17.36}$?

From the Logarithm of . . . 1 = 0.000000
Subtract the Logarithm of . . . 1736 = 3.244524

there remains the }
Log. of $\frac{1}{1736} = ,00056947 = 6.755476$

Exam. III. What is the Logarithm of $\frac{100}{1357}$?

From the Logarithm of . . . 100 = 2.000000
Take the Logarithm of . . . 1357 = 3.132580

there remains the Log of $\frac{100}{1357} = ,073692 = 8.867420$

Exam. IV. What is the Logarithm of $\frac{3273}{1000000}$?

The

The Logarithm of 5973=3.776192
 From which take the Log. of 1000000=6.000000

there rem. the Log. of $\frac{5973}{1000000}=,005973=7.776192$

2. If the Fraction be a *mix'd one*, it must be reduced to an *improper Fraction*; and then proceed with it as before.

Example I. What is the Logarithm of $13\frac{1}{7}$?

Here $13\frac{1}{7}=\frac{96}{7}$;

Therefore from the Logarithm of 96=1.982271
 take the Logarithm of 7=0.845098

there remains the Log. of $13\frac{1}{7}=13.714=1.137173$

Exam. II. What is the Logarithm of $193\frac{11}{73}$?

Here $193\frac{11}{73}=\frac{14140}{73}$;

Then from the Logarithm of 14140=4.150449
 take the Logarithm of 73=1.863323

there rem. the Log. of $193\frac{11}{73}=193.69862=2.287126$

3. If the *mix'd Fraction* consists of *large Numbers*, it may be most easily reduc'd by Logarithms, thus; supposing the Example be $2145\frac{17}{59}$;

To the Log. of the *Integral Part* 2145=3.331427
 Add the Log. of the Denominator 59=2.770115

The Sum is the Log. of . . 1263405=6.101542
 to which add the Numerator . . 57

Then you have $\left\{ \begin{array}{l} \text{the new Num. } 1263462 \\ \text{the Denomin. } 57 \end{array} \right\}$ the Fract. sought.

4. To multiply *Vulgar Fractions* by Logarithms, add the Logarithms of the Numerators for the Logarithm of a new Numerator; and the Logarithms of the Denominators for the Logarithm of a new Denominator.

Example. What is the Product of $\frac{35}{173}$ into $\frac{7}{9}$?

Add the Log^s. of the Numerators { $35 = 1.544068$
 $7 = 0.845098$

The Log. of the new Numerator $245 = 2.389166$

Add the Log^s. of the Denominat^s. { $173 = 2.238046$
 $9 = 0.954242$

The Log. of the new Denomin. $1557 = 3.192288$

Therefore $\frac{35}{173} \times \frac{7}{9} = \frac{245}{1557}$, the fractional Product required.

5. The Logarithm of the Product of several Fractions multiplied into one another is thus obtain'd; viz. Add the Logarithms of all the Numerators and the *Arithmetical Complements* of the Logarithms of all the Denominators together; the Sum is the Logarithm required.

Example. What is the Logarithm of $\frac{87}{97} \times \frac{33}{46} \times \frac{7}{9}$?

Add together { the Logarithms of { $87 = 1.939519$
 $33 = 1.518514$
 $3 = 0.477121$
 { the Arith. Comp. of the { $97 = 8.014228$
 Logarithms of { $46 = 8.337242$
 $7 = 9.154902$

The Log. of $\frac{87}{97} \times \frac{33}{46} \times \frac{7}{9} = \frac{8611}{31234} = 27639 = 9.441526$

6. To divide Vulgar Fractions by Logarithms, do thus; Add the Logarithm of the *Denominator* of the *Divisor* to the Logarithm of the *Numerator* of the *Dividend*; the Sum is the Logarithm of a *new Numerator*; and the Sum of the Logarithms of the other *two Factors*, is the Logarithm of the new *Denominator* of the *Quotient* required.

Example. Divide $\frac{245}{1557}$ by $\frac{7}{1557}$.

Add the Logarithms of { $\begin{array}{r} 9=0.954242 \\ 245=2.389166 \end{array}$

The Log. of the new Numerator $\begin{array}{r} 2205=3.343408 \end{array}$

Add the Logarithms of { . . . $\begin{array}{r} 7=0.845098 \\ 1557=3.192288 \end{array}$

The Log. of the new Denom. $\begin{array}{r} 10899=4.037386 \end{array}$

Therefore $\frac{7}{1557} \div \frac{245}{1557} (= \frac{2205}{10899} = \frac{35}{173}$. See Art. 4.

7. The Logarithm of this Quotient may be found by one Addition, in like manner as directed in Art. 5. thus;

Add together { $\begin{array}{l} \text{the Logarithm of } \left\{ \begin{array}{r} 9=0.954242 \\ 245=2.389166 \end{array} \right. \\ \text{the Arith. Comp. of } \left\{ \begin{array}{r} 7=9.154902 \\ 1557=6.807712 \end{array} \right. \\ \text{the Logarithms of } \end{array}$

The Log. of . . $\frac{7}{1557} \div \frac{245}{1557} (= \frac{35}{173} = .20231 = 9.306022$

8. The *Extraction* of the *Roots* of *Vulgar Fractions* by Logarithms is thus performed. Divide the Logarithm both of the Numerator and Denominator of the given Fraction by the *Index* of the *Root*; the *Quotients* shall be the Logarithms of the *Numerator* and *Denominator* of the *Fractional Root* required.

Examp.

Examp. I. What is the *Square Root* of the Fraction $\frac{1849}{10201}$?

Divide the Logarithm of 1849=3.266937
by the Index of the Root 2

The Log. is the Num. of the Root $43=1.633468$

Again, divide the Log. of . . . 10201=4.008643
by 2

The Log. of the Denom. of the Root $101=2.004321$

Therefore $\sqrt{\frac{1849}{10201}} = \frac{43}{101}$ the Root required.

Example II. What is the *Cube Root* of $86\frac{291}{343}$?

This reduced to an improper Fraction, is $\frac{29791}{343}$;

Therefore the Logarithm of 29791=4.474085
 $\frac{1}{3}$ thereof is the Log. of the *new Num.* $31=1.491362$

Again, the Logarithm of 343=2.535294
 $\frac{1}{3}$ of which is the Log. of the *new Denom.* $7=0.845098$

Consequently $\sqrt[3]{86\frac{291}{343}} = \frac{31}{7} = 4\frac{3}{7}$, the Cube Root sought.

9. To find the *Logarithm* of the *Root* of any Fraction; add the Logarithm of the *Numerator* to the *Arithmetical Complement* of the Logarithm of the *Denominator*, and divide that Sum by the *Index* of the *Root*; the Quotient shall be the Logarithm sought.

Example I. What is the Logarithm of the *Square Root* of the Fraction $\frac{1849}{10201}$?

The

The Log. of the Numerator . . . 1849 = 3.266937
 Comp. Arith. of the Log. }
 of the Denom. } . . 1020 = 5.991357

The Sum is 9.258284
 which divide by the Index of the Root 2

The Quot. is }
 the Log. of } $\sqrt{\frac{1849}{1020}} = \frac{43}{101} = .42574 = .9.629147$

Exam. II. What is the Logarithm of the *Cube Root*
 of the mix'd Fraction $86\frac{293}{343}$, or its equal
 $\frac{29791}{343}$?

Add { the Logarithm of 29791 = 4.474085
 { the Ar. Com. of the Log. of 343 = 7.464706

The Sum is the Log. of 68 $\frac{293}{343}$ = 1.938791
 which divide by the *Index* of the Root . . 3

The Quot. is }
 the Log. of } $\sqrt[3]{68\frac{293}{343}} = 4\frac{1}{3} = 4.4285 = 0.646263$



C H A P. VIII.

DUODECIMAL ARITHMETIC *performed by* LOGARITHMS.

1. **S**INCE this kind of Arithmetic is so very *common*, and yet in the common Way so very *difficult*; I hope 'twill not be unacceptable to the *young Artificer* to be convinced with how much more Ease and Pleasure he may compute his Dimensions in this Way by the help of Logarithms. And as it is proper to reduce them first to *Decimals*, I have here sub-

X

joined

joined a Table shewing by Inspection the *Decimal Parts* of a Foot (in this Case the *Integer*) answering to any Number of *Primes*, *Seconds*, and *Thirds*; which are the Parts or Divisions of a Foot made use of in this kind of Mensuration.

2. *The Duodecimal Table.*

Duodecimals.	Decimal Parts.		
	Primes	Seconds	Thirds
1,	083333	,006944	,000578
2,	166666	,013888	,001157
3,	25	,020833	,001736
4,	333333	,027777	,002314
5,	416666	,034722	,002893
6,	5	,041666	,003472
7,	583333	,048611	,004051
8,	666666	,055555	,004629
9,	75	,0625	,005208
10,	833333	,069444	,005787
11,	916666	,076388	,006365

3. The foregoing Table is too easy to need Description, I mean for any Person concern'd in *Duodecimal Mensuration*; and therefore I shall proceed to exemplify and illustrate the Operations this Way by Logarithms in all the Rules of *Multiplication*, *Division*, *Involution*, and *Extraction* of Roots.

4. *Multiplication of DUODECIMALS by LOGARITHMS.*

Examp. I. Suppose a Plane be 9^f 10ⁱ in Length, and 8^f 8ⁱ in Breadth; *Quare* the Content or Area?

Add { the Log. . . of 9^f 10ⁱ = 9.8333 = 0.992704
 { the Log. of . . 8^f 8ⁱ = 8.6666 = 0.937849

The Sum is the Log. of . . . 85,222 = 1.930553
 That is, 85,222 Feet, = 85^f 2ⁱ 8ⁱⁱ. the Area required.

Examp.

Examp. II. What is the Product of $40^f. 9'. 10''$ by $11'. 9''$?

Add { the L. of $40^f. 9'. 10'' = 40.8194 = 1.610866$
 { the Log. of $11'. 9'' = 0.97916 = .9.990854$

The Sum is the Log. of $39,969 = 1.601720$

Therefore $39,969 = 39^f. 11'. 7''. 6'''$. the *Product*,
 or *Area* required.

Examp. III. What is 175 Feet $00'. 04''$. by $8'''$?

Add { the L. of $175^f. 0'. 4'' = 175.027 = 2.243107$
 { the Log. of $8''' = .004629 = .7.665487$

The Sum is the Log. of . . , $87111 = .9.908594$

Therefore $0,81111 = 0^f. 9'. 8''. 9'''$ = the *Area*
 fought.

Examp. IV. What is $17^f. 9'. 2''. 6'''$ by 6^f ?

Add { the L. of $17^f. 9'. 2''. 6''' = 17,76736 = 1.247417$
 { the Log. of $6 = 0.778151$

The Sum is the Log. of . . $106,60416 = 2.025568$

Thus $106.60416 = 106^f. 7'. 3''$. the *Area* fought.

Examp. V. What Number of *solid Feet* is in a *Cellar*
 $21^f. 2'$. long, $11^f. 10'. 8''$. broad, and $7^f. 3'$.
 deep?

Add { the Log. of . . $21^f. 2' = 21,1666 = 1.325659$
 { the Log. of $11^f. 10'. 8'' = 11,888 = 1.074109$
 { the Log. of $7^f. 3' = 7,25 = 0.860338$

The Sum is the Log. of . . . $1820,19 = 3.260106$

Therefore $1820,19 = 1820^f. 2'. 3''. 4'''$, the *Solidity*
 required.

5. DIVISION OF DUODECIMALS by LOGARITHMS.

Examp. I. What is $85^f. 2'. 8''$. divide by $8^f. 8'$?

The Log. of . . . $85^f. 2'. 8'' = 85,72 = 1.930553$
 Subduct the Log. of $8^f. 8' = 8,666 = 0.937849$

The Diff. is the Log. of . . . $9.8733 = 0.992704$

So that $9.8733 = 9^f. 10'$, the Answer.

Examp. II. What is $9'. 8''. 9'''$. divided by $8'''$?

From the Log. of $9'. 8''. 9''' = 0.87111 = 9.908594$
 Subtract the Log. of $8''' = 0.004629 = 7.665487$

The Diff. is the Log. of . . . $175,027 = 2.243107$

Therefore the Answer is $175,027 = 175^f. 0'. 4''$.

Examp. III. Divide $39^f. 11'. 7''. 6'''$. by $40^f. 9'. 10''$.

From the }
 Log. of } . . . $39^f. 11'. 7''. 6''' = 39.9687 = 1.601720$
 Subd. the Log. of $40^f. 9'. 10'' = 40.8194 = 1.610866$

The Diff. is the Log. . . of $0,97916 = 9.990854$

But $,97916 = 11'. 9''$. the Quotient required.

Examp. IV. Divide $106^f. 7'. 3''$. by 6.

From the L. of $106^f. 7'. 3'' = 106,60416 = 2.205568$
 Subtract the Log. of 6 = 0.878151

The Diff. is the Log. of $17,76736 = 1.247417$

Thus $17,76736 = 17^f. 9'. 2''. 6'''$. the Quotient sought.

6. INVOLUTION *of* DUODECIMALS *by* LOGARITHMS.

Examp. I. What is the Area of that *Square* whose Side is $12^f. 9'. 7''. 10'''$?

The Log. of $12^f. 9'. 7''. 10''' = 12.80439 = 1.107359$
Multiply by . . . 2

The Product is the Log. of $163.9524 = 2.214718$

Therefore $163.9524 = 163^f. 11'. 5''. 1'''$. the *Area* required.

Examp. II. What is the Solidity of a *Cube* whose Side is $1^f. 2'. 9''. 11'''$?

The Log. of $1^f. 2'. 9''. 11''' = 1.23553 = 0.091854$
which multiply by . . . 3

The Product is the Log. of $1.8774 = 0.273562$

Therefore $1.8774 = 1^f. 10'. 6''. 4'''$. the Solidity sought.

7. EXTRACTION *of* ROOTS *of* DUODECIMALS *by* LOGARITHMS.

Examp. I. What is the Side of that *Square* whose Area is $163^f. 11'. 5''. 1'''$?

The L. of $163. 11'. 5''. 1''' = 163.9524 = 2.214718$
Divide by . . . 2

The Quotient is the Log. of $12.80439 = 1.107359$

Thus $12.80439 = 12^f. 9'. 7''. 10'''$. the Side sought.

Examp. II. What is the *Side* of that *Cube*, whose *Solidity* is $1^f. 10'. 6''. 4'''$?

The

The Log. of 1. 10'. 6". 4''' = 1.8774 = 0.273562
 Divide by 3

The Quotient is the Log. of 1.23553 = 0.091854

But 1,23553 = 1^f. 2'. 9". 11''' the *Side* of the *Cube* required.

These few Examples abundantly shew with how much more *Ease*, *Brevity* and *Expedition* the Operations of *Duodecimals* are performed by *Logarithms*, than by the *ordinary Method*.



C H A P. IX.

The OPERATION of the common RULES of ARITHMETIC by INSTRUMENTS; viz. the LOGARITHMIC SCALE; and GUNTER'S LINE, with the COMPASSES, and on the SLIDING-RULE.

I. **H**AVING in the *Theory* shewn the *Nature* and *Construction* of the *Logarithmic Scale*, and *Gunter's Line*; I shall here briefly exemplify their *Uses* in the *Operation* of the common *Rules of Arithmetic* thereby; and in doing of this I shall observe this *Method*; first, to give the *Operation* by *Logarithms* in *Numbers*. Secondly, to perform the same by the *Logarithmic Scale*. Thirdly, to work the same Case on the *Gunter* with the *Compasses*; and fourthly, to do the same thing on the *Sliding-Gunter*. In this *Method*, the *Analogy* or *Agreement* between the *Numerical* and *Instrumental Operations* will more easily appear; and the *Nature* and *Reason* of the latter be much better understood by young *Learners*.

2. MULTIPLICATION.

Examp. I. Multiply 9 by 7.

First, by Logarithms.

Add {	the Logarithmi of	9=0.954242
	the Logarithm of	7=0.845098

The Sum is the Log. of the Prod. = 63 = 1.799340

3. Secondly, by the Logarithmic Scale.

(*Note.* If the smallest Divisions in the Line AB represent *Numbers*, the Logarithms begin from Cc in the Line Ae; if the middle Divisions in AB be *Numbers*, the Logarithms begin from G; but if the largest Divisions in AB be taken for *Numbers*, then the Logarithms begin from H, in the said Line Ae. And since the *smallest Divisions* are too *small*, and the *largest Divisions* too *large* for Examples, we must consequently chuse the *mean Divisions* in AB to represent the Numbers 1, 2, 3, 4, &c. or 10, 20, 30, 40, &c. or 100, 200, 300, &c. and so the Logarithms begin from G. Therefore) Set one Foot of the Compasses in G, and extend the other to the Logarithm of the Multiplier 7=ag, which you'll find to be Ga=845, and since Gb=954 is the Logarithm of the Multiplicand 9=bh; therefore with the same Extent Ga in the Compasses, set one Foot in b, the other will fall on d; therefore Gd is the Logarithm of the Product dm=63 in A B, the Number sought.

4. Thirdly, by the Gunter with Compasses.

Set one Foot of the Compasses in the Beginning of the Line at 1, and extend the other to 7; with that Extent in the Compasses set one Foot in 9, the other will fall on 63, the Product required.

Note.

Note. When the Numbers are *small* the *larger Divisions* may be used, as in the present Case; but if the Numbers be *large*, the *lesser Divisions* must be used.

5. Fourthly, by the *Sliding-Gunter*.

In this Case, there is one Line of Numbers on the *Rule*, and another on the *Slider*, both mark'd with N, at the End. And it is easy to conceive that by sliding one of these by the other, the same Effects are produced as before with the *Compasses*; that is, any Part of the Line on the *Rule* is transfer'd to, or compar'd with any other Part of the said Line by means of the sliding Line.

Therefore set 1 on the *Slider* to 7 in the Line on the *Rule*; then against 9 on the *Slider*, you find 63 on the *Rule*, and that is the *Product* sought.

6. *Examp. II.* By the *Gunter*. What is the *Product* of 27 by 18?

Here the *lesser Divisions* must be used, and the greater ones reckoned 10, 20, 30, &c. on the *first Radius*; and consequently on the *second Radius* they will be 100, 200, 300, &c. if the *double Radius* be used. For then it will be $10 : 180 :: 27 :$ the *Product* sought. But since $10 : 180 :: 1 : 18$; therefore if you make $1 : 18 :: 27 :$ the *Product*; the *single Radius* will give the Answer in the same manner; only remembering that the *fourth Number* sought will be of the same Denomination with the *second*, which in this Case is *Hundreds*.

Therefore set one Foot of the *Compasses* in the Beginning of the Line, and extend the other to 18, the same Extent will reach from 27 to 486, the *Product* sought.

And by the *Sliding-Rule*, thus; set 1 on the *Slider* to 18 on the *Rule*, and then against 27 on the *Slider* you

you find 486 on the Rule, which is the same Product as before.

7. Examp. III. What is the Product of 257 by 34?

Take in your Compasses the Distance from 1 to 34 on the Line of Numbers, the same Extent will reach from 257 to 8738, the Product required.

By the Sliding-Rule, thus; Set 1 on the Slider to 34 on the Rule, and against 257 on the Slider, you see 8738 on the Rule, which is the Product as before.

8. Examp. IV. What is the Product of 215 by 108?

With the Compasses, take the Distance from 1 to 108 on the *Gunter*, the same Extent of the Compasses will reach from 215 to 23220, the Product required.

By the Sliding-Rule, thus; Set 1 on the Slider to 108 on the Rule, and against 215 on the Slider you find 23220 on the Rule, the Product sought.

9. When the Product becomes so large, it must be a very large Line of Numbers indeed to shew it near the Truth; the Use of these Lines being principally where the Numbers are small; or where great Exactness is not required. They who understand the foregoing Doctrine of Logarithms can never be at any loss to know how many Places of Figures are contained in the Number sought, in this, or any of the following Rules.

10. DIVISION.

Examp. I. What is the Quotient of 63 divided by 9?

First, by Logarithms.

Y

From

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From the Logarithm of . . . $63=1.799340$
 Subduct the Logarithm of . . . $9=0.954242$

The Diff. is the Log. of the Quot. $=7=0.845098$

11. Secondly, by the Logarithmic Scale.

From $Gd=1799$ the Logarithm of $dm=63$, take $Gb=954$ the Logarithm of $bh=9$; and there will remain $Ga=845$, the Logarithm of $a g=7$, the Quotient sought.

12. Thirdly, by *Gunter's* Line and Compasses.

Set one Foot of the Compasses in 1, and extend the other to 9, and then with that Extent of the Compasses set one Foot in 63, the other will fall (towards the beginning of the Line) on 7, the Quotient sought.

13. Fourthly, by the Sliding-Rule.

Because $9 : 63 :: 1 : \text{the Quotient}$; therefore set 9 on the Slider to 63 on the Rule, and then against 1 on the Slider is 7 on the Rule, which is the Quotient sought.

14. Examp. II. What is 486 divided by 18?

By the *Gunter* and Compasses.

Extend the Compasses from 1 to 18, that Extent will reach from 486 (downward) to 27, the Quotient required.

By the Sliding-Rule.

Set 18 on the Slider to 486 on the Rule, then against 1 on the Slider you find 27 on the Rule, the Quotient sought.

All other Operations of Division being performed in the very same manner, 'tis needless to add any more Examples of this kind.

15. INVOLUTION.

Examp. I. What is the Square of 9?

First, by Logarithms.

The Logarithm of . . .	9=0.954242
Multiply by the Index . . .	2

The Prod. is the Log. of the Square = 81 = 1,908484

16. Secondly, by the Logarithmic Scale.

Let the Logarithms begin from G in the Line A e, as before; then with the Compasses take the Distance g b = 954 the Logarithm of b h = 9; and with one Foot remaining in b, turn the Compasses, the other Foot will fall on n; then shall G n = (2Gb =) 1908 the Logarithm of n o = 81, which therefore is the *square Number* sought.

17. Thirdly, by the *Gunter* and Compasses.

Set one Foot in 1, and extend the other to 9, where keep it fix'd, and turn the Compasses, the other Foot will fall on 81, the *square* sought.

18. Fourthly, by the Sliding-Rule.

Because 1 : 9 :: 9 : the Square required, therefore set 1 on the Slider to 9 on the Rule, then against 9 on the Slider is 81 on the Rule; which is the *square Number* sought.

19. Examp. II. What is the Cube of 9?

By the *Gunter* and Compasses.

Extend the Compasses from 1 to 9, that Extent will reach from 9 to 81, and again from 81 to 729, the Cube Number required.

By the Sliding-Rule.

Set 1 on the Slider to 9 on the Rule, then against 9 on the Slider is 81 on the Rule, and against 81 on the Slider (remaining unmov'd) is 729 on the Rule, the Cube Number required.

20. Examp. III. What is the Square and Cube of the Number 37?

By the Gunter and Compasses.

Extend the Compasses from 1 to 37, that Extent will reach from 37 to 1369 the *Square*; and the same Extent will reach from 1369 to 50653 the *Cube*; both as required.

By the Sliding-Rule.

Set 1 on the Slider to 37 on the Rule, then against 37 on the Slider is 1369 on the Rule, which is *Square*; and against 1369 on the Slider (remaining unmov'd) is 50653 on the Rule, which is *Cube* of 37; both as before.

21. EXTRACTION of ROOTS.

Examp. I. What is the *Square Root* of 81?

First, by Logarithms.

The Logarithm of	81=1.908484
which divide by the Index	2

The Quotient is the Log. } = 9 = 0.954242
of the *Square Root*

22. Secondly, by the Logarithmic Scale.

Bisect Gn=1908 the Logarithm of n=81, in b; then shall Gb=954 be the Logarithm of the *Square Root*, viz. bh=9, the Number sought.

23. Thirdly, by the *Gunter* and *Compasses*.

Take with the *Compasses* the Distance between 1 and 81, and bisection it; then take one Half in the *Compasses*, and it will reach from 1 to 9, the *square Root* sought.

24. Fourthly, by the *Sliding-Rule*.

Move the *Slider* forwards and backwards till you make the same Number on the *Rule* answer 1 on the *Slider*, as on the *Slider* answers 81 on the *Rule*; which Number will be the *square Root* sought, and in the present Case will be found 9.

25. Examp. II. What is the *Cube Root* of 50653?

Divide the Distance between 1 and 50653 into 3 equal Parts; the first Division will fall on 37, the *Cube Root* required.

Note, In the double Line of Numbers, if the grand Divisions be esteem'd *Units* in the first Radius, those in the second Radius will be *Tens*; if those in the first be *Tens*, viz. 10, 20, 30, &c. those of the second will be *Thousands*, as 1000, 2000, 3000, &c. with regard to *square Numbers*: and consequently in *Extraction*; if the Number whose square Root is sought be less than 100, yet greater than 10, the Number it self will be found on the *second Radius*; and its Root a Number of *Units* on the *first Radius*. But if the *Square* be less than 10, both it self and Root will be found in the *first Radius*. Again, if the *square Number* be between 1000 and 10000, the Number it self will be found on the *second Radius*, and its Root a Number of *Tens* on the first Radius. But if it be between 100 and 1000, both the Number and its Root of *Tens* will be found on the *first Radius*. After the same manner you may reason concerning the *Cube Number* and its *Root*.

26. Since the Logarithm of the Square is double the Logarithm of the Root; and the Logarithm of the Cube triple the Logarithm of the Root: therefore if a Line of Numbers of a *single Radius*, be equal to another of a *double Radius*, and these two appositely laid together, beginning from the same Point; then against any Number on the *single Radius*, you see its correspondent Square on the *double Radius*; and such Lines you have on some *Sliding-Rules*.

Also if a Line of *single Radius* were made equal to another of a *triple Radius*, and these exactly and properly placed together, then the Numbers on the latter would be the Cubes of those on the former; and so the Square and Cube Roots of any Number; and *vice versa*, would be discoverable by *Inspection*.

27. Moreover by means of a *single* and *double Line* of Numbers made to slide by each other, 'tis very easy to find a *mean Proportional* between any two given Numbers; as suppose 13 and 23. Thus; set 13 on the *double Line* to 13 on the *single one*, then against 23 on the *double Line* is 17,35 on the *single one*, which is the Mean required between 13 and 23. Or if you set 23 to 23, then against 13 on the *double* you find 17,35 on the *single Line*, the Mean required as before.

28. In like manner, by means of a *single* and a *triple Line* of Numbers, *two mean Proportionals* may be easily found between any two Numbers, as 2 and 54; thus; set 2 on one Line to 2 on the other, then against 54 on the triple Line is 6 on the single one, which is the *first Mean*; then set 2 on the *triple Line* to 6 on the *single one*, and against 54 is 18, the *second Mean*, on the single Line; so the four Numbers are $2:6::18:54$. And thus you may find *two Means* between any other two Numbers, which in many Cases is a most useful Problem.

29. In the foregoing Operations I have made no mention of *Decimals*, because they are to be respected

as Whole Numbers in the Management of them by *Instrumental Operations*, in the same manner as they were by *Numerical Logarithms*; the Number of *decimal Places* in any *Product*, *Quotient*, *Power*, *Root*, &c. being always determined here, as in all other Methods of working them, by the Rules proper to *Decimal Arithmetic*.

30. Thus it appears what *Similarity*, *Coherence*, and mutual Relation there is between the foregoing Methods of solving *Arithmetical Questions* by *Logarithms*, both *Numerical* and *Instrumental*; and that they are all one in Nature, and differ only in the *Modus operandi*, or Manner and Form of working. By this Chapter, I presume, 'twill be easy for the Learner to observe how any common Question in *Arithmetic*, or the Mensuration of *Artificers Work*, as *Joinery*, *Masonry*, *Carpentry*, *Painting*, *Timber-Measure*, *Gauging*, &c. may be most readily performed by the *Line of Numbers*, with *Compasses*, or by the *Sliding-Rule*, which is much the best Way.

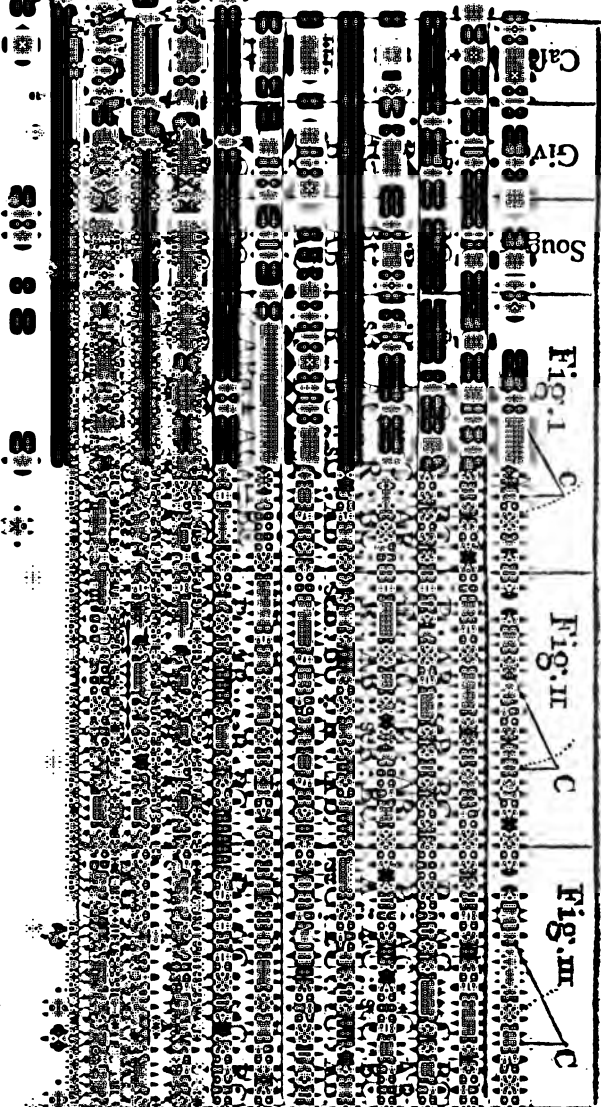


CH A P. X.

Sheweth the ANALOGIES or PROPORTIONS for the SOLUTION of all the CASES of Plain and Spherical TRIANGLES, both Right and Oblique angled.

1. **I**F any desire to be thoroughly instructed in the *Theory of Plain and Spherical Trigonometry*, I must refer them to my *Young Trigonometer's Guide*; since all I intend here is only to shew the *great and most excellent Use of Logarithms* in the practical Resolution of *Plain and Spherical Triangles*. A Syn-

plain Tri-



2. A Synopsis of the Analogies for the Solution of all the Cases of Oblique-angled plain Triangles.

Cases	Given	Sought	Fig. I.	Fig. II.
I	AB C, A, B.	AC. BC.	$sc : AB :: sb : AC$ $sc : AB :: sa : BC.$	
II	AC BC A	AB C, B	$BC : sa :: AC : sb.$ $sb : AC :: sc : AB.$	
III	AC BC C	A, B, AB.	$AC + CB : AC - CB :: t \frac{A+B}{2} : t \frac{A-B}{2}.$ Whence the Angles are known.	
IV	AB AC CB	A, B, C.	$AB : AC + CB :: AC - CB : AD - DB.$ In the first Triangle, 'tis $AD - DB \}$ AG in the second Triangle, $AD + DB \}$ Then $AB - AG = GB = 2DB.$ Thus each Tri- angle is reduced to two Right ones, viz. ADC, and BDC; in either of which two Sides are known whence the Angle C may be easily found.	

SOLUTION of Plain

3. A Synopsis of the Canons, and the Analogies (form'd from them for the Solution of all the Cases of Right-angled Spherical Triangles.

Cases	Given	Sought	Canons.	Analogies.	The Triangle.
I	AB B	AC BC C	sBE : sBA :: tFE : tAC. sDE : sDF :: tAE : tCF.	R : sBA :: tB : tAC. R : sB :: ctAB : ctBC. R : csAB :: sB : csC.	
II	AC B	AB BC C	tEF : tAC :: sBE : sAB. sEF : sAC :: sBF : sBC. sCD : sCH :: sDF : sHI.	tB : tAC :: R : sAB. sB : sAC :: R : sBC. csAC : R :: csB : sC.	
III	BC B	AB AC C	sDF : sDE :: tCF : tAE. sBF : sBC :: sEF : sAC. sCF : sCI :: tDF : tHI.	R : csB :: tBC : tAB. R : sBC :: sB : sAC. csBC : R :: ctB : tC.	
IV	AB AC	BC B C	sAD : sDC :: sAE : sCF. sAB : sBE :: tAC : tEF.	R : csCA :: csAB : csBC. sAB : R :: tAC : tB. sAC : R :: tAB : tC.	
V	BC AC	AB B C	sDC : sDA :: sCF : sAE. sBC : sBF :: sAC : sEF. tEF : tDH :: sGI : sGH.	csAC : R :: sDC : csAB. sBC : R :: sAC : sB. tBC : R :: tAC : csC.	
VI	B C	AB AC BC	sHI : sDF :: sCH : sCD. tHI : tDF :: sCI : sCF.	sB : csC :: R : csAB. sC : csB :: R : csAC. tC : ctB :: R : csBC.	

Cafes	Given	Sought	I. Analogies.	II. Analogies.	The Triangle.
B	CD		SBC : P :: CB : BCA	SD : SBC :: SB : SCD.	
			CSB : SBCA :: CSD : SDCA.		

5. Being thus furnished with Proportions, we shall soon see with what incomparable Pleasure and Ease the several Cases of Triangles before going are resolved by the Canon of Logarithmical *Sines*, *Tangents*, and *Secants*; and also by the *Line of Numbers*, both with the *Compasses* and by the *Sliding-Rule*; I say, we shall see in the two next Chapters, with how much more Ease and Pleasure they are resolved by *these two Methods*, above what is attainable by any other Way yet invented for this Purpose.



CHAP.



C H A P. XI.

*The SOLUTION of PLAIN TRIANGLES by the
CANON of LOGARITHMICAL SINES and
TANGENTS ; by GUNTER'S SCALE and
COMPASSES ; and by the SLIDING RULE.*

1. **I** Have already described and taught the Use of the Logarithmic Canon, so far as to find the Logarithms of any *Number, Sine, Tangent, or Secant* proposed. I shall therefore here only observe, that what I call *Gunter's Scale* is such a *Plane Scale* as hath upon it *Gunter's Line of Numbers*, and of *Artificial Sines and Tangents* ; whose Nature, Construction, and Design, have been before discoursed of in the Theory. The *Sliding-Rule* has the same Lines, which are contrived to slide by one another as you please ; and to avoid Repetitions, I shall call the Line of Numbers on the Rule it self, A ; and that on the Slider, B ; also I shall call the Lines of Sines and Tangents on the Rule Sr, Tr ; and those on the Slider Ss, and Ts. You must know also that the End of each Line marked 10, 90, 45, is here called Radius. Having premised these things, we proceed immediately to the Solution of

2. *Right-angled Plain Triangles.*

Case I. In the Right-angled Triangle ABC, there is given the Base AB=285, and the Angle at Base B=32° 48' ; to find the Perpendicular AC, and the Hypothenuse BC. The Hypoth. Radius, Fig. I.

The Analogy for AC is, sC : AB :: sB : AC.

In

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In Words ;

As the Sine of the Angle $C=57^{\circ} 12' = 9.924572$

is to the given Side or Base $AB=285=2.454845$ } add
So is the Sine of the Ang. $B=32^{\circ} 48' = 9.733765$ }

12.188610

To the Perpendicular $AC=183,67=2.264038$

The Analogy for BC is, $sB : AC :: R : BC$.

In Words ;

As the Sine of . . . $B=32^{\circ} 48' = 9.733765$
is to the Perpendicular $AC=183,67=2.264038$
So is Radius $90^{\circ} 00' = 10.000000$

to the Hypothenufe . . . $BC=339,06=2.530273$

3. *By Scale and Compasses. To find AC.*

Set one Foot of the Compasses to $32^{\circ} 48' = B$, in Line of Sines, and extend the other upwards to $57^{\circ} 12' = C$; the same Extent will reach from $285 = AB$ downwards in the Line of Numbers to $183,67 = AC$, the Perpendicular sought.

To find BC.

Extend the Compasses from $32^{\circ} 48' = B$ to Radius 90° in the Line of Sines; the same Extent will reach (in the Line of Numbers) from $183,67 = AC$, to $339,06 = BC$, the Hypothenufe sought.

4. *By the Sliding-Rule. To find AC.*

Make the Line of Numbers slide by the Line of Numbers, then will the Lines of Sines slide by each other, and also the Line of Tangents by the Line of Tangents. Having thus prepared the Rule; set
 57°

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57° 12' on Ss to 32° 48' on SR; then against 285 on B is 183,67 on A, the Perpendicular sought.

To find BC.

Set 32° 48' on Ss to Radius 90° on SR, then against 183,67 on B is 339,06 on A, which is the Hypothenufe required.

5. Case II. Given the Angles $B=32^{\circ} 48'$, and $C=57^{\circ} 12'$, and the Side $AC=183,67$; to find AB, and BC.

The Base made Radius. Fig. II.

As the Tangent of the Ang. $B=32^{\circ} 48'=9.809193$
 is to the Side $AC=183,67=2.264038$
 So is Radius $90^{\circ} 00'=10.000000$

 to the Side or Base . . . $AB=285=2.454845$

To find BC.

As Radius $90^{\circ} 00'=10.$
 is to the Side $AB=285=2.454845$
 So is the Secant of the Angle $B=32^{\circ} 48'=10.075428$

 to the Side or Hypoth. $BC=339,06=2.530273$

Note, I have wrought this last in *Secants* for Variety sake, and that the Reader may see the Conclusions are the same every Way. But this Case is much better resolved by making BC Radius, as in Fig. I.

6. *By the Plane Scale and Compasses.*

As there is no *Line of Artificial Secants* on the Scale or *Sliding-Rule* (as being usefess) so this Case will be best performed Instrumentally by the Analogies of Fig. I. where BC is made Radius.

Therefore

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Therefore, extend the Compasses from $32^{\circ} 48'$ to $57^{\circ} 12'$ in the Line of Sines, the same Extent will reach from 183,76 to 285 in the Line of Numbers; thus $285=AB$, the Side required.

Or thus, By the first Analogy of this Case of Fig. II. extend the Compasses from $32^{\circ} 48'$ to Radius 45 in the Line of Tangents, the same will reach from 183,67 to $285=AB$ (as before) in the Line of Numbers.

To find BC.

Extend the Compasses from $32^{\circ} 48'$ to Radius 90 in the Line of Sines, the same will reach from 183,67 to $339,06=BC$; in the Line of Numbers.

7. *By the Sliding-Rule.*

To find AB.

Set $32^{\circ} 48'$ on Ts to Radius 45° on TR; then against 183,67 on B is $285=AB$, on A.

To find BC.

Set $32^{\circ} 48'$ on Ss to Radius 90° on SR; then against 183,67 on B is $339,06=BC$, on A.

8. Case III. Given the Angles $B=32^{\circ} 48'$ and $C=57^{\circ} 12'$, and the Hypothenufe $BC=339,06$; to find the Sides AB and AC.

The Hypothenufe made Radius. Fig. I.

As Radius $90^{\circ} 00' = 10$.
is to the Hypothenufe $BC=339,06=2.530273$
So is the Sine of the Angle $C=57^{\circ} 12'=9.924572$

To the Side or Base $AB=285=2.454845$

And so is the Sine of the Ang. $B=32^{\circ} 48'=9.733765$

to the Side $AC=183,67=2.264038$

Or

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Or thus ; AB made Radius, Fig. II. To find AC.

As Radius	90° 00=10.
is to the Side	AB=285=2.454845
So is the Tangent of	B=32° 48'=9.809193
to the Side	AC=183,67=2.264038

9. *By the Scale and Compaffes.*

Extend the Compaffes from Radius 90 to 57° 12' in the Line of Sines, the fame Extent will reach from 339,06 to 285=AB, in the Line of Numbers.

And then again, extend them from 90 to 32° 48' in the Line of Sines, the fame will reach from 339,06 to 183,67=AC, in the Line of Numbers.

10. *By the Sliding-Rule.*

This Cafe may be folved by once fetting the Rule, thus ; Make the Line of Sines to flide by the Line of Numbers: Then fet Radius 90 on Ss to 339,06 on A ; thus againft 57° 12' on Ss you fee 285=AB, and againft 32° 48' is 183,67=AC, on A. Such is the great Conveniency of this fmall Inftrument.

11. Cafe IV. Given the two Sides, AB=285 and AC=183,67 ; to find the Side BC, and the Angles B and C.

The Bafe made Radius, Fig. II. To find B.

As the Side	AB=285=2.454845
is to Radius	10.
So is the Side	AC=183,67=2.264038

To the Tangent of the Angle B=32° 48'=9.809193

Or thus, making AC Radius, Fig. III. To find C.

A a

As

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As the Side $AC=183,67=2.264038$
 is to Radius $10.$
 So is the Side $AB=285=2.454845$

To the Tang. of the Angle $C=57^{\circ}48'=10.190707$

12. To find the Side BC,

This may be done directly with the Secants, or by finding the Angles first, with the Sines; as is manifest from the Synopsis. But since if the required Side BC be made Radius, it can't be found by the common Analogies of *Trigonometry*; I have given an Equation formed on the Principles of *Geometry* for that purpose, viz. $\sqrt{ABq+ACq}=BC$; which I shall resolve by Logarithms, as follows.

The Log. of the Side . . . $AB=285=2.454845$
 the Double thereof } . . . $ABq=81225=4.909690$
 is the Log. of }

Also the Log. of the Side $AC=183,67=2.264038$
 The Double of which } $ACq=33734,66=4.528076$
 is the Log. of }

Add the Square $ABq=81225$

The Log. of } $ABq+ACq=114959,66=5.060546$
 the Sum }

Half which is the Log. of

$\sqrt{ABq+ACq}=BC=339,06=2.530273$
 the Side required.

13. *By Scale and Compasses.* To find the Ang. B.

Extend the Compasses from 285 to 183,67 in the Line of Numbers, the same will reach from Radius 45° to $32^{\circ}48'$, in the Line of Tangents, the Angle B required.

By

By the Sliding-Rule.

Set 285 on A to 183,67 on B, then against Radius 45° on Tr is 32° 48' = the Angle B, on Ts.

Having thus found the Angles, the Side BC is found as in the foregoing Cases.

14. Case V. Given the Hypotenuse $BC=339,06$ and the Side $AB=285$; to find the Angles B and C, and the Side AC.

To find the Angle C. Fig. I. BC Radius.

As the Side $BC=339,06=2.530273$
is to Radius 10.

So is the Side $AB=285=2.454845$

To the Sine of the Angle $C=57^{\circ} 12'=9.924572$

To find the Side AC.

As Radius 10.
is to the Side $BC=339,06=2.530273$

So is the Sine of the Angle $B=32^{\circ} 48'=9.733765$

to the Side $AC=183,67=2.264038$

Note, The Side AC may be found Geometrically, as taught in Art. 12. the Equation being $\sqrt{BC^2 - AB^2} = AC$.

15. *By Scale and Compasses.* To find the Angle C.

Extend the Compasses from 339,06 to 285 in the Line of Numbers, the same will reach from Radius 90° to 57° 12' = C, in the Line of Sines.

By the Sliding-Rule.

Set 339,06 on A to 285 on B, then against Radius 90° on Sr is 57° 12' = C, on Ss.

A 2 2

The

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The Angles being thus found, the Side AC may be found by *Scale* or *Sliding-Rule*, as before.

16. Case VI. Given the Side AC=183,67 and BC=339,06; to find the Angles B, C, and the Side AB.

As this Case is, in the Nature of it, the same as the last, so the Solution is in all respects the same, and needs not be repeated.

17. *Of Oblique-angled Plain Triangles.*

- Case I. There is given the Angles C=82° 30', B=60° 00'; and the Side AB=365; to find the other two Sides AC and BC.

Note, The first Triangle in the Synopsis of Oblique Plain Triangles is that which I have regard to here, and is acute-angled; the Difference between this and the obtuse-angled one, Fig. II. will be taken notice of as I go along.

To find the Side AC. Fig. I. Co. Ar.

As the Sine of the Angle C=82° 30'=0.003732
is to the Side AB=365=2.562293
So is the Sine of the Angle B=60° 00'=9.937531

to the Side sought AC=318,82=2.503556

To find the Side BC. Co. Ar.

As the Sine of the Angle C=82° 30'=0.003732
is to the Side AB=365=2.562293
So is the Sine of the Angle A=37° 30'=9.784447

to the Side required BC=224,11=2.350472

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18. *By the Scale and Compasses.* To find AC.

Extend the Compasses from $82^{\circ} 30'$ to 60° in the Line of Sines, and the same Extent will reach from 365 to 318,82 in the Line of Numbers; therefore $318,82=AC$.

To find BC.

Extend the Compasses from $82^{\circ} 30'$ to $37^{\circ} 30'$ in the Line of Sines, the same will reach from 365 to 224,11=BC, in the Line of Numbers.

19. *By the Sliding-Rule.*

To find AC.

Set $82^{\circ} 30'$ on SR to $60^{\circ} 00'$ on Ss; then against 365 on A, you have $318,82=AC$, on B.

To find BC.

Set $82^{\circ} 30'$ on SR to $37^{\circ} 30'$ on Ss; then against 365 on A, you have $224,11=BC$, on B.

But since all the Angles are known, both the unknown Sides are found at once setting the Rule thus;

Let the *Line of Numbers* slide by the *Line of Sines*; and set $182^{\circ} 30'$ on SR to 365 on B, then against $60^{\circ} 00'$ is $318,82=AC$; and against $37^{\circ} 30'$ is $224,11=BC$, on the Line of Numbers.

20. Case II. Given two Sides, $AC=318,82$ and $BC=224,11$; and the opposite Angle $A=37^{\circ} 30'$; to find the Angle B, and the other Side AB.

To find the Angle B.

Co. Ar.

As the Side $BC=224,11=7.649528$
is to the Sine of the Angle $A=37^{\circ} 30'=9.784447$
So is the Side $AC=318,82=2.503556$

to the Sine of the Angle . . $B=60^{\circ} 00'=9.937531$

The

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The Angles being thus known, the Side AB is found as were the Sides AC and BC in the foregoing Case.

21. *By the Scale and Compasses.*

To find the Angle B.

Extend the Compasses from 224,11 to 318,82 in the Line of Numbers, the same Extent will reach from $37^{\circ} 30'$ to $60^{\circ} 00' = B$, on the Line of Sines.

By the Sliding-Rule.

Set 224,11 on A to 318,82 on B, then against $37^{\circ} 30'$ on Sr is $60^{\circ} 00' = B$, the Angle sought, on Ss.

Note, In this Case of the Obtuse-angled Triangle, Fig. II. 'tis obvious the Angle here found is the outward Angle CBG, whose Complement therefore to 180 Degrees is equal to the internal obtuse Angle $ABC = 120^{\circ} 00'$.

22. Case III. Given two Sides $AC = 318,82$ and $BC 224,11$; and the included Angle $C = 82^{\circ} 30'$; to find the Angles A, B, and the Side AB.

To find the Angles A and B.

The Sum of the given Sides is $AC + BC = 542,93$; their Difference is $AC - BC = 94,71$; the Sum of the unknown Angles $A + B = 97^{\circ} 30'$; therefore the half Sum is $\frac{A+B}{2} = 48^{\circ} 45'$; whence by the Analogy in the Synopsis, say;

A,

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Co. Ar.

As the Sum of } ... $AC + BC = 542,93 = 7.265256$
 the Sides }
 is to their Difference $AC - BC = 94,71 = 1.976396$
 So is the Tangent of }
 the half Sum of the } $\frac{A+B}{2} = 48^\circ 45' = 10.057012$
 unknown Angles }

to the Tangent of half } $\frac{A-B}{2} = 11^\circ 15' = 9.298664$
 their Difference }

Then to the half Sum of the Angles $48^\circ 45'$
 add the half Difference $11^\circ 15'$

The Sum is the greater Angle $B = 60^\circ 00'$

But the Diff. is the lesser Ang. $A = 37^\circ 30'$

23. *By Scale and Compasses.*

To find the Half Difference of the Angles $\frac{A-B}{2}$.

Having prepared the Work as above, proceed thus ;

Extend the Compasses from the Sum of the Sides 542,93 to their Difference 94,71 on the Line of Numbers ; with this Extent let one Foot of the Compasses in Radius 45° , and pitch the other downwards in the Line of Tangents, where fix it while you bring the other Foot from 45° to $48^\circ 45'$; then with this Extent apply one Foot in 45° , the other will reach to $11^\circ 15' =$ the half Difference of the Angles A and B ; which therefore may be found as before.

Or if you have a Line of Tangents continued beyond 45° , then the Extent from 94,71 to 542,93 in the Line of Numbers will reach from $48^\circ 45'$ to $11^\circ 15'$ in the Tangent Line.

24. *By the Sliding-Rule.*

Set 542.93 on A to 94.71 on B; then against 45° on TR observe the Degree and Minute on Ts, and bring that Point to $48^{\circ} 45'$ on TR; then against 45° on TR you have $11^{\circ} 15'$ on Ts, which is the half Difference as before, of the two enquired Angles A and B.

Or thus, if the Line of Tangents be continued on the Slider beyond 45° ; set 542.93 on A to $48^{\circ} 45'$ on Ts, then against 94.71 on A is $11^{\circ} 15'$ on Ts.

Having therefore the *Half Sum*, and *Half Difference* of those Angles, they are found as in Art. 22. and then the Side AB will be found to be 365 as in Case I. hercof.

25. Case IV. Given all three Sides $AB=365$, $AC=318,82$, and $CB=224,11$, to find the Angles.

In order to this 'twill be necessary to reduce the oblique Triangle into two Right-angled ones ADC, and BDC, thus; Find the Sum of any two Sides $AC+CB=542,93$; and their Difference $AC-CB=94,71$; esteeming the other Side AB the Base, the Difference of whose Segments $AD-DB=AG$, is first of all to be found by this Proportion, viz.

	Co. Ar.
As the Base	$AB=365=7.437707$
is to the Sum of	
the two Sides	} $AC+BC=542,93=2.734744$
So the Diff. of the	
two Sides	} $AC-BC=94,71=1.976396$
to the Diff. of	
the Segments	} $AD-DB=AG=141=2.148847$

Therefore

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Therefore $AB - AG = 224 = GB = 2BD$; therefore $BD = 112$, and $AD = 253$; and so the whole Triangle ACB is reduced to two Right-angled ones ADC and BDC , in each of which there is two Sides given AD and AC , DB and BC ; by which means the Angles ACD and DCB may be found, which together are equal to the Angle ACB ; and this being known, the other two are found with Ease by Calculation. And thus the whole oblique Triangle is resolved.

Note, When the Perpendicular falls without, as in the obtuse-angled Triangle ABC , Fig. II. then it will be the Sum of the Segments $AD + DB = AG$; and the Difference of the Angles $ACD - BCD = ACB$, the Angle required.

26. And since the several Problems of *Navigation*, whether in the *Plain*, *Mercator's*, *Middle Latitude*, *Oblique*, or *Traverse Sailing* ; as also of measuring *Heights* and *Distances*, *accessible* and *inaccessible* ; of *Fortification*, *Gunnery*, and divers Parts of *Astronomy*, &c. are all resolved by the *Doctrine of Plain Trigonometry*, as in the Method before-going ; it must be very easy for any who understands the *Solution of Plain Triangles*, to apply it to any practical Cases that may occur in any of the aforesaid Arts, without any farther Instructions or Examples. Yet those who would see the *Theory of Plain Trigonometry*, and its *Application*, in the largest Extent, may find it in the first Vol. of my *Young Trigonometrer's Guide*.





C H A P. XII.

*The SOLUTION of SPHERICAL TRIANGLES by
LOGARITHMS, by GUNTER'S SCALE, and
by the SLIDING RULE.*

1. **A**S in the foregoing Chapter I have exemplified the Resolution of *Plain Triangles* both by the *Canon of Logarithms*, and *Logarithmical Instruments*, so I shall pursue the same Method here with respect to *Spherical Triangles* of both kinds; in each of which there are *six different Cases*, which in all their *Varieties* are resolvable according to the *Analogies* assigned in the *Synopsis*; on which, and on the Figure of the Triangle there, the Reader is desired to have his eye, thro' the whole Course of Examples. To begin then with.

2. *Right-angled Spherical Triangles.*

Case I. Given the Base $AB=38^{\circ} 15'$, and Angle at Base $B=39^{\circ} 56'$ to find the Perpendicular AC , the Hypothenuse BC , and the Angle C .

1. To find the Perpendicular AC .

As Radius $90^{\circ} 00' = 10.000000$
is to the Sine of the Side $AB=38^{\circ} 15' = 9.791757$
So is the Tang. of the Ang $B=39^{\circ} 56' = 9.922787$

to the Tang. of the Perp. $AC=27^{\circ} 23' 4'' = 9.714544$

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2. To find the Hypothenufe BC.

As Radius $90^{\circ} 00' = 10.000000$
 to the Co-Sine of the Angle B $= 39^{\circ} 56' = 9.884677$
 So is the Co-T. of the Side AB $= 38^{\circ} 15' = 10.103288$
 to the Co-T. of the Hypoth. BC $= 45^{\circ} 48' = 9.988065$

3. To find the Angle C.

As Radius $90^{\circ} 00' = 10.000000$
 to the Co-Sine of the Side AB $= 38^{\circ} 15' = 9.895045$
 So is the Sine of the Angle B $= 39^{\circ} 56' = 9.807465$
 to the Co-Sine of the Angle C $= 59^{\circ} 44' = 9.702510$

3. *By Scale and Compasses:*

1. To find AC.

Extend the Compasses from 90° to $39^{\circ} 56'$ in the Line of Sines, that will reach from $38^{\circ} 15'$ in the Line of Tangents to $27^{\circ} 23\frac{1}{4}' = AC$, the Side required.

2. To find the Hypoth. BC.

Extend the Compasses from Radius 90° to the Co-Sine of B, $50^{\circ} 04'$ in the Line of Sines; then apply that Extent from 45° in the Line of Tangents downwards, where fix that lower Foot, and bring the other to $51^{\circ} 45'$ the Co-Tangent of AB, this last Extent will reach from 45° to $44^{\circ} 12'$ the Co-Tangent of BC $= 45^{\circ} 48'$, the Side required.

3. To find the Angle C.

Extend the Compasses from 90° to $51^{\circ} 45'$ the Co-Sine of AB, the same will reach from the Sine $39^{\circ} 56'$ to $30^{\circ} 16'$ the Co-Sine of C $= 59^{\circ} 44'$.

4.

By the Sliding-Rule.

1. To find AC.

Set 90° on SR to $38^\circ 15'$ on Ss; then against $39^\circ 56'$ on TR is $27^\circ 23\frac{1}{2}' = AC$, on Ts.

2. To find BC.

Set 90 on SR to $50^\circ 04'$ the Co-Sine of B, on Ss; and mark the Degree and Minute on Ts against 45° on TR, bring that Point to $51^\circ 45'$ (the Co-Tangent of AB) on TR, on which against 45° , you have $44^\circ 12'$ on Ts, the Co-Tangent of $BC = 45^\circ 48'$, as required.

3. To find the Angle C.

Set 90° on SR to the Co-Sine of AB, $51^\circ 45'$ on Ss; then against $39^\circ 56'$ on SR is $30^\circ 16'$ on Ss, the Co-Sine of $C = 59^\circ 44'$, as required.

5. Case II. Given the Perpendicular $AC = 27^\circ 23'$, and the opposite Angle $B = 39^\circ 56'$; to find the Side AB, the Hypothenufe BC, and Angle C.

To find AB.

As the Tangent of $B = 39^\circ 56' = 9.922787$
is to the Tangent of . . . $AC = 27^\circ 23\frac{1}{2}' = 9.714544$
So is Radius $90^\circ 00' = 10$.

to the Sine of the }
Side required } $AB = 38^\circ 15' = 9.791757$

The Operations of this Case being only the Converse of the foregoing, needs no further Examples, in Numbers.

6.

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6. *By Scale and Compasses.*

1. To find AB.

Extend the Compasses from $39^{\circ} 56'$ to $27^{\circ} 23\frac{1}{2}'$ on the Tangents, the same will reach from 90° on the Sines, to $38^{\circ} 15' = AB$.

2. To find BC.

Extend the Compasses from $39^{\circ} 56'$ to $27^{\circ} 23\frac{1}{2}'$ on the Sines, the same will reach from 90° to (the Sine of $BC =$) $45^{\circ} 48'$, as required.

3. To find the Angle C.

Extend the Compasses from (the Co-Sine of AC) $62^{\circ} 36\frac{1}{2}'$ to 90° , the same will reach from (the Co-Sine of B) $50^{\circ} 04'$ to $59^{\circ} 44'$ the Sine of the Angle C required.

7. *By the Sliding-Rule.*

1. To find AB.

Set $39^{\circ} 56'$ on TR to $27^{\circ} 23\frac{1}{2}'$ on Ts , then against 90° on SR is $38^{\circ} 15' = AB$, on Ss .

2. To find BC.

Set $39^{\circ} 56'$ on SR to $27^{\circ} 23\frac{1}{2}'$ on Ss ; then against 90° on SR is $45^{\circ} 48' = BC$, on Ss .

3. To find the Angle C.

Set (the Co-Sine of AC) $62^{\circ} 36\frac{1}{2}'$ on SR to 90° on Ss ; then against (the Co-Sine of B) $50^{\circ} 04'$ on SR is $59^{\circ} 44' = C$, on Ss .

8. Case III. Given the Hypothenuse $BC = 45^{\circ} 48'$, and an acute Angle $B = 39^{\circ} 56'$, to find the Legs AB , AC , and the Angle C .

1.

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1. To find AB.

As Radius $90^{\circ} 00' = 10.000000$
 to the Co-Sine of . . . $B = 39^{\circ} 56' = 9.884677$
 So is the Tangent of . . $BC = 45^{\circ} 48' = 10.012129$
 to the Tang. of the Side } $AB = 38^{\circ} 15' = 9.896806$
 sought

2. To find AC.

As Radius $90^{\circ} 00' = 10.$
 is to the Sine of . . . $BC = 45^{\circ} 48' = 9.855465$
 So is the Sine of . . . $B = 39^{\circ} 56' = 9.807465$
 to the Sine of the Side } $AC = 27^{\circ} 23' = 9.662930$
 sought

3. To find the Angle C.

As the Co-Sine of . . . $BC = 45^{\circ} 48' = 9.843336$
 is to Radius $90^{\circ} 00' = 10.$
 So is the Co-Tangent of $B = 39^{\circ} 56' = 10.077213$
 To the Tangent of . . . $C = 59^{\circ} 44' = 10.233877$

9. *By Scale and Compasses.*

1. To find AB.

Extend the Compasses from Radius 90° to (the Co-sine of B) $50^{\circ} 04'$ in the Sines, then set one Foot in 45° in the Tangents, and pitch the other downward ; where fix it, while you bring the former from 45° to $45^{\circ} 48'$, then will this last Extent reach from 45° to the Tangent of $38^{\circ} 15' = AB$, the Side required.

2. To find AC.

Extend the Compasses from 90° to $45^{\circ} 48'$ in the Sines,

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Sines, the same will reach from $39^{\circ} 56'$ to $27^{\circ} 23\frac{1}{2}'$ $\equiv AC$, in the same Line.

3. To find the Angle C.

Extend the Compasses from (the Co-Sine of BC) $44^{\circ} 16'$ to Radius 90° ; the same applied to (the Co-Tangent of B) $50^{\circ} 04'$, will reach to the Tangent $59^{\circ} 44' \equiv C$, the Angle required.

10.

By the Sliding-Rule.

1. To find AB.

Set Radius 90° on S_R to $50^{\circ} 04'$ (the Co-Sine of B) on S_s , and against 45° on T_R mark the Point on T_s , bring that Point to $45^{\circ} 48'$ on T_R ; and now against 45° on T_R you have $38^{\circ} 15' \equiv AB$, on T_s .

2. To find AC.

Set Radius 90° on S to $45^{\circ} 48'$ on S_R , then against $39^{\circ} 56'$ on S_s , you'll see $27^{\circ} 23\frac{1}{2}'$ on S_R , the Side AC required.

3. To find the Angle C.

Set (the Co-Sine of BC) $44^{\circ} 16'$ on S_s to Radius 90° on S_R ; then against (the Co-Tangent of B) $50^{\circ} 04'$ on T_R , is $59^{\circ} 44' \equiv C$, on T_s .

11. Case IV. Given the Legs, $AB = 38^{\circ} 15'$, and $AC = 27^{\circ} 23\frac{1}{2}'$; to find the rest.

To find the Hypothenuse BC.

As Radius $90^{\circ} 00' = 10$.
 is to the Co-Sine of . . $AC = 27^{\circ} 23\frac{1}{2}' = 9.948388$
 So is the Co-Sine of . . $AB = 38^{\circ} 15' = 9.895045$
 to the Co-Sine of . . . $BC = 45^{\circ} 48' = 9.843433$

As

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As there is nothing new in finding the Analogies B and C, I shall pass them by.

12. *By Scale and Compasses.*

1. To find BC.

Extend the Compasses from 90° to (the Co-Sine of AC) $62^\circ 37'$; the same applied to (the Co-Sine of AB) $51^\circ 45'$ will extend to (the Co-Sine of BC) $44^\circ 16'$; therefore $BC = 45^\circ 48'$.

2. To find the Angle B.

Extend the Compasses from $38^\circ 15'$ to 90° in the Sines, the same will reach from $27^\circ 23\frac{1}{4}'$ to $39^\circ 56' = B$, in the Line of Tangents.

3. To find the Angle C.

Extend the Compasses from $27^\circ 23\frac{1}{4}'$ to 90° in the Sines; the same will reach from 45° to a Point in the Tangent-Line, where fix the Compasses while you bring the Leg from 45° to $38^\circ 15'$, then will this last Extent reach from 45° to $59^\circ 44' = C$, in the Tangents.

13. *By the Sliding-Rule.*

1. To find BC.

Set 90° on Ss to (the Co-Sine of AC) $62^\circ 36\frac{1}{4}'$ on SR, then against (the Co-Sine of AB) $51^\circ 45'$ on Ss is (the Co-Sine of BC) $44^\circ 16'$ on SR; wherefore $BC = 45^\circ 48'$, as required.

2. To find the Angle B.

Set $38^\circ 15'$ on Ss to 90° on SR; then against $27^\circ 23\frac{1}{4}'$ on Ts is $39^\circ 56'$ on TR, the Angle B required.

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3. To find the Angle C.

Set $27^{\circ} 23'$ on Ss to 90° on SR; and mark the Point on Ts against 45° on TR; bring that Point to $38^{\circ} 15'$ on TR; then against 45° on TR you see $59^{\circ} 44' = C$, on Ts.

14. Case V. Given the Hypothemuse $BC = 45^{\circ} 48'$, and one Side $AC = 27^{\circ} 23'$; to find the rest.

To find the Angle C.

The Tangent of . . . $BC = 45^{\circ} 48' = 10.612129$
is to Radius . . . $90^{\circ} 00' = 10.$

As the Tangent of $AC = 27^{\circ} 23' = 9.714544$

is to the Co-Sine of the Angle $C = 59^{\circ} 44' = 9.702419$

15. By the Scale and Compasses.

Extend the Compasses from 45° to $27^{\circ} 23'$ in the Tangents; then set one Foot in $45^{\circ} 48'$, and pitch the other downward, where fix it while you move the other again from $45^{\circ} 48'$ to 45° ; then shall this last extent reach from Radius 90° to (the Co-Sine of C) $30^{\circ} 16'$ in the Line of Sines; whence the Angle $C = 59^{\circ} 44'$, as required.

16. By the Sliding-Rule.

Set 45° on TR to $27^{\circ} 23'$ on Ts; and mark the Point in Ts against $45^{\circ} 48'$ on TR; then bring that Point to 45° on TR; lastly, against 90° on SR, you have $30^{\circ} 16'$ on Ss, which is the Co-Sine of $C = 59^{\circ} 44'$, as required.

Note, The Proportions for AB and the Angle B, contain nothing new or difficult either in Numbers or by Instrument; therefore shall give no Examples in them.

17. Case VI. Given the Angles $B=39^{\circ} 56'$, and $C=59^{\circ} 44'$, to find the Sides AB, AC, BC.

1. To find AB.

As the Sine of $B=39^{\circ} 56'=9.807465$
 is to the Co-Sine of . . . $C=59^{\circ} 44'=9.702452$
 So is Radius $90^{\circ} 00'=10.$

To the Co-Sine of the Side $AB=38^{\circ} 15'=9.894987$

2. To find AC.

The Analogy is the same as for AB, by which you will find $AC=27^{\circ} 23\frac{1}{2}'$ nearly.

3. To find BC.

As the Tangent of . . . $C=59^{\circ} 44'=10.233905$
 is to the Co-Tangent of $B=39^{\circ} 56'=10.077213$
 So is Radius $90^{\circ} 00'=10.$

to the Co-Sine of the Side $BC=45^{\circ} 48'=9.843308$

18. *By Scale and Compasses.*

1. To find AB.

Extend the Compasses from $39^{\circ} 56'$ to (the Co-Sine of C) $30^{\circ} 16'$, the same will reach from Radius 90° to (the Co-Sine of AB) $51^{\circ} 45'$, in the Line of Sines; therefore $AB=38^{\circ} 15'$. In the same manner you find $AC=27^{\circ} 23\frac{1}{2}'$.

2. To find BC.

Extend the Compasses from $59^{\circ} 44'$ to (the Co-Tangent of B) $50^{\circ} 04'$ in the Tangents, the same Extent will reach from Radius 90° to (the Co-Sine of BC) $44^{\circ} 12'$ in the Sines; therefore $BC=45^{\circ} 48'$.

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19.

By the Sliding-Rule.

1. To find AB.

Set $39^{\circ} 56'$ on Ss to (the Co-Sine of C) $30^{\circ} 16'$ on SR; then against 90° on Ss is (the Co-Sine of AB) $51^{\circ} 45'$; consequently $AB=38^{\circ} 15'$.

In like manner, you find $AC=27^{\circ} 23\frac{1}{2}'$.

2. To find BC.

Set $59^{\circ} 44'$ on Ts to (the Co-Tangent of B) $50^{\circ} 04'$ on TR; then against 90° on SR is $44^{\circ} 12'$ (the Co-Sine of BC) on Ss. Wherefore $BC=45^{\circ} 48'$.

20.

Of Oblique Triangles.

Oblique Spherical Triangles admit also of fix different Cases, as follow.

Case I. Given two Angles, $B=34^{\circ} 30'$, and $D=48^{\circ} 00'$, and an opposite Side $BC=38^{\circ} 45'$; to find the other two Sides DC and BD; and Angle C.

Let fall a Perpendicular CA from the unknown Angle C to its opposite Side BD. Then is the oblique Triangle reduced to two Right ones, BAC and DAC. Then say by the first Analogy; To find the Angle C.

As the Co-Sine of $BC=38^{\circ} 45'=9.892039$
is to Radius $90^{\circ} 00'=10.$

So is the Co-Tangent of $B=34^{\circ} 30'=10.162866$

to the Tangent of . . $BCA=61^{\circ} 48\frac{1}{2}'=10.279836$

Again, by the second Analogy, say;

C c 2

Aa

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Co. Ar.

As the Co-Sine of . . . $B=34^{\circ} 30' = 0.084006$
 is to the Sine of . . . $BCA=61^{\circ} 48' = 9.945159$
 So is the Co-Sine of . . . $D=48^{\circ} 00' = 9.825511$

to the Sine of the Angle $DCA=45^{\circ} 41' = 9.854676$

Now since the Perpendicular CA falls within the Triangle, 'tis plain the Sum of the two Angles now found makes the Angle sought, viz. $BCA + DCA = BCD = 45^{\circ} 41' + 61^{\circ} 48' = 107^{\circ} 30'$.

To find the Side BD,

21. By the first Analogy, say ;

As Radius $90^{\circ} 00' = 10.$
 to the Co-Sine of . . . $B=34^{\circ} 30' = 9.915994$
 So is the Tangent of . . . $BC=38^{\circ} 45' = 9.904497$

to the Tangent of the Side $BA=33^{\circ} 29' = 9.820485$

Again, by the second Analogy, say ;

Co. Ar.

As the Tangent of . . . $D=48^{\circ} 00' = 9.954437$
 is to the Tangent of . . . $B=34^{\circ} 30' = 9.837134$
 So is the Sine of . . . $AB=33^{\circ} 29' = 9.741698$

to the Sine of . . . $AD=19^{\circ} 57' = 9.533269$

Now 'tis evident, the Sum of the two Arches $AB + AD = BD = 33^{\circ} 29' + 19^{\circ} 57' = 53^{\circ} 26'$, the Side required.

22. To find the Side CD. Co. Ar.

As the Sine of the Angle $D=48^{\circ} 00' = 0.128927$
 is to the Sine of the Side $BC=38^{\circ} 45' = 9.796571$
 So is the Sine of the Angle $B=34^{\circ} 30' = 9.753128$

to the Sine of the Side sought $DC=28^{\circ} 30' = 9.678626$

Thus

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Thus the whole Triangle is resolved according to the Data of this Case.

23. Case II. Given two Angles $B=34^{\circ} 30'$, and $C=107^{\circ} 30'$; and a Side included $BC=38^{\circ} 45'$; to find the rest.

You let fall the Perpendicular CA, and find the Angle $BCA=61^{\circ} 48\frac{1}{2}'$, as in Art. 20. Then from the given Angle $C=107^{\circ} 30'$, take the Angle $BCA=61^{\circ} 48\frac{1}{2}'$, and there will remain the Angle $DCA=45^{\circ} 41\frac{1}{2}'$, by which you will find the Angle $D=48^{\circ} 00'$, according to the second Analogy of this Case. These Things being known, we may proceed

To find DC.

Co. Ar.

Thus, as the Co-Sine of $DCA=45^{\circ} 41\frac{1}{2}'=0.155821$
is to the Co-Sine of $BCA=61^{\circ} 48\frac{1}{2}'=9.674329$
So is the Tangent of . . . $BC=38^{\circ} 45'=9.94491$

To the T. of the Side sought $DC=28^{\circ} 30'=9.734641$

24.

To find BD.

From the given Angle B let fall the Perpendicular BN to the unknown Side DC produced; then by the first Analogy, find the Angle CBN, thus;

As Radius $90^{\circ} 00'=10$.
to the Co-Sine of $BC=38^{\circ} 45'=9.892030$
So is the Tangent of $CBN=72^{\circ} 30'=10.501278$

to the Co-Tangent of $CBN=22^{\circ} 00'=10.393308$

Then by the second Analogy, say;

As the Co-Sine of . . $DBN=56^{\circ} 30'=0.258208$
is to the Co-Sine of $NBC=22^{\circ} 00'=9.967151$
So is the Tangent of . . . $BC=38^{\circ} 45'=9.94491$

to the T. of the Side sought $DC=53^{\circ} 26'=10.129850$

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25. Case III. Given two Sides $BC=38^{\circ} 45'$, and $CD=28^{\circ} 30'$, and an Angle opposite to one of them, $B=34^{\circ} 30'$; to find the rest.

This Case being but the Converſe of Caſe I. I ſhall not repeat the Examples.

Caſe IV. Given two Sides $BC=38^{\circ} 45'$, and $BD=53^{\circ} 26'$, and the Angle included $B=34^{\circ} 30'$; to find the reſt.

This Caſe contains nothing difficult if what goes before be well underſtood; the Analogies being plain and eaſy for the Operations, I ſhall leave them to the Learner's Exerciſe; and paſs on to

26. Caſe V. Given all the three Sides, $BC=38^{\circ} 45'$, $DC=28^{\circ} 30'$, and $BD=53^{\circ} 26'$; to find the Angles.

To find the Angle C.

The Difference of the Legs containing the Angle ſought, is $BC-CD=AM=10^{\circ} 15'$; then $\frac{BD+AM}{2}=31^{\circ} 50\frac{1}{2}'$, and $\frac{BD-AM}{2}=21^{\circ} 35\frac{1}{2}'$; wherefore, according to the Analogy for this Caſe in the Synopſis, proceed thus:

The Sine of the Side . . . $BC=38^{\circ} 45'=9.796571$
Add the Sine of the Side $DC=28^{\circ} 30'=9.678663$

The Sum is the Log. of $sBC \times sDC=19.475234$

Again, the Sine of $\frac{BD+AM}{2}=31^{\circ} 50\frac{1}{2}'=9.722283$

Add the Sine of $\frac{BD-AM}{2}=21^{\circ} 35\frac{1}{2}'=9.565836$

The Sum is $s \frac{BD+AM}{2} \times s \frac{BD-AM}{2}=19.288119$

Then

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Then say ;

As $sBC \times sDC = 19.475234$
is to the Square of Radius . . . $Rq = 20.$

So is $s \frac{BD+AM}{2} \times s \frac{BD-AM}{2} = 19.288119$

to the square Sine of $\frac{1}{2}$ the } . . $Sq \frac{1}{2} C = 19.812885$
Angle sought

The half whereof is $s \frac{1}{2} C = 53^\circ 44' = 9.906442$

Wherefore the Angle sought is $C = 107^\circ 28'$; or
more compendiously thus, by the Arithmetical Com-
plement of the Sides BC, and DC.

27. The Sine of $BC = 38^\circ 45' = 0.203429$ Co. Ar.
The Sine of $DC = 28^\circ 30' = 0.321337$ Co. Ar.
The Sq. of Radius $Rq = 20.000000$

The Sine of $\frac{BD+AM}{2} = 31^\circ 50' \frac{1}{2} = 9.722283$

The Sine of $\frac{BD-AM}{2} = 21^\circ 35' \frac{1}{2} = 9.565836$

The Sum of all $sq \frac{1}{2} C = 19.812885$

Therefore $s \frac{1}{2} C = 53^\circ 44' = 9.906442$

Consequently . . . $C = 107^\circ 28'$; the same as before.

Having found one Angle, the others are easily found
by the former Cases.

Note, By Case I. the Angle C was found to be
 $107^\circ 30'$, which is but $2'$ different from what it is
found by this Case ; whence the Reader may observe
the wonderful Certainty and Agreement of the most
different Methods of Calculation, and from the great-
est Diversity of Data.

28. The sixth Case is but the same with this, if the three Angles given be changed into Sides, taking for the greatest Angle in that Triangle and greatest Side in this, their Supplements to 180. Degrees.

29. Having thus shewn the best Methods, both for ~~Exactness and Ease~~, of resolving the several Cases of *Right and Oblique-angled Triangles*, I shall leave the Application thereof to the various Problems of *Astronomy, Geography, Dialling, Orthodromics, &c.* for the Learner's own Recreation; as being in it self very easy, if what is here taught be understood; and also because I have both demonstrated the Theory, and very closely applied the Doctrine of *Spherical Trigonometry*, in the 2d Vol. of my *Young Trigonometer's Guide*, printed for Mr. J. Noon, at the *White Hart*, in *Christchurch*; and which I recommend to all unacquainted with, and who would have a good Notion of the noble and most useful Art of *Trigonometry*.



C H A P. XIII.

MERCATOR'S SAILING performed by the CANON of LOGARITHMIC TANGENTS, without the MERIDIONAL PARTS.

THE Property of *Mercator's Chart* is its having the Degrees of Latitude increased in the same Proportion as a Degree of Longitude decreases from the Equator to the Pole. Which Proportion is that of *Radius* to the *Co-Sine* of Latitude; or, of the *Secant* of the Latitude to the *Radius*; which is thus demonstrated.

2. Let AEB (Fig. VIII.) be a Sector in the Plane of the Equator, made by the Intersections of the Planes of two Meridians therewith, viz. AE, and

IX

B

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BE, whose Inclination, or Angle BEA, that is, the Arch of the Equator AB is = 1 Degree. Also let EC be the Radius of any Parallel of Latitude; then shall DC be an Arch in that Parallel similar to (AB) one Degree of Longitude in the Equator, or it is that Degree diminished. Now (from the Elements) the Arch AB is to the same diminish'd in the given Parallel DC, as the Radius of the Equator AE to the Radius of that Parallel CE; but since AE=EC (AC being the given Latitude) and because of similar Triangles EcC and ESA, therefore Ec (EA) : EC :: ES : EA :: AB : DC. But EC=Ec, the Co-Sine of the Latitude AC, and ES the Secant thereof; therefore, &c. which was to be demonstrated.

3. And because in *Mercator's Projection* or *Chart*, the Meridians and Parallels are all represented by *parallel Right-Lines*, the Arch CD in every Parallel is ever equal to that in the Equator AB; therefore that the true Proportion between Longitude and Latitude might be preserved on this Chart, as on the Globe it self, 'twas necessary the several Degrees of Latitude should be such as belong to those Circles whose Radii are severally equal to the Secants of those Latitudes.

4. Let A = the Arch of one Degree in any Latitude; a, b, c, &c. = the several enlarg'd Degrees on the Meridian; R = Radius; and S, s, s, &c. the Secants of those Latitudes. Then it will

$$\text{be } \left\{ \begin{array}{l} \text{A 1st : } a :: R : S. \\ \text{A 2d : } b :: R : s. \\ \text{A 3d : } c :: R : s, \text{ \&c.} \end{array} \right\} \begin{array}{l} \text{Therefore we shall have} \\ 3A : a+b+c :: 3R : S+s \\ \hspace{10em} s+s. \end{array}$$

That is, the Sum of the Secants of 1, 2, 3 Degrees is always equal to the Distance of the Parallel of those three enlarg'd Degrees from the Equator. And consequently by a continual Addition of the Secants of $\frac{1}{2}$, $1\frac{1}{2}$, $2\frac{1}{2}$, $3\frac{1}{2}$, &c. or their Doubles 1', 3', 5', 7', &c. the Line or Table of *Meridional Parts* may be made; and by some has been made in this manner

D d

for

for every Minute of the Quadrant; but a truer way is that delivered in Chap. X. of the Theory, which see.

5. If then you draw any Line AB (Fig. X.) to represent an Arch of the Equator, as suppose 60 Degrees; and on every tenth Degree be erected Perpendiculars, these shall be the *Meridians*; on which if the Degrees of Latitude, enlarg'd in the Manner and Proportion above describ'd, be set off; and those Divisions join'd by Right-Lines, representing the Parallels of Latitude, *Mercator's Chart* will be constructed for the given Longitude and Latitude; and a *Meridian* thus graduated, is what is called the *Nautical or Meridian Line*. This Chart, as here drawn, includes 60° Longitude, and 80° Latitude, that the Reader may have a perspicuous Idea thereof.

6. Now suppose F be the Latitude whence you sail, and C the Latitude you arrive to, on the Rhumb FC; 'tis plain there will be formed a Right-angled plain Triangle FCD, wherein FD is the *enlarg'd Difference of Latitude*, and CD the *true Difference of Longitude*, and the Angle CFD the *Course or Rhumb*; and therefore any two of these being given, the rest are found by the Analogies of plain Triangles before-going.

7. Since every Degree is equal to 60 Minutes or Nautical Miles, let the *proper Difference of Latitude* be reduc'd to these Parts, and set (off the same Scale as AB was laid down by) from F to E, and draw EK parallel to DC; so shall there be form'd another Right-angled plain Triangle FEK; in which FE is the *proper Difference of Latitude*; EK the *Departure*; KF the *Distance* sail'd on the Rhumb or Course EFK. Any two of which Parts being given, the others are to be found as aforesaid.

8. And these two Triangles FCD, FKE, comprehending all the Particulars of *Mercator's Sailing*; 'tis obvious enough how they are all resolv'd, either
by

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by *Projection* or *Trigonometrical Calculation*; but my Purpose is here to shew, that without those *common Methods*, they are all to be resolv'd by the Canon of Logarithm-Tangents only; in order to which, the following Articles must be premis'd and duly observ'd, *viz.*

9. First, That it has been shewn, the *Nautical or Meridian Line*, or Scale of *Mercator's Chart*, is no other than a Scale of Logarithm-Tangents of the Half-Complements of the Latitude. Secondly, that such Logarithm-Tangents of Mr. *Brigg's Form*, (or those in common Use) are a Scale of the Differences of Longitude upon the Rhumb, which makes an Angle of $51^{\circ} 38' 9''$ with the Meridian. Thirdly, the Differences of Longitude on differing Rhumbs, are as the Tangents of the Angles of those Rhumbs with the Meridians; as is evident from the Triangle FCD (Fig. X.) And fourthly, that the Logarithm-Tangent of the Angle $51^{\circ} 38' 9''$, *viz.* 10.1015104 is a constant Factor in these Computations. On these Premises 'tis easy to operate the Propositions of *Mercator's Sailing*, as follows.

10. In Fig. 11. let F represent the *Lizard's Point*, whose Latitude North $DF = 49^{\circ} 55'$; also let K, represent *Barbadoes* in Latitude North $13^{\circ} 10' = CK$; then is FE = the Difference of Latitude; CD the Difference of Longitude; and EFK the Angle of the Rhumb, FK with the Meridian. Lastly, let FO be the Rhumb making an Angle of $52^{\circ} 38' 9''$ with the Meridian; or that on which the Difference of Longitude QD is the Difference of the Logarithms of the Tangents of half the Complements of the Latitude PF, PK, or PO: Then,

11. Case I. Given the Latitude of the *Lizard*, $49^{\circ} 55' N.$ and that of *Barbadoes* $13^{\circ} 10' N.$ and their Difference of Longitude $53^{\circ} 00' =$

D d 2

3180

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3180 Nautic Miles : I demand the Course
and Distance fail'd?

$$\left. \begin{array}{l} \text{The Lat.} \\ \text{CK}=13^{\circ} 10' \\ \text{DF}=49^{\circ} 55' \end{array} \right\} \begin{array}{l} \text{Comp.} \\ \text{PK}=76^{\circ} 50' \\ \text{PF}=40^{\circ} 05' \end{array} \left\{ \begin{array}{l} \text{the Halfs.} \\ 38^{\circ} 25' \\ 20^{\circ} 2\frac{1}{2}' \end{array} \right\}$$

Then the Log. Tangent of $\left\{ \begin{array}{l} 38^{\circ} 25' = 9.899308 \\ 20^{\circ} 2\frac{1}{2}' = 9.562048 \end{array} \right\}$

The Difference of which is . . . $\text{QD}=3372,60$

(Note, The four first Figures are *Integers*, according
to the Theory)

Then say, for the Course KFE;

As the Diff. of Log. Tang. $\text{QD}=3372,6=3.527965$

to the given Diff. of Long. $\text{CD}=3180=3.502427$

So is the const. Tang. of OFE $=51^{\circ} 38' 9''=10.101510$

3.603937

to the Tangent of the } KFE $=49^{\circ} 59'\frac{1}{2}=10.075972$
Course sought

Secondly, For the Distance FK.

The Difference of Latitude is $49^{\circ} 55' - 13^{\circ} 10' =$
 $36^{\circ} 45' = 2205$ Miles;

Say, as Radius

10.

to the Diff. of Lat. . . . EF $=2205=3.343409$

So is the Secant } KFE $=49^{\circ} 59' 10''=10.191807$
of the Course

to the Distance fail'd FK $=3429,38=3.535216$

By this Proposition you estimate the Course a Ship
must steer, and the Distance of her Port.

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12. Case II. Given the Latitude of the *Lizard* $49^{\circ} 55'$ N. and of *Barbadoes* $13^{\circ} 10'$ N. and the Course $49^{\circ} 59' \frac{1}{2}$, to find the Difference of Longitude, and Distance sailed.

Things being prepared as before, say ;

As the Log. Tang. of OFE $= 51^{\circ} 38' 9'' = 10.101510$

to the Tang. of } KFE $= 49^{\circ} 59' 10'' = 10.075972$
the Course }

So is the Diff. of the Log. } QD $= 3372,6 = 3.527965$
Tang. of the $\frac{1}{2}$ Comp. }
of the Latitudes

3.603937

to the Diff. of Long. required CD $= 3180 = 3.502427$

Which converted into Degrees makes $53^{\circ} 00'$. And so much is the Difference of Motion or Time between these two Places. The Distance on the Rhumb will be found $3429,38 = FK$, as before.

13. Case III. Given the Latitude of the *Lizard* $49^{\circ} 55'$ N. the Distance sailed $3429,38$ Miles on a Course $49^{\circ} 59' 10''$ Southwesterly, 'tis required to find the Latitude and Longitude of the Place to which the Ship is arrived.

As Radius 10.

to the Distance sailed FK $= 3429,38 = 3.535216$

So is the Co. Sine of } KFE $= 49^{\circ} 59' 10'' = 9.808193$
the Course }

to the Diff. of Latitude FE $= 2205 = 3.343409$

Then from the given Lat. of the *Lizard* $49^{\circ} 55'$
Subd. the Diff. of the Lat. $2205 = 36^{\circ} 45'$

There remains the Lat. sought . . . $= 13^{\circ} 10'$

And

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And having thus obtained the Latitudes, the Difference of Longitude will be found, as *per* Case II. to be 3180 Miles or 53 Degrees; and so the Operation needs not to be repeated.

14. If it so happen that the Ship passes the *Equator*, and consequently has one Latitude *North*, the other *South*; then observe two things; first, that both the Complements of the Latitudes are to be estimated from the same Pole of the World. And therefore, secondly; suppose you sail from a *Northern* to a Southern Latitude, you must add 90° to the *former*, and subtract the latter from 90° ; then subtract this Sum and Remainder from 180° , and take the Difference of the Logarithm-Tangents of half the Remainders, as before.

15. Case IV. Suppose I sail from Latitude $48^{\circ} 30' N.$ to Latitude $23^{\circ} 45' S.$ on a Course $43^{\circ} 50'$ Southwesterly; required the *Difference, Longitude, and Distance* sailed?

Then $48^{\circ} 30' + 90 = 128^{\circ} 30'$; and $90 - 23^{\circ} 45' = 66^{\circ} 15'$. And

$180 - 66^{\circ} 15' = 113^{\circ} 45'$ } $\frac{1}{2}$ { $56^{\circ} 52\frac{1}{2}' = 10.185410$
 $180 - 128^{\circ} 30' = 51^{\circ} 30'$ } $\frac{1}{2}$ { $25^{\circ} 45' = 9.683356$

The Diff. of those Tangents is . . . 5020,54

Therefore say, for the Difference of Longitude;

As the Log. Tangent of $51^{\circ} 38' 9'' = 10.101510$

is to the Diff. of the Log. Tang. 5020,54 = 3.700750

So is the Tang. of the Course $43^{\circ} 50' = 9.982309$

3.683059

to the Diff. of Long. required $3815,5 = 3.581549$

That is, in Degrees, $= 63^{\circ} 35\frac{1}{2}'$.

To

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To find the Distance fail'd ; say,
 As Radius 10.
 to the Sum of the Lat. $72^{\circ} 15' = 4335 = 3.636989$
 So is the Secant of the Course $43^{\circ} 50' = 10.141849$
 to the Dist. fail'd in Miles, $6009,5 = 3.778838$

16. Thus you have all the *Practical Cases* of *Mercator's Sailing* performed by the *Canon of Logarithm-Tangents* only, without the *Meridional Parts* or *Chart*, as in the common Way. And since this is the most exact, and natural Method of Navigation (next to the *Globular Chart* it self) and wholly resolvable by *Logarithms*, it adds not a little to the (before invaluable) Estimation of those excellent Numbers ; and renders their Use to Navigators more necessary than before.



C H A P. XIV.

Of the MENSURATION of SUPERFICIES and SOLID BODIES by LOGARITHMS.

I. **A**mongst the Variety of Methods for measuring the *Surfaces* and *Solidity* of *Bodies*, I intend here to shew the Excellency of that by *Logarithms* ; which may justly be allow'd the *Preference* to all others in point of *Ease* and *Brevity*, Advantages none of the least in common Estimation. And since these Operations consist altogether in *Multiplication* and *Division*, I need not here repeat them by the *Instruments*, as having already largely shewn the Manner thereof in a *Chapter* particularly on that Head.

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2. *To measure a SQUARE, Fig. XII.*

The Logarithm of the Side multiplied by 2, gives the Logarithm of the *Area* or *superficial Content*.

Examp. Let the Side of the Square be $AB=31,57$.

Then the Logarithm of $AB=31,57=1.499275$
which multiply by 2

The Product is the } $ABCD=996,66=2.998550$
Area of the Square }

3. *To measure a PARALLELOGRAM, Fig. XIII.*

The Sum of the Logarithms of the Length and Breadth is the Logarithm of the Area.

Examp. The Length $AB=41,5$ and Breadth $BC=31,57$.

To the Log. of the Length $AB=41,5=1.618048$
add the Log. of the Breadth $BC=31,57=1.499275$

The Sum is the Log. } $ABCD=1312,155=3.117323$
of the Parallelog. }

4. *To measure a RHOMBUS ABCD, Fig. XIV.*

The Sum of the Logarithms of a Side and the Perpendicular Height, is the Logarithm of the Area.

Examp. Let the Side $AB=15,5$; and the Perpendicular $BE=13,42$.

Then to the Log. of the Side $AB=15,5=1.190332$
add the Log. of the Perpend. $BE=13,42=1.127752$

The Log. of the Area of } $\dots = 208,01 = 2.318084$
the Rhombus }

5. *To measure a RHOMBOIDES, ABCD, Fig. XV.*

The Sum of the Logarithm of the longest Side, and perpendicular Height is the Logarithm of the Area.

Exam. Let the longest Side $AB=19,5$, and perpendicular Height $BE=6,07$

Then to the Log. of the Side $AB=19,5=1.290035$
Add the Log. of the Perpend. $BE=6,07=0.783189$

The Log. of the Area of the }
Rhomboides } $=118,365=2.073224$

6. *To measure a TRIANGLE ABC. Fig. XVI.*

The Sum of the Logarithm of the Base, and of half the perpendicular Height (or *vice versa*) is the Logarithm of the Area.

Exam. Let the Base $BC=65,25$; and the perpendicular Height $AG=21,5$; then $\frac{1}{2} AG=10.75$.
Therefore,

To the Log. of the Base . . . $BC=65,25=1.814580$
Add the Log. of half the Perp. $\frac{1}{2} AG=10,75=1.031408$

The Log. of the Area $=701,4375=2.845988$

7. *To measure a CIRCLE ABCD. Fig. XVII.*

In order to this, let $\begin{cases} D=\text{the Diameter.} \\ P=\text{the Periphery.} \\ A=\text{the Area.} \end{cases}$

Then the Rules or Theorems for finding those several Parts are as follows ;

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Theo. I. $3.1416D=P$. Th. II. $0.7854DD=A$.

Theo. III. $0.3183P=D$. Th. IV. $0.07957PP=A$.

Theo. V. $\sqrt{1.2732A}=D$. Th. VI. $\sqrt{12.5664A}=P$.

8. Therefore the Diameter AB being given, suppose $= 20.15$, to find the Periphery P? *per Theor. I.*

Thus, to the Log. of the }
Diameter } $AB=20.15=1.304275$
Add the Log. of the con- }
stant Number, ... } $3.1416=0.497151$

The Log. of the Periphery $P=63.303, \&c.=1.801426$

9. Given the Periphery of a Circle $P=63.303, \&c.$ to find the Diameter D? *per Theor. III.*

To the Log. of the Pe- }
riphery } $P=63.303, \&c.=1.801426$
Add the Log. of the }
constant Number... } $0.3183, \&c.=9.502837$

The Log. of the Diameter D or $AB=20.15=1.304263$

10. Having the Diameter given, suppose $= 20.15$, to find the Area A of the Circle? *per Theor. II.*

The Log. of the Diameter .. $AB=20.15=1.304275$
Multiply by 2

The Product is the Log. of $DD=2.608550$
To which add the Log. }
of the constant Numb. } $0.7854=9.895091$

The Log. of the A- }
rea of the Circle } $ACBD=318.89, \&c.=2.503641$

11. Or thus supposing the Periphery given, = 63,303,
Ec. per Theor. IV.

To twice the Log. of $P=63.303=\left\{ \begin{array}{l} 1.801426 \\ 1.801426 \end{array} \right.$
 Add the Log. of the constant }
 Number } $0.07957=8.900749$

The Sum is the Log. of } $A=318.89, Ec.=2.503601$
 the Area }

12. To measure the SECTOR of a CIRCLE ACB,
Fig. XVIII.

The Sum of the Logarithms of the *Radius*, and
 $\frac{1}{2}$ the *Arch* (or of the Arch and half the *Radius*) is
 the Logarithm of the *Area*.

Examp. Suppose the Radius $AC=12.36$, and the
 Arch AB 10,12; then $\frac{1}{2}AB=5.11$.

Then to the Log. of Radius $AC=12.36=1.092018$
 Add the Log. of $\frac{1}{2}AB=5.11=0.707570$

The Log. of the Area } $ACB=63.2596=1.799588$
 of the Sector }

13. To measure the SEGMENT of a Circle, as
 AFBG, *Fig. XIX.*

The best way is to find the Centre C; as by this
 Theorem, viz. $\overline{FB}^2 - FG = N$, then $\frac{N+FG}{2} = CG$;
 whence C is given; and finding the Area of the
 whole Sector ACBG, (as *per* last Article,) and the
 Area of the Triangle ABC, (*per* Art. 6.) if the latter
 Area be subducted from the former, it will leave
 the Area of the Segment required, AFBG,

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14. To measure a SPHERICAL TRIANGLE, ABC. Fig. XX.

From the Sum of the three Angles A, B, C, take 180 Degrees; then from the Logarithm of the Remainder subduct the Logarithm of the constant Number 720; to that Remainder add the Logarithm of the Superficies of the whole Sphere; the Sum shall be the Logarithm of the Area required.

Exam. Let the $\begin{cases} A=37^{\circ} & 30' \\ B=92 & 30 \\ C=60 & 45 \end{cases}$
Angles be

Their Sum is $190^{\circ} & 45'$

Subtract $180 & 00$

There remains $10^{\circ} & 45' = 10,75 = 1.031408$

Subduct the Logarithm of $720 = 2.857332$

8.174076

Suppose the Superficies of the } $= 1257,3 = 3.099439$
Sphere }

The Sum is the Log. of the } $ABC = 23,623 = 1.273515$
Area of the Triangle . . }

15. To measure an ELLIPSIS, as ACBD, Fig. XXI.

The Sum of the Logarithms of the Transverse Diameter AB, the Conjugate Diameter CD, and the constant Number 0.7854; is the Logarithm of the Area.

Exam. Let $AB=61,6$, and $CD=44,4$.

Then add the Logarithms of $\begin{cases} AB=61,6=1.789581 \\ CD=44,4=1.647383 \\ N. a. 7854=9.895091 \end{cases}$

The Log. of the Area } $= 2148,1004, \&c. = 3.332055$
of Ellipsis }

16.

16. To measure a PARABOLA, as ACD, Fig. 22.

From the Sum of the Logarithms of the Base AB, the perpendicular Height CD, and the Number 2, subtract the Logarithm of 3; the Remainder is the Logarithm of the Area.

Exam. Let $AB=61,6$, and $CD=44,4$; as in the Ellipse.

Then add the Logarithms of $\left\{ \begin{array}{l} AB=61,6=1.789581 \\ CD=44,4=1.647383 \\ \text{and N. 2}=0.301030 \end{array} \right.$

3.737994

Subtract the Logarithm of $\dots\dots\dots 3=0.477121$

The Log. of the Area required $=1823,36=3.260873$

17. To measure any REGULAR POLYGON, Fig. XXIII.

In order to this the following Table will be very expedient.

	Sides.	Names.	Numbers.
A Table for the more ready find- ing the A- rea of any Regular Polygon.	5	Pentagon.	1.72048
	6	Hexagon.	2.59808
	7	Heptagon.	3.63896
	8	Octagon.	4.82843
	9	Enneagon.	6.18183
	10	Decagon.	7.69421
	11	Endecagon.	8.51425
	12	Dodecagon.	9.33012

Then the Sum of double the Logarithm of the Side of the Polygon, and of the Number in the Table proper to it, is the Logarithm of the Area.

Exam. Let the *Pentagon* ABCDE be proposed, and let its Side be $AB=14,6$.

Then

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Then twice the Logarithm of $AB=14,6$ $\left\{ \begin{array}{l} 1.164353 \\ 1.164353 \end{array} \right.$
 Add the Log. of the *tabular Numb.* $1.72048=0.235649$

The Log. of the Area required $=366,74=2.564355$

In the same manner you find the Area of any other Polygon mention'd in the Table.

18. Of the MENSURATION of SOLIDS.

To measure a CUBE ABCDFGE, *Fig.* XXIV.

Three times the Logarithm of the Side is the Logarithm of the Solidity.

Exam. Let the Side $AB=31.57=1.499275$
 Multiply by 3

The Log. of the Solidity ... $31464,81=4.497825$

19. To measure a PARALLELOPIPEDON AD, *Fig.* XXV.

The Sum of the Logarithms of the *Breadth*, *Depth*, and *Length*, is the Logarithm of the Solidity.

Exam. Let the Width $AB=21,56$; the Length $AG=31,57$; and Depth $GF=9,03$.

Then add the Logarithms of $\left\{ \begin{array}{l} AG=31,57=1.499275 \\ AB=21,56=1.333649 \\ GF=9,03=0.955688 \end{array} \right.$

The Log. of the Solidity } $=6146,2623=3.788612$
 required

20. To measure a PRISM ABCDEF, *Fig.* XXVI.

First find the *Area* of the *Base*, whether a *Triangle*, *Square*, &c. Then the Sum of the Logarithms of the

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the said *Area*, and Length of the *Prism*, is the Logarithm of its *Solidity*.

Exam. Suppose a *Prism* of a *triangular Base*, as in the Figure, then let its *Area* be $ABC=701,4375$, and its Length $BD=70,15$.

Then add the Loga- $\left\{ \begin{array}{l} ABC=701,4375=2.845988 \\ \text{rithms of both} \dots \end{array} \right. \left. \begin{array}{l} BD=70,15=1.846028 \end{array} \right.$

The Log. of the Solidity $=49205,84, \&c.=4.692016$

21. To measure a PYRAMID ABCDE. Fig. XXVII.

First find the *Area* of its *Base*, whether *triangular*, *quadrangular*, &c. then the Sum of the Logarithms of the *Area* of the *Base*, and $\frac{1}{3}$ of the *perpendicular Height*, is the Logarithm of the *Solidity*.

Exam. Let the *Prism* have a *quadrangular Base* $ABCD=342,25$; and the *perpendicular Height* $GE=180$; then $\frac{1}{3}GE=60$.

Therefore add the Lo- $\left\{ \begin{array}{l} ABCD=342,25=2.534343 \\ \text{garithms of} \dots \end{array} \right. \left. \begin{array}{l} \frac{1}{3}GE=60=1.778151 \end{array} \right.$

The Log. of the Solidity $\dots =20535=4.312494$

22. To measure a CYLINDER ACBDEF;
Fig. XXVIII.

The Sum of the Logarithms of the *Area* of its *Base*, and of the *Height*, is the Logarithm of the *Solidity*.

Exam. Let the *Area* of the *Base* be $380,15=AGBH$; and the *Height* $BD=50,05$.

Then add the Loga- $\left\{ \begin{array}{l} AGBH=380,15=2.579955 \\ \text{rithms of} \dots \end{array} \right. \left. \begin{array}{l} BD=50,05=1.699404 \end{array} \right.$

Log. of the Solidity $\dots =19026,5075=4.279359$

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22. To measure a CONE AEBFD, *Fig. XXIX.*

The Sum of the Logarithms of the *Area* of the *Base*, and $\frac{1}{3}$ of the perpendicular Height, is the Logarithm of the *Solidity*.

Exam. Let the Area of the Base be $100.75 = \text{AEBF}$;
and the perpendicular Height $CD = 19.95$; and
therefore $\frac{1}{3}CD = 6.65$.

Then add the Loga- $\left\{ \begin{array}{l} \text{AEBF} = 100.75 = 2.003245 \\ \text{rithms of } \dots\dots\dots \left\{ \begin{array}{l} \frac{1}{3}CD = 6.65 = 0.822822 \end{array} \right. \end{array} \right.$

The Sum is the Log. of the } $669.9875 = 2.826067$
Solidity $\dots\dots\dots$

23. To measure the FRUSTUM of a Pyramid. *Fig. XXX.*

If it be a square $\left\{ \begin{array}{l} D = \text{Side of the greater Base AB.} \\ \text{Base as ABCD,} \left\{ \begin{array}{l} d = \text{Side of the lesser Base EF.} \\ \text{then put } \dots \left\{ \begin{array}{l} D - d = x \text{ and } H = \text{the Height GO.} \end{array} \right. \end{array} \right.$

Then we have this Theorem $\overline{Dd} + \frac{1}{3}xx \times H = \text{the Solidity.}$

Exam. Suppose $D = 50$, $d = 21$; and $H = 105.6$;
then $D - d = x = 29$.

Then add the Logarithms of $\left\{ \begin{array}{l} D = 50 \dots = 1.698970 \\ d = 21 \dots = 1.322219 \\ H = 105.6 = 2.023664 \end{array} \right.$

The Logarithm of $\dots DdH = 110880 = 5.044853$

Again, add the Logarithms of $\left\{ \begin{array}{l} \frac{1}{3}xx = 280.3 = 2.447673 \\ H = 105.6 = 2.023664 \end{array} \right.$

The Logarithm of $\dots \frac{1}{3}xxH = 29603.2 = 4.471337$
To which add $\dots DdH = 110880$

The Sum is $\dots \overline{Dd} + \frac{1}{3}xxH = 140483.2 = \text{the Solidity required.}$

24. But if the Bases of the Fruſtum be *triangular*; the Theorem is $Dd + \frac{1}{3}xx \times 0.433H = \text{the Solidity}$. Again, if the Baſe be any of the regular Polygons; then put $N = \text{the Number in Table at Art. 17. proper to the Polygon}$, and the Theorem will be $Dd + \frac{1}{3}xx \times NH = \text{the Solidity}$.

25. To meaſure the Fruſtum of a RIGHT CONE.

Fig. XXXI.

Let $D = \text{the Diameter of the great Baſe AB}$, and $d = \text{the Diameter CD of the leſſer Baſe}$; $D - d = x$, and $H = \text{the perpendicular Height, as before}$; then the Theorem for the *Solidity* will be $\frac{Dd + \frac{1}{3}xx \times H}{1.27323}$, or thus $Dd + \frac{1}{3}xx \times 0.7854H = \text{Solidity}$.

Exam. Let $AB = 16$, $CD = 12$, and $GO = 9$: then $D - d = x = 4$; and $\frac{1}{3}xx = 5.3$: alſo $Dd = 192$; and ſo $Dd + \frac{1}{3}xx = 197.3$.

Then add the Logarithms of $\left\{ \begin{array}{l} Dd + \frac{1}{3}xx = 197.3 = 2.295199 \\ 0.7854 = 9.895091 \\ H = 9 = 0.954242 \end{array} \right.$

The Log. of the Solidity $\therefore = 1394.8704 = 3.144532$

26. To meaſure a GLOBE or SPHERE ACBD,
Fig. XXXII.

The Sum of the triple Logarithm of the Diameter, and the Logarithm of the conſtant Number 0.5236 is the Logarithm of the Solidity of the Sphere.

Exam. Suppose the Diameter of a Sphere $AB = 50.37$.

F f

Then

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$$\text{Then add the Logarithms } \left\{ \begin{array}{r} AB=50,37=1.702172 \\ 1.702172 \\ 1.702172 \\ \hline 0,5236=.9.718999 \end{array} \right.$$

$$\text{The Log. of the Solidity } \left\{ \begin{array}{r} \text{of the Sphere} \dots\dots\dots \end{array} \right\} = 66913,8 = 4.825515$$

27. To measure the Superficies of a Sphere.

The Sum of Logarithm of double the Diameter, and the constant Number 3.1416, is the Logarithm of the Superficies of the Sphere.

Exam. Let the Diameter $AB=50.37$.

$$\text{Then add the Logarithms of } \left\{ \begin{array}{r} AB=50.37=1.702172 \\ 1.702172 \\ 3,1416=0.497151 \\ \hline \end{array} \right.$$

$$\text{The Log. of the Superficies} \dots\dots\dots 7970,7 = 3.901495$$

28. To measure the SEGMENT of a Sphere,
Fig. XXXIII.

Let $\left\{ \begin{array}{l} D = \text{The Axis or Diameter of the Sphere CD.} \\ C = \text{Half the Diam. of the Segment's Base EB.} \\ H = \text{The Height of the Segment ED.} \end{array} \right.$

Then we have the two following Theorems for finding the Solidity.

$$\text{viz. } \left\{ \begin{array}{l} \text{Theor. I. } \frac{3CCH + H^3}{\times 0.5236} = \text{the Solidity.} \\ \text{Theor. II. } \frac{3DH^2 - 2H^3}{\times 0.5236} = \text{the Solidity.} \end{array} \right.$$

29. To measure a SPHEROID ACBDA,
Fig. XXXIV.

The Sum of the *double Logarithm* of the *Lesser*, the *Logarithm* of the Greater Diameter, and the *constant*

constant Number 0.5236, is the Logarithm of the solid Content of the *Spheroid*.

Exam. Let the lesser Diameter $CD=33$; and the greater Diameter $AB=55$.

Then the double Log. of . . . $CD=33 \begin{cases} =1.518514 \\ =1.518514 \end{cases}$

The Logarithm of $AB=55=1.740363$

The Logarithm of 0.5236=9.719000

The Log. of the Solidity ..= $31361,022=4.496391$

30. To measure the SEGMENT of a SPHEROID.

As the Solidity of the Sphere $AFBE$ is to the Solidity of its Segment AGK ; so is the Solidity of the Spheroid $ACBD$ to its like Segment AHI .

31. To measure a PARABOLIC CONOID $ACBD$,
Fig. XXXV.

The Sum of the double Logarithm of the *Diameter* of the *Base*, the Logarithm of the *Height*, and *constant Number* 0.3927; is the Logarithm of the Solidity of the *Conoid*.

Exam. Let the Diameter of the Base $AB=55$; and its Height $CD=33$.

Then the double Log. of . . . $AB=55 \begin{cases} =1.740363 \\ =1.740363 \end{cases}$

The Logarithm of $CD=33=1.518514$

The Logarithm of 0.3927=9.594061

The Logarithm of the Solidity 39201,4=4.593301

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32. To measure the FRUSTUM of a PARABOLIC CONOID, *Fig. XXXVI.*

To this end, let $\begin{cases} D = \text{Diameter of the greater Base AB.} \\ d = \text{Diameter of the lesser Base CD.} \\ H = \text{the Height of the Frustum FE.} \end{cases}$

Then we have this Theorem $\overline{DD + dd} \times 0.3927 H = \text{the Solidity.}$

33. To measure a PARABOLIC SPINDLE, *Fig. XXXVII.*

The Sum of the double Logarithm of its Thickness, the Logarithm of its Length, and the Logarithm of the constant Number 0.41888, is the Logarithm of the Solidity of the Spindle.

Exam. Let the Diameter of its greatest Circle, or Thickness $AB = 43.45$; and the Length $CD = 50.075$.

Then the double Log. of $AB = 43.45 \begin{cases} = 1.637990 \\ = 1.637990 \end{cases}$

The Logarithm of $CD = 50.075 = 1.699621$

The Logarithm of 0.41888 = 9.622090

The Log. of the Solidity . . = 39599.6 = 4.597691

34. To measure any of the FIVE REGULAR BODIES, *Fig. XXXVIII.*

For this purpose the following Table is necessary.

Names.	Superficies.	Solidity.
Tetrahedron	1.73205	0.11785
Octahedron	3.46410	0.47140
Hexahedron	6.00000	1.00000
Icosahedron	8.66025	2.18169
Dodecahedron	20.64573	7.66312

Exam.

Exam. Suppose the Side $AB=12$, of the *Icosahedron*,
Fig. XXXVIII.

The Sum of the Logarithm of the *tabular Number*, and double Logarithm of the Side is the Logarithm of the Superficies; and the Sum of the Logarithm of the *tabular Number*, and triple Logarithm of the Side, is the Logarithm of the Solidity.

Thus the double Log. of . . . $AB=12 \left\{ \begin{array}{l} =1.079181 \\ =1.079181 \end{array} \right.$

The Log. of the *tab. Number* .. $8,66025=0.937530$

The Log. of the *Superficies* $=1247,0688=3.095892$

Again the Logarithm of . . . $AB=12=1.079181$
3

Add the Log. of the *tab. Numb.* $2.18169=0.338793$
3.237543

The Log. of the Solidity . . $=3769,9458=3.576336$

35. These Propositions are sufficient for measuring any *Superficies* or *Solid Body* in common Use; and the great Ease and Conciseness of performing the same by Logarithms, is abundantly manifest from these Examples.





C H A P. XV.

The DOCTRINE of MENSURATION applied to GAUGING, MEASURING TIMBER, and SURVEYING; wherein the PRACTICAL USE of the PLAIN SCALE and SLIDING RULE, for these Purposes, is clearly explain'd.

1. **I**N the preceeding Chapter you have the *practical Method of absolute or general Mensuration of the Content of Superficies and Solids*, laid down in divers Propositions; my Business is here to apply that general Doctrine to particular Uses as those of *Gauging, Timber-Measure, and Surveying*. For tho' I there shew'd how the *Dimensions* of Bodies might be express'd in *Numbers*; yet since these Numbers represent *divers kinds of Quantities*, as *Inches, Feet, Yards, Poles, Chains, &c.* at pleasure, so they are equally subservient to those several Arts above-mention'd, and must be reduced to those *Denominations or Measures* which are *peculiar* to each. I shall begin therefore with

2.

Gauging.

In this Art the *Dimensions* are taken in *Inches*, and *Decimal Parts* thereof (or must be reduced to such;) and by *Inches* is here to be understood *solid or cubic Inches*; and that as well in *superficial* as *solid Measure*. For though it be improper, *geometrically speaking*, to ascribe *Thicknes*s to a *Superficies*, the *Gaugers* always consider them as *one Inch deep*, and accordingly compute the *superficial Content* in *Gallons*, or *solid Measure*.

3.

3. As *Inches* are the *lineal Dimensions* of *Gaugers*, so *Gallons* are the common *Quantities* of their *solid Measure*, of *liquid Substances* especially; and *Busbels* for *solid dry Measure*, as *Malt*, *Corn*, &c. Now the *standard Gallons* and *Busbel*, as used in *England*, are as follows: the *Ale* or *Beer Gallon* = 282 *solid Inches*; the *Wine Gallon* = 231; the *Corn Gallon* = 268,8; and the *Corn Busbel* = 2150,42 *Cubic Inches*.

4. From whence it follows, that, supposing the *Dimensions* of the *Superficies* in the foregoing Chapter taken in *Inches*, the several *Area's*, or *superficial Contents*, will by the *Gauger* be understood to be so many *solid* or *cubic Inches*; which therefore if he divide by 282, 231, 268,8, or 2150,42, the *Quotients* will be the several *Contents* in the respective *Gallons* or *Busbels*. For instance, in Fig. XII. the Side of the Square AB = 31,57 *Inches*, and the *Area* was therefore found (Art. 2.) to be 996,66 *solid Inches*.

Conse- quently {	282	)996,66(=3,53	Ale Gallons.
	231	)996,66(=4,31	Wine Gallons.
	268,8	)996,66(=3,707	Corn Gallons.
	2150,42	)996,66(=0,46	Corn Busbels.

5. But since *Multipliers* are the *Reciprocals* of *Divisors*, therefore *Unity* divided by these *Divisors* will produce so many *Multipliers*. Thus,

282	)1.00000(=0.003546	the Mult. for A. Gall.
231	)1.00000(=0.004329	the Mult. for W. Gall.
268,8	)1.00000(=0.003722	the Mult. for C. Gall.
2150,42	)1.00000(=0.000465	the Mult. for C. Busb.

Consequently, if any *Area* be

Divided by .. {	282	} or multi- plied by {	{ 0.003546 } the Area {	{ 0.004329 } will be {	{ 0.003722 } expressed {	{ 0.000465 } in . . . {	A. Gall.
	231						W. Gall.
	268.8						C. Gall.
	2150,42						C. Busb.

6. In order to work these Dimensions by the *artificial Line of Numbers*, either on the *Plain Scale*, or *Sliding-Rule*; put B=Breadth, L=Length, D=Depth or Thickness, G=Standard Gallon; and A=Area, and S=Solid Content in those Gallons, &c. Then for *Superficies*; $\frac{LB}{G} = A$; and so $LB = AG$; therefore $G : B :: L : A$. That is, the *Ratio*, or *Logarithm* of the *Ratio* of the *Standard Gallon* to the *Breadth*, is equal to that of the *Ratio* of the *Length* to the *Area* in Gallons. Wherefore supposing $B=31,57$, $L=41,5$ (as in the *Parallelogram* Fig. XIII. Art. 3.) if you set one Foot of the *Compasses* in 282, and extend the other to 31.57, that Extent will reach from the Length 41.5 to $4\frac{1}{5}$ Gallons, Ale Measure. Or, on the *Sliding-Rule*, set 282 on the Rule to 31.57 on the Slider, then against 41.5 on the Rule is $4\frac{1}{5}$ on the Slider, the *Area* in Ale Gallons, as before; and thus the *Area* is found in *Wine* or *Corn Gallons*.

7. Again, Since $AD=S \times 1$, therefore $1 : A :: D : S$. That is, the *Logarithm* of the *Ratio* of Unity 1 to the *Area* in Gallons A, is equal to that of the *Ratio* of the *Depth* (or *Height*) D to the *Solidity* (or *Capacity*) S, in Gallons of the same kind. Wherefore if $B=21,56$, $L=31,57$, $D=9,03$, as in *Parallelopipedon*, Fig. XXV. then $G=282 : B=21,56 :: L=31,57 : A=2\frac{1}{5}$ the *Area* in Ale Gallons; consequently $1 : A=2\frac{1}{5} :: D=9,03 : S=21\frac{1}{5}$, the *solid Content* in Ale Gallons. Where extend the *Compasses* from Unity to the *Area* $2\frac{1}{5}$; the same Extent will reach from the *Depth* 9,03 to the *solid Content* $21\frac{1}{5}$ Gallons. Or, on the *sliding Rule*, set Unity 1 on the Rule to the *Area* $2\frac{1}{5}$ on the Slider, then against the *Depth* 9,03 on the Rule you have $21\frac{1}{5}$ the *solid Content* in Ale Gallons on the Slider. Thus the Method of finding the *Content* or *Capacity* of square or rectilinear *Area's* or *Bodies* in Ale;
Wine;

Wine, or Corn Gallons is exceeding plain and easy by the *Single Gunter and Sliding-Rule*.

8. In case of *circular Area's*; since they are all in the Ratio of the Squares of their Diameters; and supposing the Diameter of a Circle 1 Inch, the Area will be 0,7854 Decimal Parts of a Cubic Inch; therefore having the Diameter of any Circle given in Inches, if the Square thereof be multiplied by 0,7854, the Product will be the Area of that Circle in Cubic Inches. Let D = the Diameter of a Circle, $a = 0,7854$, G = Standard Gall. and A = Area of the Circle in such Gallons, as before. Then $1 : a :: DD : DDa = \text{Area in Inches}$; therefore $DDa = GA$; and so $G : a :: DD : A = \text{Area in Gallons}$. But $a : G :: 1 : \frac{G}{a} = \text{the Square of the Diameter of that Circle, whose Area is } G$. Wherefore since $DD = \frac{G}{a} A$,

therefore if the Square of the Diameter DD be divided by $\frac{G}{a} =$

0,7854	282.0000	(= 359,05
0,7854	231.0000	(= 294,12
0,7854	268.8000	(= 342,24
0,7854	2150,4200	(= 2737,92

or the several Divisors 359,05; 294,12, &c. then these Quotients will be the Area of the Circle in *Ale, Wine, &c. Gallons*.

9. Or thus, since $DD \times \frac{a}{G} = A$; therefore if the Square of the Diameter DD

be multiplied by $\frac{a}{G} =$

282	) 0,7854	(0,002785
231	) 0,7854	(0,003389
2688	) 0,7854	(0,002922
2150,42	) 0,7854	(0,0000036

the several Products will be the Area in *Ale, Wine, &c. Gallons*, as before. Suppose the Diameter of a Circle $D = 50$ Inches, then $DD = 2500$; and put the constant Divisor $\frac{G}{a} = dd = 359,05$; 294,12, &c.

G g

Then

Then since $1 \times DD = ddA$, therefore $dd : DD :: 1 : A =$ the Area in Gallons. Therefore with the Compasses set one Foot in 359.05 extend the other to 2500, the same Extent will reach from 1 to 6 $\frac{2}{3}$ = the Gallons in the Area of the proposed Circle.

10. Otherwise thus; put $\frac{1}{G} = dd = 0.002785$; 0.002389 , &c. (see Art. 9.) then because $DDdd = A \times 1$; therefore we have $1 : DD :: dd : A$; or $1 : dd :: DD : A$. If then you set 1 on the Rule to 0.00278 on the Slider, you will see against 2500 on Rule 6 $\frac{2}{3}$ on the Slider, the Gallons of Ale in the Area of that Circle; and thus you find the Gallons of Wine, Corn, &c. both by the *Sliding-Rule* and *Plain Scale*.

11. Again, putting $H =$ Height of a Cylinder, and $D =$ the Diameter of its circular Base; also $S =$ the solid Content or Capacity in Gallons; then $dd : DD :: (1 : A ::) H : S$. Now let $D = 50$ Inches, and $H = 15$, the Area of such a Cylinder will be thus found by the *Sliding-Rule*. Set 359.05 on the Slider, to 2500 on the Rule; then against 15 on the Slider, is 104 on the Rule; and so many Gallons of Ale would such an hollow Cylinder contain.

12. But since $dd : 1 :: DD : A$, (Art. 9.) therefore $d : 1 :: D : \sqrt{A}$; and for the same Reason $d : \sqrt{H} :: D : \sqrt{S}$. Consequently, if a *single Line of Numbers* be made to slide by a *double one*, if you set d on the *single one* to 1 on the *double one*, then against D on the *former*, you have A on the *latter*. Also, if against d on the *single Line*, you set H on the *double one*; then against D on the *former*, is S on the *latter*. By this means therefore, having only the *Diameter* and *Height* of a *Cylinder*, the *Area* of the *Base*, and *solid Content* of the said *Cylinder*, is immediately known. The Reason of the Method here

here delivered, appears from what has been already said in the Chapter of *Instrumental Operations*.

13. Now $d =$

$$\left\{ \begin{array}{l} \sqrt{359,05} = 18,95 \\ \sqrt{294,12} = 17,15 \\ \sqrt{342,24} = 18,5 \\ \sqrt{2737,92} = 52,32 \end{array} \right\} \text{Which Numbers } \left\{ \begin{array}{l} \text{Ale Gall.} \\ \text{Wine Gall.} \\ \text{Gauge Points} \\ \text{for } \dots\dots\dots \end{array} \right\} \left\{ \begin{array}{l} \text{Corn Gall.} \\ \text{Corn Bush.} \end{array} \right.$$

And accordingly on the single Line of Numbers against those *Gauge Points* there is a small Stroke ('), by which are set the Capitals AG, WG, MB, to signify they are the *Gauge Points* for *Ale Gallons*, *Wine Gallons*, and *Malt Bushels*.

14. Therefore in the foremention'd Cylinder, whose Height H was $= 15$ Inches; and the Diameter of its Base $D = 50$; in order to find the Area of the Base, and Capacity of the Cylinder in Gallons, by the Sliding-Rule; set 1 on the Slider to the *Gauge Point* for Ale 18.95 on the single Line, then against the Diameter $D = 50$, on the single Line is $6 \frac{7}{8}$ on the double Line on the Slider, the Gallons of Ale in the Area of the Base.

Secondly, Set the *Gauge Point* 18.95 to the Height 15 on the Slider, then against the Diameter 50 on the single Line is 104 = S on the double Line or Slider; therefore the Capacity of the Cylinder is 104 Gallons, *Ale-Measure*, as before.

15. Or thus, with the single Line and Compasses only. Set one Foot in the *Gauge Point* 18,95, and extend the other to $D = 50$, this Extent turn'd twice over from 1 will fall on $6 \frac{7}{8} = A =$ the Area of the Base in Gallons of Ale. The solid Content, in this Case, will fall beyond the single Line, or else might be performed in the same manner.

16. With the double Lines of Numbers, on the Sliding Rule, thus. Set the *Gauge-Point* 18.95 on the Rule to the Diameter 50 on the Slider, and mark the Number on the Slider against 1 on the Rule;

G g 2

then

then against that Number on the Rule you have $6\frac{1}{2} = A$ the Area of the Base on the Slider. Also mark the Number on the Slider against the Height 15 on the Rule, then against that Number on the Rule is 104 on the Slider, the Capacity of the Cylinder as before; and thus both the Area and Solidity are found at once setting the Rule. The same is performed by the double Line and Compasses, thus; Set one Foot in the Gauge Point 18.95, and extend the other to the Diameter 50, the same Extent will reach at twice from 1 to $6\frac{1}{2}$ the Area; and from the Height 15 to 104 solid Capacity in Ale Gallons.

17. In this manner may Gauge-Points be found for right-lined Areas: for let any such Area given in *Cubic Inches* be called a ; then (by Art. 6.) $1 \times a = GA$, and so $G : a :: 1 : A$; or thus $G : 1 :: a : A$; therefore $\sqrt{G} : 1 :: \sqrt{a} : \sqrt{A}$.

Therefore $\sqrt{G} = \begin{cases} \sqrt{282} = 16.79 = \\ \sqrt{231} = 15.19 = \\ \sqrt{2150.42} = 46.36 = \end{cases}$ the Gauge $\begin{cases} \text{Ale G.} \\ \text{W. G.} \\ \text{Bush.} \end{cases}$ Point for

Call these Gauge Points d ; then $d : 1 :: \sqrt{a} : \sqrt{A} =$ the Area in Gallons. Also $d : \sqrt{H} :: \sqrt{a} : \sqrt{S} =$ the Solidity in Gallons. Whence you may observe; that *right-lined Areas*, as well as *circular ones*, may be found in Gallons, by the *single and double Line* of Numbers sliding by each other; but not the *Solidity* of such Solids, there being *three Terms* of the *four variable*, in the Analogy for that.

18. But notwithstanding this, there is a Method whereby the Solidity or Capacity of Solids or Vessels may be found (without knowing the Areas at all) by the *Breadth*, *Length*, and *Depth* only; which call B , L , D ; and $S =$ Solidity, and $G =$ the Gallon or Bushel, as before. Now since $BDL = GS$, therefore $BD = \frac{G}{L} S = G \times \frac{1}{L} \times S$; and so $G \times \frac{1}{L} : B :: D$

:: D : S. But because $\frac{1}{L}$ is the Reciprocal of L, therefore the Line of Numbers on which you seek L must be *inverted*; and because the Logarithm of G is added, the Number G must always begin the *inverted Line*, or be placed equal to 1 on the *direct Line*. These things premised, 'tis plain that if B on the *Slider* be set to L on the *inverted Line* order'd as before, then against D on the *direct Line* you have S on the *Slider*.

19. For Example; Suppose a Cistern be 80 Inches long, 50 broad, and 40 deep. Quære the Content in Bushels?

Here $G = 2150,42$, and for the Purpose of *Malt-Gauging* there is on some Rules an *inverted Line of Numbers* fix'd on one Side the *Slider*; beginning at 2150,42 as before said; then on such a Rule set $50 = B$ on the *Slider* to $80 = L$ on the *inverted Line*, and against $40 = D$ on the *direct Line* on the Rule, you have $74 = S =$ the Number of Bushels the *Cistern* will contain. And thus you might proceed for Gallons, had you *inverted Lines* beginning at 282, 231, 268.8.

20. In *gauging Casks*, the principal Consideration is the *Curvature* of the *Staves*; as A B D. Fig. XXXIX. which *Gaugers* reduce to four Degrees, or Varieties, viz.

Variety I. Those Casks whose Staves are most curved or bent, are consider'd as the *middle Zone* or *Frustrum* of a *Spheroid*, such as Fig. XXXIV.

Variety II. If the Staves are not quite so much arching or bent, the Cask is supposed to be the *middle Zone*, or *Frustrum* of a *Parabolic Spindle*, as Fig. XXXVII.

Variety III. When the Staves of Casks are but very little curved, they are reputed to be in the Form of the *Frustrums* of two equal *Parabolic Conoids*,

voids, join'd together at the widest Bases. See *Fig. XXXVI.*

Variety IV. When the Staves are strait from the Bung to the Head, as the prick'd Lines AB, BD, (or very nearly so) then 'tis plain such a Cask consists of the *Frustums* of two equal right *Cones*, set together at the greater Bases. *Fig. XXXIX.*

21. The Casks being reduced to these four Varieties, if you multiply the Difference between the Head and Bung Diameters

$$\text{By } \left\{ \begin{array}{l} 0.7 \\ 0.65 \\ 0.6 \\ 0.55 \end{array} \right\} \text{ for the } \left\{ \begin{array}{l} \text{I.} \\ \text{II.} \\ \text{III.} \\ \text{IV.} \end{array} \right\} \text{ Variety,}$$

and then add the Product to the Head Diameter, then that Sum shall be a *mean Diameter*, or that of a *Cylinder*; whose *Height* and *Capacity* is equal to that of the *Cask*, as near as possible.

22. For Example; Let there be a Cask ADEG, whose *Bung-Diameter* BF = 31.5 Inches, and *Head-Diameter* AG = 24.5; then their Difference is BF - AG = 7.

$$\text{Consequently } \left\{ \begin{array}{l} 7 \times 0.55 = 28.35 \\ 7 \times 0.6 = 28.7 \\ 7 \times 0.65 = 29.05 \\ 7 \times 0.7 = 29.4 \end{array} \right\} \begin{array}{l} \text{The mean Diameter} \\ \text{of the Cylinder} \\ \text{equal to the Cask.} \end{array}$$

$$24.5 + \left\{ \begin{array}{l} 28.35 \\ 28.7 \\ 29.05 \\ 29.4 \end{array} \right\}$$

Having found the Areas (by Art. 8, 9, 10, 12) to be 2,2385; 2,2941, &c. Ale Gallons; and supposing the Length of the Cask 42 Inches, = HL.

$$\text{Then the Contents } \left\{ \begin{array}{l} 2,2385 \times 42 = 94.03 \\ 2,2941 \times 42 = 96.35 \\ 2,3504 \times 42 = 98.71 \\ 2,4073 \times 42 = 101.10 \end{array} \right\} \begin{array}{l} \text{Ale} \\ \text{Gallons.} \end{array}$$

according to the several Varieties will be :

And

And these being all the principal Articles, wherein the Use of *instrumental Logarithms*, or the *artificial Line of Numbers* is directly concerned, I shall say no more on the Head of *Gauging*; but proceed to the next Article of

Timber Measure.

23. Every Piece of *Timber* is a Solid, like to some one or other of those, whose *absolute Mensuration* was shewn in the last Chapter, viz. The *Frustum* of a *Cone*, a *Cylinder*, the *Frustum* of a *Pyramid*, a *Parallelopipedon*, *Prism*, &c.

24. In whatever Form the Piece happens to be, find the *Content* or *Solidity* in Inches, as there taught; which divide by 1728, (the *Solid Inches* in one *Foot* of *Timber*) the *Quotient* is the *Solid Content* in *Feet*. But that, the Analogies for Operation, by Instruments may be evident, I shall make use of the foregoing Method in *Gauging*, by putting B = the Breadth, D = Depth, L = Length, $G = 12$, and S = Solidity, all in Inches. Then since $\frac{BDL}{G^3} = S$,

therefore $BDL = G^3 S$; but $\frac{L}{12} =$ the Length in Feet, which let be F , then $BD \frac{L}{12} = GGS = BDF$; and consequently $GG : BD :: F : S$.

25. Now if the Piece of *Timber* be in form of a *square Prism*, then the Base BD is a *square Number*, which call gg ; whence $GG : gg :: F : S$; and therefore $G : g :: \sqrt{F} : \sqrt{S}$. Wherefore, having a *single* and *double Line* of *Numbers*, by the *Sliding-Rule*, set $G = 12$ on the *single Line*, to the Length in Feet F on the double one; then against g on the *single Line* is S = Solidity on the double Line on the Slider. Example, Suppose a Piece of *Timber*, the Side of whose square Base is $g = 15$ Inches,

Inches, the Length 18 Feet $\pm$ F; Quere the solid Content S?

Set the constant Point $G = 12$ on the *single Line* to the Length in Feet $F = 18$; then against the Side of the given *square Base* $g = 15$ Inches, on the former, is $28 = S$ = the Number of the solid Feet in the Piece, on the latter. Or with the Compasses, extend from 12 to 15 on the *single Line*, the same will reach from 18 to 28 $= S$ in the double Line, the Answer as before.

26. But if you have no *single Line*, proceed with the double Line on the *Sliding-Rule* thus: Set the constant Number $GG = 144$ on the Rule to the given Square $gg = 225$ on the Slider; then against the Length in Feet $F = 18$, on the Rule is $28 = S$ = the solid Feet on the Slider. Or, with the Compasses, extend from 144 to 225, the same Extent will reach from 18 to 28 the solid Feet, as before. Otherwise by the Analogy $G : g :: \sqrt{F} : \sqrt{S}$; thus, set the constant Point $G = 12$, on the Rule to the given Side $g = 15$, on the Slider, and mark the Point on the Slider against 15 on the Rule; bring that Point to 12 on the Rule; then against $F = 18$ on the Rule, is $28 = S$ on the Slider, the *Solidity* as before. But much better with the Compasses, thus; Extend from 12 to 15, that Extent turn'd twice over will reach from 18 to 28 $= S$, as before.

27. If the Piece be in the Form of a *Parallelopipedon*, that is, hath its Breadth and Depth unequal; then the common way is to add the *Depth* and *Breadth* together; and to take half that Sum for the Side of a mean Square, viz. $\frac{B+D}{2} = g$, and then they imagine $gg = BD$, and so measure the Piece as before. But this is a very erroneous way; and the more so, as the Difference between the Breadth and Depth is greater. For since, (in order to reduce this Piece to a *square Prism*) $BD = gg$; 'tis evident

$g =$

$g = \sqrt{BD}$; that is, a *mean Proportional* between B and D, (for $B : \sqrt{BD} :: \sqrt{BD} : D$) and not their *half Sum* $\frac{B+D}{2}$; as they ignorantly suppose.

28. For Example; Suppose a piece of *beam Timber* in form as aforesaid, whose Breadth is 22,5 Inches $= B$, the Depth or Thickness 10 Inches $= D$, and Length 18 Feet $= F$; what is the solid Content?

29. Find a *mean Proportional* between $B = 22,5$ and $D = 10$, (as heretofore taught) it shall be $g = 15 = \sqrt{BD}$; and therefore $G^2 = 144 : g^2 22,5 :: F = 18 : S = 28$, as before. But according to the common false way, $g = \frac{B+D}{2} = 16,25$, and so $gg = 264,0625$, which is greater than the *true Area* of the Base $BD = 225$ by 39,0625 *square Inches*. Also set $G = 12$ on the *single Line* of Numbers to the Length $F = 18$ Feet on the double, then against (the false) $g = 16,25$ on the *former*, is $324 = S$ on the latter. But $324 - 28 = 44$ Feet more than is really in the Piece.

29. Because $B \cdot D \cdot F = G^2 \cdot S$; therefore $BD = \frac{G^2}{F} S$; and so we have $G^2 \propto \frac{1}{F} : B : D : S$. If then you have an *inverted Line* of Numbers beginning from $G^2 = 144$ placed on one side the Slider, with a *direct Line* on the other; then may the *solid Content* in Feet be found by the B, D, and F; as taught in Art. 18. hereof; thus; Set $B = 22,5$ on the Slider, to $F = 18$, on the *inverted Line*; then against $D = 10$, on the *direct Line* is $28 = S$, on the Slider, the solid Feet as before.

30. If the Timber be in Form of the *Frustum* of a *square Pyramid*, as Fig. XXX. the Theorem for its Solidity in Inches is $H D d + \frac{1}{3} H x x$, as per last Chap. Art. 23. Therefore $H D d + \frac{1}{3} H x x = G^2 S$ (where $S =$ Solidity in *cubic Feet*) and since $\frac{1}{3} H = F =$ the Length in Feet, therefore $F D d + \frac{1}{3} F x x =$

H h
G

$G^2 S$, consequently $G^2 : F :: Dd + \frac{1}{3}xx : S$. Example; Suppose a Piece of Timber 25 Inches square at the greatest end, 9 Inches square at the lesser End, and 20 Feet long, how many solid Feet is there in such a Tree?

Here $F = 20$, $Dd = 225$, and $\frac{1}{3}xx = 85,3$; therefore $Dd + \frac{1}{3}xx = 310,3$; wherefore $G^2 = 144$, $F = 20$; $310,3 : 43,1 = S$, the Number of solid Feet as required. Note, the common way, by supposing the Square of $\frac{D+d}{2} = Dd + \frac{1}{3}xx$ is very false; and in this Instance would not give the Solidity above 40, 1 Feet, which is three Feet less than the Truth.

31. If the Bases of the *Frustum* be *Parallelograms*, as is the Case of most Pieces of *bowed Timber*; then they may be reduced to *Frustums of square Pyramids*, thus; Let $A =$ the Area of the greater Base, $a =$ Area of the lesser. Then, $\sqrt{A} = D$, and $\sqrt{a} = d$; and $\sqrt{A \times a} = Dd$; also since $D^2 + 2 Dd + dd = xx$, therefore $A = 2 \sqrt{A \times a} + a = xx$; and so $\frac{3Dd + \frac{1}{3}xx}{3} = \frac{A + \sqrt{A \times a} + a}{3}$; consequently, $G^2 : F ::$

$\frac{A + \sqrt{A \times a} + a}{3} : S =$ the Number of solid Feet in such a Tree.

32. For Example; Suppose a Piece of squared Timber be 32 Inches broad, and 20 deep at the largest End; and 10 broad and 6 deep at the lesser End; the Length 18 Feet. Quære the solid Content?

Here $A = 32 \times 20 = 640$, and $a = 10 \times 6 = 60$; and $\sqrt{A \times a} = \sqrt{38400} = 195,959$; therefore

$\frac{A + \sqrt{A \times a} + a}{3} = 298,653$. Then $G^2 = 144$; $F = 18 :: 298,653 : S = 37,33$ the solid Feet required. Note, the Product of $\frac{1}{3}$ of the Girth of each End taken for a *mean Area* is very false; and cannot be practised without considerable Error, tho' it be the common or customary way.

33. If the Timber be in form of a Cylinder, *Fig. XXVIII.* then putting $a = 0.7854$, and $G = 12$ Inches, we have (as per Art. 8.) this Analogy; $1 : a :: DD : DDa$ = the Area of the Cylinder's Base in Inches (supposing D = Diameter in Inches;) let H = Height in Inches; then $HDDa$ = the Inches solid, whereof $1728 = G^3$ = one *solid Foot*. Consequently, if S = solid Content in Feet, $HDDa = G^3 S$; and $\frac{H}{12} = F$ = the Length in Feet, therefore $FDDa = G^3 S$; and $FDD = \frac{G^3}{a} S$; and putting $\frac{G^3}{a} = dd = 183.34$; then $FDD = ddS$; wherefore

$$dd : DD :: F : S, \text{ or } d : D :: \sqrt{F} : \sqrt{S}.$$

34. Or with the Length in Inches, to find the Solidity in Feet, thus; since $HDDa = G^3 S$, therefore putting $\frac{G^3}{a} = dd$, we shall have $HDD = ddS$; consequently $dd : DD :: H : S$ = Solidity in Feet; or $d : D :: \sqrt{H} : \sqrt{S}$. Wherefore, if there be a *single Line* of Numbers on your Sliding Rule; set the constant Number $d = 46.9$ on the *single Line* to H = the Height in Inches, on the *double Line* of the Slider; then against D = the Diameter of the Cylinder's Base in Inches on the *single Line*, is S = the Number of solid Feet on the Slider as required. And if the Length be given in Feet; then the Solidity is found, as above, Art. 13.

35. If you have no *single Line* on your Rule, you must work with the Analogies $dd : DD :: F : S$; or $dd : DD :: H : S$. In the first $dd = 183.34$, and $dd = 2200.152$; therefore $d = 13.54$ = Diameter of a Circle, when the Area is 144; and $d = 46.9$ = the Diameter of a Circle, whose Area is 1728 = the Inches in a solid Foot. And these Numbers being constant in all Operations, and the Method of operating such kind of Analogies every way on the ar-

ficial Lines of Logarithms already sufficiently exemplified in the preceeding Articles; I shall not here again repeat it.

36. The common way of measuring round Timber, is by girting them about the *Middle* with a String, and taking $\frac{1}{4}$ of the Girt for the Side of a Square equal to a mean circular Area, such as would reduce the proposed Piece to a Cylinder. But this is also very false and ungeometrical. For, since the Area of that Circle, whose Circumference is 1, is 0.07958; and the Area of that Square, whose Side is $\frac{1}{4}$ ($=\frac{1}{4}$ of the said Circumference) is 0.0625, and the Solidities being in proportion to these Numbers, viz. as 0.07958 : 0.0625; that is, as 23 to 18; 'tis evident, the Content by this false way is above $\frac{1}{4}$ less than what it really is; which Error, if it be not considerable enough to be regarded and corrected, is great pity indeed.

37. Therefore to measure a Piece of round tapering Timber truly, it must be considered as the *Frustum* of a right Cone; whose solid Content in Inches is found by the Theorem $Dd + \frac{1}{3}xx \times 0.7854H$, as per Art. 25. of the last Chapter. Now putting 0.7854 $\equiv a$, and $G^3 = 1728$, the cubic Inches in a solid foot; the Theorem will become $Dd + \frac{1}{3}xx \times aH = G^3S$; and again since $\frac{1}{3}H = F$, the Length in Feet; therefore $Dd + \frac{1}{3}xx \times aF = G^3S$; also put $\frac{G^3}{a} = dd$; and then $Dd + \frac{1}{3}xx \times F = ddS$; wherefore we have $dd : Dd + \frac{1}{3}xx :: F : S =$ the solid Content in Feet.

38. For Example; Suppose a Piece of round Timber be 36 Inches Diameter at one End, and 9 Inches Diameter at the other; and 24 Feet long; quare the Solidity in Feet?

Here $D = 36$, $d = 9$, $D - d = x = 27$, $Dd = 324$, $\frac{1}{3}xx = 243$, $Dd + \frac{1}{3}xx = 567$, and $F = 24$. There-

Therefore the Analogy is as $dd = 183,34 : 567 :: F = 24 : S = 74,22$ the solid Feet required; as may be wrought by the *Lines of Numbers* in any of the before-mention'd ways.

Of MEASURING LAND.

39. What I principally design here, is to shew how the true Area or Content of a Plot of Land is to be found by the *artificial Line of Numbers*, in Acres and Decimal Parts. The Dimensions of a Field are commonly taken in Rods or Poles; each containing $16\frac{1}{2}$ Feet. Of these Poles, 40 in Length and 4 in Breadth make an Acre; or an Acre is $= 160$ square Poles. Some (and indeed most) use a Chain, called *Gunter's Chain*, in taking Dimensions; which consisteth of 100 Links, and the whole in Length $= 4$ Poles or Rods. And so 10 of these *square Chains* make an Acre.

40. The Field being measur'd with the Pole, if it be in Form of a *Parallelogram*, put $L =$ Length, and $B =$ Breadth; and then it will be $\frac{LB}{160} = A =$ the Number of Acres; therefore $LB = 160 A$; and so we have $160 : L :: B : A =$ the Acres. For Example, suppose a Field be 35 Pole broad, and 185 Pole in length; how many Acres doth it contain? Set 160 on the Rule to 185 on the Slider, then against 35 on the Rule is $40\frac{1}{2}$ nearly, the Number of Acres required.

41. If the Field be in Form of a Triangle, and measur'd with a Rod or Pole; then, having plotted it and measur'd the *Base* and perpendicular Height, which call B and H ; then $B \times \frac{1}{2}H = 160A$; and so $160 : B :: \frac{1}{2}H : A =$ the Acres as before.

42. If the Field be in Form of a *Trapezium*, as Fig. XL. then because, by drawing the Diagonal AB , it is reduced to two Triangles ACB , ADB ,
and

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and calling the two Perpendiculars Cc and Dd, H and b; and the common Base AB, B; we have $\frac{BH}{2} + \frac{bA}{2} = \overline{H + b} \times \frac{1}{2}B = 160A$. Therefore $160 : \frac{1}{2}B :: H + b : A =$ the Acres contain'd in the Trapezium ACBD.

43. If the Field be of a *multangular Form*, it must, when plotted, be reduced to several *Triangles* and *Trapezia*; and then measur'd, as per Art. 41, 42. In case you take the Measures with a Chain of 4 Rods or 100 Links, then the Analogies for Operation will be the same as above; only instead of 160 you must use 10; thus $10 : L :: B : A$. Art. 40. and $10 : B :: \frac{1}{2}H : A$. Art. 41, &c. All which is so easy as to need no Example, nor any thing more to be said concerning it.



C H A P.



C H A P. XVI.

The Practical Use of the LOGISTICAL
LOGARITHMS.

1. **T**HE Nature and Construction of Logistical Logarithms, having been sufficiently inculcated in Chap. IV. it remains now that I shew their Use in some Cases of practical Astronomy, for in other respects it is very little. But in finding the Places, Distances, &c. of the *heavenly Bodies*, the Calculation of Eclipses, &c. they are very necessary for finding the proportional Parts, as will, in some Degree, appear by the following Examples.

2. **Example 1.** Admit the *mean Anomaly* of the *Sun* be $4^{\circ} 7' 48'' 14'''$ what is the *true Equation*, and Logarithm of his Distance from the Earth?

Mean A-	$4^{\circ} 7'$	Equa-	$1^{\circ} 33' 49''$	Lo-	$4.99.5607$
nomalies	$\{ 4 \ 8 \}$	tions	$\{ 1 \ 32 \ 36 \}$	gar.	$\{ 4.99.5501$
Differences	$1=60'$		$1 \ 13$		106

Then for the *Proportional Parts* of the Equation, say by the Logistical Logarithms,

If one Degree, or	$60'$	$=$	100
give the Difference	$1 \ 13$	$=$	16930
what gives the Anomaly	48.214	$=$	0948

Answer, the prop. Part $0 \ 59 = 17878$

This subtracted, (because the Equation is decreasing) from the Equation $1^{\circ} 32' 49''$ (answering the

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the Anomaly $4^{\circ} 7'$ leaves the *true Equation* $= 1^{\circ} 32' 50''$, as was requir'd.

3. Then for the proportional Part of the Logarithm say

One Degree, or $\dots\dots\dots 60' 00'' = 0$
 gives the Difference of Logarithms. $106 = 15310$
 what gives the Anomaly $\dots\dots\dots 48' 14'' = 948$

Answer, the Proportional Part. $\dots\dots\dots 85 = 16258$

Which subduct from the Logarithm 4.995607 , answering to the Anom. $4^{\circ} 7'$ there remains 4.995522 , the *true Logarithm* of the Sun's Distance from the Earth, as required.

4. Example II. Suppose the Moon's *annual Argument* be $29^{\circ} 51' 37''$, what is the Equation of the *Apogee* and *Eccentricity* of her Orb?

Annual { 29° } Equa- { $8^{\circ} 53' 8''$ } Eccen- { 60392
 Argum. { 30 } tion { $9\ 07\ 14$ } tricity { 61045

Differences. $1 = 60'$ $14\ 6$ 653

Then for the Equation of the Apogee, say

If one Degree, or $\dots\dots\dots 60' 00'' = 0$
 give the Difference $\dots\dots\dots 14' 6'' = 6289$
 what gives $\dots\dots\dots 51' 37'' = 654$

Answer, the Prop. Part $\dots\dots\dots 12' 8'' 6943$

To which add the Equation $8^{\circ} 53' 8''$, agreeing to the an. Arg. 29° ; the Sum is $\dots\dots\dots 9^{\circ} 05' 16''$ the *true Equation* of the Apogee as required.

5. Then for the true Eccentricity, say

$\dots\dots\dots$ if

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If one Degree, or 60' 00" = 0
 give the Difference of Eccentricity . . 653 = 7414
 what will the Part 51' 37 = 654

Answer, the Proportional Part 562 = 8068
 To which add the Eccentricity . . . 60392

The Sum is the *true Eccentricity* . . 60954, as required.

6. In like manner the Equation of the Moon's Center, Latitude, Inclination of Limit, and Logarithmic Distance from the Earth may be found. Also the same things are in like manner found for any of the Planets, of which there needs no more Examples.

7. Example III. In an Eclipse of the Moon, admit her horary Motion be 30' 31", and the Sun's 2' 27"; then the horary Motion of the Moon from the Sun will be 28' 4": and suppose the Moon hath pass'd the Sun by the Distance 1° 19' 4" = 4744"; What is the Time requisite for that Motion? Say thus,

As the horary Motion of the Moon	{	28'	4" = 3300
from the Sun			
is to one Hour		60	0 = 0
So is the Distance pass'd above	{	56'	
8", viz.		22'	56" = 4177

To the Time above 2 Hours, viz. . . 49' 00" = 877

Therefore 2^h 49' were pass'd since the *true Opposition*, or Moment of the Eclipse.

Note, because in this Case the Motion of the Moon from the Sun, performed in 1 Hour, is 28' 4"; therefore 56' 8" will be passed in 2 Hours: But the present Distance of the Moon from the Sun is

79' 4", which because it exceeds the Table of *Logistical Logarithms*, therefore subduct the Motion of two Hours, viz. 79' 4" — 56' 8" = 22' 56"; and this Excess of Motion will give the Excess in Time above 2 Hours; as in the Example. And thus you proceed when the Distance of the Moon from the Sun exceeds one Degree, or the Table.

8. Example IV. Suppose, in a *Lunar Eclipse*, the *Semidiameter* of the Moon be 15' 15", and the *Difference* between the *Moon's Latitude*, and *Sum* of the *Semidiameters* of the Moon and the Earth's Shadow be 9' 01"; Quære the Digits eclipsed?

Note, the *Semidiameter* of the *Moon* is always supposed to be 6 Digits, or equal to 6 Degrees. Therefore say,

As the Semidiameter of the Moon	15' 15" =	5949
Is to six Digits	6° 00' 00" =	10000
So is the said Difference	9' 1" =	8231

To the Digits eclips'd 3° 32' 51" = 12282

9. Example V. In a *Lunar Eclipse*, let the *Scruples of Incidence* be 30' 17", and the *horary Motion* of the Moon from the Sun be 28' 47"; to find the Time of *half Duration*?

Here because 28' 47" : 60' :: 30' 17" : a fourth Number greater than 60', and so consequently beyond the Extent of the Table; therefore (as in Art. 7.) from 30' 17" subduct the Motion for an Hour, viz. 28' 47", and to the *Remainder* 1' 30" find the Time thus;

As the Motion	28' 47" =	3190
Is to an Hour	60' 00" =	0
So the remaining Scruples	1' 30" =	16021

To the Time 3' 8" = 12831
The

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The Time sought therefore $1^h 3' 8''$; the double of which is $2^h 6' 16''$, the Time of the whole Duration of the Eclipse.

10. Example VI. If the diurnal Motion of the Sun be $59' 8''$, what is the Motion for 7 Hours $15'$?

$$\begin{array}{rcl}
 \text{Say, as one Day} & \dots\dots\dots 24^h 00' 00'' = & 0 \\
 \text{Is to its Motion} & \dots\dots\dots 59' 8'' = & 63 \\
 \text{So is the Time} & \dots\dots\dots 7^h 15' 0'' = & 5197 \\
 \hline
 \text{To the Motion therein} & \dots\dots\dots 17' 52'' = & 5260
 \end{array}$$

11. Example VII. If the mean Motion of the Moon in one Hour be $32' 56''$, how far doth she move in $17^h 45'$?

$$\begin{array}{rcl}
 \text{Say, as one Hour} & \dots\dots\dots 1^h 00' 00'' = & 13802 \\
 \hline
 \text{Is to its Motion} & \dots\dots\dots 32' 56'' = & 20392 \\
 \text{So is} & \dots\dots\dots 17^h 45' 00'' = & 1309 \quad \left. \vphantom{\begin{array}{l} 1309 \\ 1309 \end{array}} \right\} \text{add} \\
 \hline
 & & 21701
 \end{array}$$

To the Motion required $\dots\dots\dots 9^\circ 44' 00'' = 7899$

Thus the Motion in 24 Hours, or one natural Day, will be found $13^\circ 10' 35''$.

12. The *Logistical Logarithms* may also be used with *common Logarithms*. Thus suppose you would find the Logarithmic Sine of $18^\circ 16' 47''$, proceed thus;

$$\begin{array}{rcl}
 \text{The Logarithmic Sine of} & \dots\dots\dots \left\{ \begin{array}{l} 18^\circ 16' = 9.496154 \\ 18 \quad 17' = 9.496537 \end{array} \right. & \\
 \hline
 \text{The Differences} & \dots\dots\dots 1' = 60'' & 383 \\
 & \text{I i 2} & \text{Now}
 \end{array}$$

244 *The PRACTICAL Use of the*

Now the Proportion is, $60'' : 383 :: 47'' : \text{the Proportional Part}$. But since the *Logistical Logarithms* of $47''$ and $60''$ are reciprocally as those Numbers, and that of 60 is nothing; therefore (the Differences of the common Logarithms being proportional to the Numbers $60''$ and $47''$) say,

As the Logistical Logarithm of $43'' = 1447$
 is to the Logistical Logarithm of . . . $60 = 0$
 So is the common Logarithm of . . . $383 = 2.5998$

To the common Logarithm of the } $285 = 2.4551$
 proportional Part }
 Which add to the common }
 Logarithm } 9.496154

The Sum is the Logarithm . . . 9.496439 of $18^\circ 16' 47''$ as required. And thus proceed in any other Case of like nature.

13. The *Logistical Logarithms* may be used likewise with the *Logarithmic Sines* and *Tangents*, as in finding the Parallaxes of the Planets, &c. Thus suppose the Horizontal Parallax of the Moon $55' 12''$; the Angle of her Orb with the Horizon $22^\circ 4'$; and her Longitude in her Orb from the Horizon $81^\circ 27'$, to find her Parallax in Longitude.

From the Logistical Logarithm of the Hor. Paral. } $00^\circ 55' 12'' = 362$

Subduct the { Sine of the } $22 \quad 4 \quad 0 = 9.5748$
 Sum of the { Angle . . . }
 { Co-Sine of } $81 \quad 27 \quad 0 = 9.1722$
 { Longitude.. }

viz. 18.7470

There remains the Logif. Logarithm of the Par. in Long. . . } $00^\circ 3' 5'' = 12892$

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14. Because when the Angle of the Moon's Orb with the Horizon is greatest $67^{\circ} 14' 20''$, and she in the Horizon, her greatest Parallax in Longitude will be then; and will be found as in the last Example, thus;

$$\left. \begin{array}{l} \text{Logist. Log. of the greatest} \\ \text{Hor. Par.} \end{array} \right\} 00^{\circ} \quad 1' \quad 24'' = 17680$$

The Sine of $67^{\circ} 14' 20'' = 9.9648$
 Radius $90^{\circ} 0 0 = 10.0000$

Logist. Log. of the gr. Par. } $00^{\circ} 56' 37'' = 18032$
in Long. }

15. To find the Moon's *Parallax* in *Latitude*.

From the Logist. Log. of } $00^{\circ} 55' 12'' = 362$
Horizontal Parallax. . . }

Subduct the Co-Sine of the
Angle of her Orb with
the Horizon } $22^{\circ} \ 4' \ 0'' = 9.9669$

there remains the Logistical } $00^{\circ} 51' 9'' = 693$
 Logarithm of }
 which is the *Parallax of Latitude* sought.

16. To find the greateſt and leaſt *Parallaxes* in
Latitude.

From the Logistical Log. of } $1^{\circ} \ 1' \ 24'' = 17680$
the gr. Hor. Par. }

Subtract the Co-Sine of the least }
Ang. of her Orb with the Hor. } $9^{\circ} 41' 40'' = 9.9937$

Logistical Logarithm of the } $1^{\circ} 00' 31'' = 17783$
greatest Parallax

Again,

Again, from the L. Log. of } $00^{\circ} 54' 59'' = 379$
 the least *Horiz. Parallax* .. }

Subtract the Co-Sine of the } $67^{\circ} 14' 20'' = 9.5876$
 greatest Angle of Orb with }
 the Horizon }

Logistical Logarithm of the } $00^{\circ} 21' 16'' \quad 4503$
 least *Parallax* }

17. By these Instances the Reader will be appriz'd of the great Usefulness and Expediency of *Logistical Logarithms* in his *Astronomical Calculations*; and as the common Logarithms are laid on a Rule, so likewise are these; and such a *Sliding-Rule* of Logistical Logarithms may be very useful to those who desire to be more *expeditious* than *exact* in their Calculations.



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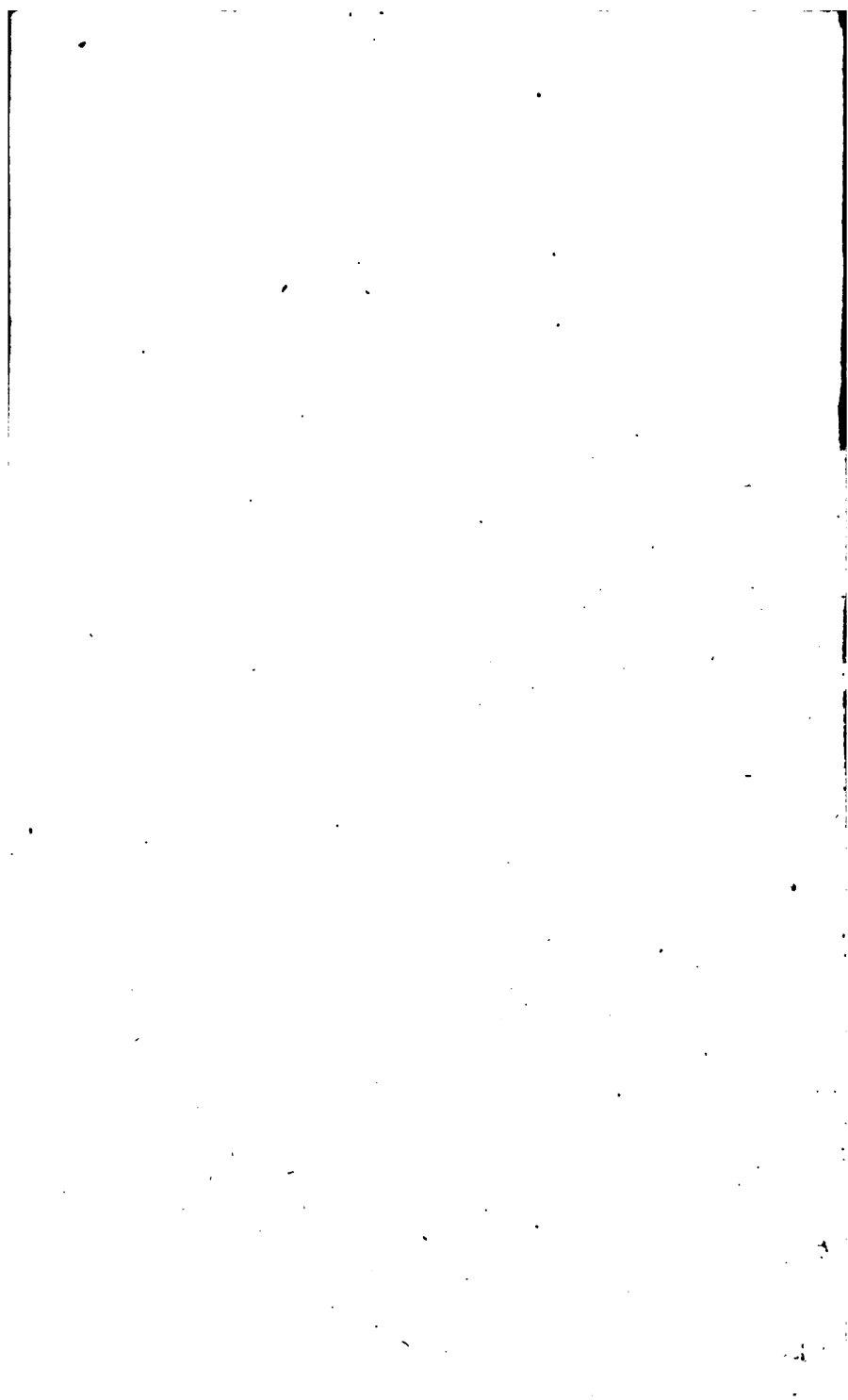
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Printed in the Year MDCCXXXIX.



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N ^o	0	1	2	3	4
0		000000	301030	477121	602060
1	000000	041393	079181	113943	146128
2	301030	322219	342423	361728	380211
3	477121	491362	505150	518514	531479
4	602060	614784	623249	633468	643453
5	698970	707570	716003	724276	732394
6	778151	785330	792392	799340	806180
7	845098	851258	857332	863323	869232
8	903090	908485	913814	919078	924279
9	954242	959041	963788	968483	973128
10	000000	004321	008600	012837	017033
11	041393	045323	049218	053078	056905
12	079181	082785	086360	089905	093422
13	113943	117271	120574	123852	127105
14	146128	149219	152288	155336	158362
15	176091	178977	181844	184691	187521
16	204120	206826	209515	212188	214844
17	230449	232996	235528	238046	240549
18	255272	257679	260071	262451	264818
19	278754	281033	283301	285557	287802
20	301030	303196	305351	307496	309630
21	322219	324282	326336	328380	330414
22	342427	344392	346353	348305	350248
23	361728	363612	365488	367356	369216
24	380211	382017	383815	385606	387390
25	397940	399674	401400	403120	404834
26	414973	416640	418301	419956	421604
27	431364	432969	434569	436163	437751
28	447158	448706	450249	451786	453318
29	462398	463893	465383	466868	468347
30	477121	478566	480007	481443	482874
31	491362	492761	494155	495544	496930
32	505150	506505	507856	509202	510545
33	518514	519828	521138	522444	523746
34	531479	532754	534026	535294	536558
35	544068	545307	546543	547775	549003
36	556302	557507	558709	559907	561101
37	568202	569374	570543	571709	572872
38	579784	580925	582063	583199	584331
39	591065	592177	593286	594393	595496

*4 The Logarithms of Nat. Numbers to 400.

	5	6	7	8	9
0	698970	778151	845098	903090	954242
1	176091	204180	230449	255272	278754
2	397940	414973	431364	447158	462398
3	544068	556302	568202	579784	591065
4	653212	662758	672098	681241	690196
5	740363	748188	755875	763428	770852
6	812913	819544	826075	832509	838849
7	875061	880814	886491	892095	897627
8	929419	934498	939519	944483	949390
9	977724	982271	986772	991226	995635
10	021189	025306	029384	033424	037426
11	060698	064458	068186	071882	075547
12	096910	100370	103834	107210	110590
13	130334	133539	136721	139879	143015
14	161368	164353	167317	170262	173186
15	190332	193125	195900	198657	201397
16	217484	220108	222716	225309	227887
17	243038	245513	247973	250420	252853
18	267172	269513	271842	274158	276462
19	290035	292256	294466	296665	298853
20	311754	313867	315970	318063	320146
21	332438	334454	336460	338456	340444
22	352182	354108	356026	357935	359835
23	371068	372912	374748	376577	378398
24	389166	390935	392697	394452	396199
25	406540	408240	409933	411620	413300
26	423246	424882	426511	428135	429752
27	439333	440909	442480	444045	445604
28	454845	456366	457882	459392	460898
29	469822	471292	472756	474216	475671
30	484300	485721	487138	488551	489958
31	498311	499687	501059	502427	503791
32	511883	513218	514548	515874	517196
33	525045	526339	527630	528917	530200
34	537819	539076	540329	541579	542825
35	550228	551450	552668	553885	555095
36	562293	563481	564666	565848	567026
37	574031	575188	576341	577492	578639
38	585461	586587	587711	588832	589950
39	596597	597695	598790	599883	600973

7	8	9
0594	0660	1723
0136	1176	2214
0428	1444	2457
0481	1474	2464
0307	1278	2246
0916	0865	1813
0317	0246	1173
0518	9428	0335
0529	8420	9309
0356	7229	8101
0008	5864	6718
0491	4330	5167
0811	2634	3456
0974	0782	1589
0798	8781	9572
0585	6634	7412
0583	4348	5112
0176	1928	2679
0863	9377	0115
05974	6701	7427
08189	3904	4617
0285	0988	1691
07267	7960	8651
04139	4821	5501
0904	1575	2245
07565	8226	8885
04126	4776	5426
0947	0589	1230
0957	7588	8219
03233	3855	4477
09419	0033	0646
05519	6124	6729
0534	2131	2727
07467	8056	8644
03321	3902	4482
09096	9669	0242
04795	5361	5926
0421	0980	1537
06975	6526	7077
01458	2003	2547
06873	7411	7948
02222	2753	3284
07505	8030	8554
02725	3244	3762
07883	8396	8908
02981	3487	3993
08019	8520	9020
03000	3494	3989
04924	8413	8902
02791	3276	3760

No.	0	1	2	3	4	5	6	7	8	9
90	95.4242	4725	5206	5688	6168	6649	7128	7607	8086	8564
91	9041	9518	9995	0471	0946	1421	1895	2369	2843	3315
92	96.3788	4260	4731	5202	5672	6142	6611	7080	7548	8016
93	8483	8950	9416	9882	0347	0812	1275	1740	2203	2666
94	97.3128	3590	4051	4512	4972	5432	5891	6350	6808	7266
95	7724	8180	8637	9093	9548	0003	0457	0912	1365	1819
96	98.2271	2723	3175	3626	4077	4527	4977	5426	5875	6324
97	6772	7219	7666	8113	8559	9005	9450	9895	0339	0783
98	99.1226	1669	2111	2553	2995	3436	3877	4317	4757	5196
99	5635	6074	6512	6946	7386	7823	8259	8695	9130	9565
100	00.0000	0434	0868	1301	1734	2165	2598	3029	3460	3891
101	4321	4751	5180	5609	6038	6466	6894	7321	7748	8174
102	8600	9026	9451	9876	0300	0724	1147	1570	1993	2415
103	01.2837	3259	3680	4100	4520	4940	5360	5779	6197	6615
104	7033	7451	7868	8284	8700	9116	9532	9947	0361	0775
105	02.1189	1603	2016	2428	2841	3252	3664	4075	4486	4896
106	5306	5715	6124	6533	6942	7350	7757	8164	8571	8978
107	9384	9789	0195	0600	1004	1408	1812	2216	2619	3021
108	03.3424	3826	4227	4628	5029	5430	5830	6229	6630	7028
109	7426	7825	8223	8620	9017	9414	9811	0207	0602	0998
110	04.1393	1787	2182	2575	2969	3362	3755	4148	4540	4931
111	5323	5714	6105	6495	6885	7275	7664	8053	8442	8830
112	9218	9606	9993	0380	0766	1152	1538	1924	2309	2694
113	05.3078	3463	3846	4230	4613	4996	5378	5760	6142	6524
114	6905	7286	7666	8046	8426	8805	9185	9563	9942	0320
115	06.0698	1075	1452	1829	2206	2582	2958	3333	3709	4083
116	4458	4832	5206	5580	5953	6326	6699	7071	7443	7814
117	8186	8557	8928	9298	9658	0038	0407	0776	1145	1514
118	07.1882	2250	2617	2985	3352	3718	4085	4451	4816	5182
119	5547	5912	6276	6640	7004	7368	7731	8094	8457	8819
120	9181	9543	9904	0266	0626	0987	1347	1707	2067	2426
121	08.2785	3144	3503	3861	4219	4576	4934	5291	5647	6004
122	6360	6716	7071	7426	7781	8136	8490	8845	9198	9552
123	9905	0258	0611	0963	1315	1667	2018	2370	2721	3071
124	09.3422	3772	4122	4471	4820	5169	5518	5866	6215	6562
125	6910	7257	7604	7951	8297	8644	8990	9335	9681	0026
126	10.0371	0715	1059	1403	1747	2090	2434	2777	3119	3462
127	3804	4146	4487	4828	5169	5510	5851	6191	6531	6870
128	7210	7549	7888	8227	8565	8903	9241	9578	9915	0253
129	11.0590	0926	1262	1598	1934	2270	2605	2940	3275	3609
130	3943	4277	4611	4944	5278	5610	5943	6276	6608	6940
131	7271	7603	7934	8265	8595	8926	9256	9586	9915	0245
132	12.0574	0903	1231	1560	1888	2216	2543	2871	3198	3525
133	3852	4178	4504	4830	5156	5481	5806	6131	6456	6781
134	7105	7429	7752	8076	8399	8722	9045	9367	9690	0012
135	13.0334	0655	0977	1298	1619	1939	2260	2580	2900	3219
136	3539	3858	4177	4496	4814	5133	5451	5768	6086	6403
137	6721	7037	7354	7670	7987	8303	8618	8934	9249	9564
138	9879	0194	0508	0822	1136	1450	1763	2076	2384	2702
139	14.3015	3327	3639	3951	4263	4574	4885	5196	5507	5818

N ^o .	0	1	2	3	4	5	6	7	8	9
140	14.6128	6438	6748	7058	7367	7676	7985	8294	8603	8991
141	9219	9527	9835	0112	0449	0756	1063	1370	1676	1982
142	15.2288	2594	2900	3205	3510	3815	4119	4424	4728	5032
143	5336	5640	5943	6246	6549	6852	7154	7457	7759	8061
144	8362	8664	8965	9266	9567	9868	0168	0468	0769	1068
145	16.1368	1667	1967	2266	2564	2863	3161	3460	3757	4055
146	4353	4650	4947	5244	5541	5838	6134	6430	6726	7022
147	7317	7613	7908	8203	8497	8792	9085	9380	9674	9968
148	17.0261	555	0848	1141	1434	1726	2019	2311	2603	2895
149	3186	3478	3769	4060	4351	4641	4931	5222	5512	5802
150	6091	6381	6670	6959	7248	7536	7825	8113	8401	8689
151	8977	9264	9552	9839	0126	0413	0699	0986	1272	1558
152	18.1844	2129	2415	2700	2985	3270	3554	3839	4123	4407
153	4691	4975	5259	5542	5825	6108	6391	6674	6956	7239
154	7521	7803	8084	8366	8647	8928	9209	9490	9771	0051
155	19.0332	0612	0892	1171	1451	1730	2010	2289	2567	2846
156	3125	3403	3681	3959	4237	4514	4792	5069	5346	5623
157	5909	6176	6452	6729	7005	7281	7556	7832	8107	8382
158	8657	8932	9206	9481	9755	0029	0303	0577	0850	1124
159	20.1397	1670	1943	2216	2488	2761	3033	3305	3577	3848
160	4120	4391	4662	4933	5204	5475	5745	6016	6286	6556
161	6826	7095	7365	7634	7903	8172	8441	8710	8978	9247
162	9515	9783	0051	0318	0586	0853	1120	1388	1654	1921
163	21.2188	2454	2720	2986	3252	3518	3783	4048	4314	4579
164	4844	5109	5373	5638	5902	6166	6430	6694	6957	7221
165	7484	7747	8010	8273	8535	8798	9060	9322	9584	9846
166	22.0108	0370	0631	0892	1153	1414	1675	1936	2196	2456
167	2716	2976	3236	3494	3755	4015	4274	4533	4792	5051
168	5309	5568	5826	6084	6342	6600	6858	7115	7372	7630
169	7881	8144	8400	8657	8913	9170	9426	9682	9938	0193
170	23.0449	0704	0960	1215	1470	1724	1979	2233	2488	2742
171	2996	3250	3504	3757	4011	4264	4517	4770	5023	5276
172	5528	5781	6033	6285	6537	6789	7041	7292	7544	7795
173	8046	8297	8548	8799	9049	9299	9550	9799	0050	0300
174	24.0549	0799	1048	1297	1546	1795	2044	2293	2541	2790
175	3038	3286	3534	3782	4030	4277	4524	4772	5019	5266
176	5513	5759	6006	6252	6499	6745	6991	7236	7482	7728
177	7973	8219	8464	8709	8954	9198	9443	9687	9932	0176
178	25.0420	0664	0908	1151	1395	1638	1881	2125	2367	2610
179	2853	3096	3338	3580	3822	4064	4306	4548	4790	5031
180	5272	5514	5755	5996	6236	6477	6718	6958	7198	7439
181	7679	7918	8158	8398	8637	8877	9116	9355	9594	9833
182	26.0071	0310	0548	0787	1025	1263	1501	1738	1976	2214
183	2451	2688	2925	3162	3399	3636	3873	4109	4345	4582
184	4818	5054	5290	5525	5761	5996	6232	6467	6702	6937
185	7172	7406	7641	7875	8110	8344	8578	8812	9046	9279
186	9513	9746	9980	0213	0446	0679	0912	1144	1377	1609
187	27.1842	2074	2306	2538	2770	3001	3233	3464	3696	3927
188	4158	4389	4620	4850	5081	5311	5542	5772	6002	6232
189	6462	6691	6921	7151	7380	7609	7838	8067	8296	8525

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0578	0806
2848	3075
5107	5332
7354	7578
9589	9812
1813	2034
4025	4246
6226	6446
8416	8635
0595	0813
2764	2980
4921	5136
7068	7282
9204	9417
1330	1542
3445	3656
5550	5760
7645	7854
9730	9938
1805	2012
3871	4077
5926	6131
7972	8176
0008	0211
2034	2236
4051	4253
6059	6260
8058	8257
0047	0246
2028	2225
3999	4196
5961	6157
7915	8110
9860	0054
1796	1989
3724	3916
5643	5834
7554	7744
9456	9646
1350	1539
3236	3424
5113	5301
6983	7169
8844	9030
0698	0883
2544	2728
4332	4565
6212	6394
8034	8216
9849	0030

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76	1656	1837
77	3456	3636
70	5249	5427
55	7034	7212
34	8811	8989
05	0582	0758
59	2345	2521
26	4101	4276
76	5850	6025
18	7592	7766
54	9327	9501
83	1056	1228
05	2777	2949
20	4492	4663
29	6199	6370
31	7900	8070
26	9595	9764
14	1283	1451
96	2964	3132
72	4639	4806
41	6308	6474
04	7970	8135
60	9625	9791
10	1275	1439
54	2918	3082
02	4555	4718
23	6186	6349
08	7811	7973
68	9429	9591
81	1042	1203
88	2649	2809
00	4249	4409
85	5844	6003
75	7433	7592
59	9017	9175
37	0591	0752
09	2166	2323
76	3732	3888
37	5293	5448
02	6842	7003
42	8397	8552
87	9941	0095
26	1479	1633
09	3012	3165
37	4540	4692
10	6062	6214
28	7579	7730
40	9091	9241
47	0597	0747
28	2200	2348

No.	0	1	2	3	4	5	6	7	8	9
290	46.2398	2548	2697	2847	2997	3146	3296	3445	3594	3744
291	3893	4042	4191	4340	4489	4639	4787	4936	5085	5234
292	5383	5532	5680	5829	5977	6126	6274	6423	6571	6719
293	6868	7016	7164	7312	7460	7608	7756	7904	8052	8200
294	8347	8495	8643	8790	8938	9085	9233	9381	9527	9675
295	9822	9969	0116	0263	0410	0557	0704	0851	0998	1145
296	47.1292	1438	1585	1732	1878	2025	2171	2317	2464	2610
297	2756	2903	3049	3195	3341	3487	3633	3779	3925	4070
298	4216	4362	4508	4653	4799	4944	5090	5235	5381	5526
299	5671	5816	5962	6107	6252	6397	6542	6687	6832	6976
300	7121	7266	7411	7555	7700	7844	7989	8133	8278	8422
301	8566	8711	8855	8999	9143	9287	9431	9575	9719	9863
302	48.0007	0151	0294	0438	0582	0725	0869	1012	1156	1299
303	1443	1586	1729	1872	2016	2159	2302	2445	2588	2731
304	2874	3016	3159	3302	3445	3587	3730	3872	4015	4157
305	4300	4442	4584	4727	4868	5011	5153	5295	5437	5579
306	5721	5863	6005	6147	6289	6430	6572	6714	6855	6997
307	7138	7280	7421	7563	7704	7845	7986	8127	8269	8410
308	8551	8692	8833	8973	9114	9255	9396	9537	9677	9818
309	9958	0099	0239	0380	0520	0661	0801	0941	1081	1222
310	49.1362	1502	1642	1782	1922	2062	2201	2341	2481	2621
311	2760	2900	3040	3179	3319	3458	3597	3737	3876	4015
312	4155	4294	4433	4572	4711	4850	4989	5128	5267	5406
313	5544	5683	5822	5960	6099	6237	6376	6514	6653	6791
314	6930	7068	7206	7344	7482	7621	7759	7897	8035	8173
315	8311	8448	8586	8724	8862	8999	9137	9275	9412	9550
316	9687	9824	9962	0099	0236	0374	0511	0648	0785	0922
317	50.1059	1196	1333	1470	1607	1744	1880	2017	2154	2290
318	2427	2564	2700	2837	2973	3109	3246	3382	3518	3654
319	3791	3927	4063	4199	4335	4471	4607	4743	4878	5014
320	5150	5286	5421	5557	5692	5828	5963	6099	6234	6370
321	6505	6640	6775	6911	7046	7181	7316	7451	7586	7721
322	7856	7991	8125	8260	8395	8530	8664	8799	8933	9068
323	9202	9337	9471	9606	9740	9874	0008	0143	0277	0411
324	51.0545	0679	0813	0947	1081	1215	1348	1482	1616	1750
325	1883	2017	2150	2284	2417	2551	2684	2818	2951	3084
326	3218	3351	3484	3617	3750	3883	4016	4149	4282	4415
327	4548	4680	4813	4946	5079	5211	5343	5476	5609	5741
328	5874	6006	6139	6271	6403	6535	6668	6800	6932	7064
329	7196	7328	7460	7592	7724	7855	7987	8119	8251	8382
330	8514	8645	8777	8909	9040	9171	9303	9434	9565	9697
331	9828	9959	0090	0221	0352	0483	0614	0745	0876	1007
332	52.1138	1269	1400	1530	1661	1792	1922	2053	2183	2314
333	2444	2575	2705	2835	2966	3096	3226	3356	3486	3616
334	3746	3876	4006	4136	4266	4396	4526	4656	4785	4915
335	5045	5174	5304	5434	5563	5692	5822	5951	6081	6210
336	6339	6468	6598	6727	6856	6985	7114	7243	7372	7501
337	7630	7759	7888	8016	8145	8274	8402	8531	8660	8788
338	8917	9045	9174	9302	9434	9559	9687	9815	9943	0072
339	53.0200	0328	0456	0584	0712	0840	0968	1095	1223	1351

The Logarithms of Natural Numbers to 2900.

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N ^o .	0	1	2	3	4	5	6	7	8	9
340	53.1479	1607	1734	1862	1990	2117	2245	2372	2500	2627
341	2754	2882	3009	3136	3263	3391	3518	3645	3772	3899
342	4026	4153	4280	4407	4534	4661	4787	4914	5041	5167
343	5294	5421	5547	5674	5800	5927	6053	6179	6306	6432
344	6558	6685	6811	6937	7063	7189	7315	7441	7567	7693
345	7819	7945	8071	8197	8322	8448	8574	8699	8825	8951
346	9076	9202	9327	9452	9578	9703	9829	9954	0079	0204
347	54.0329	0455	0580	0705	0830	0955	1080	1205	1330	1454
348	1579	1704	1829	1953	2078	2203	2327	2452	2576	2701
349	2825	2950	3074	3199	3323	3447	3571	3696	3820	3944
350	4068	4192	4316	4440	4564	4688	4812	4936	5060	5183
351	5307	5431	5554	5678	5802	5925	6049	6172	6296	6419
352	6543	6666	6789	6913	7036	7159	7282	7405	7529	7652
353	7775	7898	8021	8144	8266	8389	8512	8635	8758	8881
354	9003	9126	9249	9371	9494	9616	9739	9861	9984	0106
355	55.0228	0351	0473	0595	0717	0840	0962	1084	1206	1328
356	1450	1572	1694	1816	1938	2060	2181	2303	2425	2546
357	2668	2790	2911	3033	3154	3276	3397	3519	3640	3762
358	3883	4004	4126	4247	4368	4489	4610	4731	4852	4973
359	5094	5211	5336	5457	5578	5699	5820	5940	6061	6182
360	6302	6423	6544	6664	6785	6905	7026	7146	7266	7387
361	7507	7627	7748	7868	7988	8108	8228	8348	8469	8589
362	8709	8828	8948	9068	9188	9308	9428	9548	9667	9787
363	9907	0026	0146	0265	0385	0504	0624	0743	0863	0982
364	56.1101	1221	1340	1459	1578	1697	1817	1936	2055	2174
365	2293	2412	2531	2650	2768	2887	3006	3125	3244	3362
366	3481	3600	3718	3837	3955	4074	4192	4311	4429	4548
367	4666	4784	4903	5021	5139	5257	5375	5494	5612	5730
368	5848	5966	6084	6202	6320	6437	6555	6673	6791	6909
369	7026	7144	7262	7379	7497	7614	7732	7849	7967	8084
370	8202	8319	8436	8554	8671	8778	8905	9023	9140	9257
371	9374	9491	9608	9725	9842	9959	0076	0193	0309	0426
372	57.0543	0660	0776	0893	1010	1126	1243	1359	1476	1592
373	1709	1825	1942	2058	2174	2291	2407	2523	2639	2755
374	2872	2988	3104	3220	3336	3452	3568	3684	3800	3915
375	4031	4147	4263	4378	4494	4610	4726	4841	4957	5072
376	5188	5303	5419	5534	5650	5765	5880	5996	6111	6226
377	6341	6456	6572	6687	6802	6917	7032	7147	7262	7377
378	7492	7607	7721	7836	7951	8066	8181	8295	8410	8525
379	8639	8754	8868	8983	9097	9212	9326	9441	9555	9669
380	9784	9898	0012	0126	0240	0355	0469	0583	0697	0811
381	58.0925	1039	1153	1267	1381	1494	1608	1722	1836	1950
382	2063	2177	2291	2404	2518	2631	2745	2858	2972	3085
383	3199	3312	3425	3539	3652	3765	3879	3992	4105	4218
384	4331	4444	4557	4670	4783	4896	5009	5122	5235	5348
385	5461	5573	5686	5799	5912	6024	6137	6250	6362	6475
386	6587	6700	6812	6925	7037	7149	7262	7374	7486	7599
387	7711	7823	7925	8047	8160	8272	8384	8496	8608	8720
388	8831	8944	9055	9167	9279	9391	9503	9614	9726	9838
389	9950	0061	0173	0284	0396	0507	0619	0730	0842	0953

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390	59.1065	1176	1287	1397	1510	1621	1732	1843	1955	2066
391	2177	2238	2399	2510	2621	2732	2843	2954	3064	3175
392	3286	3397	3508	3618	3729	3840	3950	4061	4171	4282
393	4393	4503	4613	4724	4834	4945	5055	5165	5276	5386
394	5495	5606	5717	5827	5937	6047	6157	6267	6377	6487
395	6597	6707	6817	6927	7037	7146	7256	7366	7476	7585
396	7695	7805	7914	8024	8134	8243	8353	8462	8572	8681
397	8790	8900	9009	9119	9228	9337	9446	9556	9665	9774
398	9883	9992	0101	0210	0319	0428	0537	0646	0755	0864
399	60.0973	1082	1190	1299	1408	1517	1625	1734	1843	1951
400	2060	2169	2277	2386	2494	2602	2711	2819	2928	3036
401	3144	3252	3361	3469	3577	3685	3794	3902	4010	4118
402	4226	4334	4442	4550	4658	4766	4874	4982	5089	5197
403	5305	5413	5521	5628	5735	5843	5951	6059	6166	6274
404	6381	6489	6586	6704	6811	6918	7026	7133	7240	7348
405	7455	7562	7677	7777	7884	7991	8098	8205	8312	8419
406	8526	8633	8740	8847	8954	9060	9167	9274	9381	9488
407	9594	9701	9808	9914	0021	0128	0234	0341	0447	0554
408	61.0610	0767	0873	0979	1086	1192	1298	1405	1511	1617
409	1723	1829	1936	2042	2148	2254	2360	2466	2572	2678
410	2784	2890	2996	3101	3207	3313	3419	3525	3630	3736
411	3842	3947	4053	4159	4264	4370	4475	4581	4686	4792
412	4897	5003	5108	5213	5319	5424	5529	5634	5740	5845
413	5950	6055	6160	6265	6370	6475	6580	6685	6790	6895
414	7000	7105	7210	7313	7420	7524	7629	7734	7839	7943
415	8048	8153	8257	8362	8466	8571	8675	8780	8884	8989
416	9093	9198	9302	9406	9511	9615	9719	9823	9928	0032
417	62.0135	0240	0344	0448	0552	0656	0760	0864	0968	1072
418	1176	1280	1384	1488	1592	1695	1799	1903	2007	2110
419	2214	2318	2421	2525	2628	2732	2835	2939	3042	3146
420	3249	3353	3456	3559	3663	3766	3869	3972	4076	4179
421	4282	4385	4488	4591	4694	4798	4901	5004	5107	5209
422	5312	5415	5518	5621	5724	5827	5929	6032	6135	6238
423	6340	6443	6546	6648	6751	6853	6956	7058	7161	7263
424	7366	7468	7571	7673	7775	7878	7980	8082	8184	8287
425	8389	8491	8593	8695	8797	8900	9002	9104	9206	9308
426	9410	9511	9613	9715	9817	9919	0021	0123	0224	0326
427	63.0423	0530	0631	0733	0834	0936	1038	1139	1241	1342
428	1444	1545	1647	1748	1849	1951	2052	2153	2255	2356
429	5427	2558	2660	2761	2862	2963	3064	3165	3266	3367
430	3468	3569	3670	3771	3872	3973	4074	4175	4276	4376
431	4477	4578	4679	4779	4881	4981	5081	5182	5283	5383
432	5484	5584	5685	5785	5886	5986	6086	6187	6287	6388
433	6488	6588	6688	6789	6889	6989	7089	7189	7289	7390
434	7490	7590	7690	7790	7890	7990	8090	8190	8289	8389
435	8489	8589	8689	8789	8888	8988	9088	9188	9287	9387
436	9486	9586	9685	9785	9885	9984	0084	0183	0283	0382
437	64.0841	0581	0680	0779	0879	0978	1077	1176	1276	1375
438	1474	1573	1672	1771	1871	1970	2069	2168	2267	2366
439	2464	2563	2662	2761	2860	2959	3058	3156	3255	3354

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43	242	340
27	226	324
09	208	306
089	187	285
067	165	262
043	140	237
016	113	210
087	084	181
056	053	150
023	019	116
888	984	080
850	946	042
810	906	002
769	864	960
725	820	916
679	774	870
631	726	821
581	676	771
529	623	718
474	569	663
18	512	607
360	454	548
299	393	487
237	331	424
173	266	359
106	199	293
038	131	224
967	060	153
895	988	080
821	913	005
744	836	929
666	758	850
586	677	769
503	595	687
419	511	602
333	424	516
245	336	427
155	246	337
063	154	245
970	060	151
874	964	055
777	867	957
667	767	857
576	666	756
473	563	652
368	457	547
261	351	440
153	242	331
042	131	220
930	019	107

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905	993
788	877
671	759
551	639
430	517
306	394
182	269
055	142
926	013
796	883
664	751
531	617
395	482
258	344
119	205
979	065
837	922
643	778
147	632
400	485
251	336
100	185
948	033
794	879
638	723
481	565
322	406
162	246
000	084
836	920
671	754
504	588
336	419
165	248
994	077
821	903
646	728
469	552
291	374
112	194
931	013
748	830
564	646
379	460
191	273
903	084
912	893
621	701
428	508
233	313

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540	732.394	474	555	635	715	790	876	956	037	117
541	733.197	277	358	438	518	598	679	759	839	919
542	999	079	159	240	320	400	480	560	640	720
543	734.800	880	960	040	120	200	279	359	439	519
544	735.599	679	758	838	918	998	078	157	237	317
545	736.396	476	556	635	715	795	874	954	033	113
546	737.192	272	352	431	511	590	670	749	828	908
547	987	067	146	225	305	384	463	542	622	701
548	738.781	860	939	018	097	177	256	335	414	493
549	739.572	651	730	810	889	968	047	126	205	284
550	740.363	442	521	599	678	757	836	915	994	073
551	741.152	230	309	388	467	545	624	703	782	860
552	939	018	096	175	254	332	411	489	568	647
553	742.725	804	882	961	039	118	196	274	353	431
554	743.510	588	666	745	823	902	980	058	136	215
555	744.293	371	449	528	606	684	762	840	918	997
556	745.075	153	231	309	387	465	543	621	699	777
557	855	933	011	089	167	245	323	401	478	556
558	746.634	712	790	868	945	023	101	179	256	334
559	747.412	489	567	645	722	800	878	955	033	110
560	748.188	266	343	421	498	576	653	731	808	885
561	963	040	118	195	272	350	427	504	582	659
562	749.736	814	891	968	045	122	200	277	354	431
563	750.508	585	663	740	817	894	971	048	125	202
564	751.279	356	433	510	587	664	741	818	895	972
565	752.048	125	202	279	356	433	509	586	663	740
566	816	893	970	047	123	200	277	353	430	506
567	753.583	660	736	813	889	966	042	119	195	272
568	754.348	425	501	578	654	730	807	883	960	036
569	755.112	189	265	341	417	494	570	646	722	799
570	875	951	027	103	179	256	332	408	484	560
571	756.636	712	788	864	940	016	092	168	244	320
572	757.396	472	548	624	700	775	851	927	003	079
573	758.155	230	306	382	458	533	609	685	760	836
574	912	987	063	139	214	290	366	441	517	592
575	759.668	743	819	894	970	045	121	196	272	347
576	760.422	498	573	649	724	799	875	950	025	100
577	761.176	251	326	402	477	552	627	702	777	853
578	928	003	078	153	228	303	378	453	528	603
579	762.679	754	829	903	978	053	128	203	278	353
580	763.428	503	568	633	707	782	857	932	007	081
581	764.176	251	326	400	475	550	624	699	774	848
582	023	998	072	147	221	296	370	445	519	594
583	765.669	743	817	892	966	041	115	190	264	338
584	766.413	487	562	636	710	784	859	933	007	081
585	767.156	230	304	378	453	527	601	675	749	823
586	898	972	046	120	194	268	342	416	490	564
587	768.638	712	786	860	934	008	082	156	230	303
588	769.377	451	525	599	673	746	820	894	968	041
589	770.115	189	263	336	410	484	557	631	705	778

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40	514
75	248
908	981
40	713
71	444
00	173
29	902
56	623
82	354
06	079
30	802
52	524
72	245
93	965
12	684
329	401
46	117
61	832
75	546
87	259
99	970
609	680
19	390
027	098
734	804
440	510
144	215
848	918
550	620
252	322
952	022
691	721
349	418
045	115
741	810
436	505
129	198
821	850
512	582
203	272
892	960
580	648
566	335
952	02
637	705
320	389
003	071
684	722
65	423
044	112

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640	806.180	248	316	383	451	519	587	655	722	790
641	858	926	993	061	129	197	264	332	400	467
642	807.535	603	670	738	805	873	941	008	076	143
643	808.211	278	346	413	481	548	616	683	751	818
644	886	953	021	088	155	223	290	358	425	492
645	809.560	627	694	762	829	896	963	031	098	165
646	810.232	300	367	434	501	568	636	703	770	837
647	904	971	038	106	173	240	307	374	440	508
648	811.575	642	709	776	843	910	977	044	111	178
649	812.245	312	378	445	512	579	646	713	780	846
650	913	980	047	114	180	247	314	381	447	514
651	813.581	648	714	781	848	914	981	048	114	181
652	814.248	314	381	447	514	580	647	714	780	847
653	913	980	046	113	179	246	312	378	445	511
654	815.578	644	710	777	843	910	976	042	109	175
655	816.241	308	374	440	506	573	639	705	771	838
656	904	970	036	102	169	235	301	367	433	499
657	817.565	632	697	764	830	896	962	028	094	160
658	818.226	292	358	424	490	556	622	688	754	819
659	885	951	017	083	149	215	281	346	412	478
660	819.544	610	675	741	807	873	939	004	070	136
661	820.201	267	333	398	464	530	595	661	727	792
662	858	924	989	055	120	186	251	317	382	448
663	821.513	579	644	710	775	841	906	972	037	103
664	822.168	233	299	364	430	495	560	626	691	756
665	822	887	952	017	083	148	213	279	344	409
666	823.474	539	605	670	735	800	865	930	996	061
667	824.126	191	256	321	386	451	516	581	646	711
668	776	841	906	971	036	101	166	231	296	361
669	825.426	491	556	621	686	751	815	880	945	010
670	826.075	140	204	269	334	399	463	528	593	658
671	722	787	852	917	981	046	111	175	240	305
672	827.369	434	498	563	628	692	757	821	886	950
673	828.015	080	144	209	273	338	402	466	531	595
674	660	724	789	853	918	982	046	111	175	239
675	829.304	368	432	497	561	625	690	754	818	882
676	947	011	075	139	204	268	332	396	460	524
677	830.589	653	717	781	845	909	973	037	102	166
678	831.230	294	358	422	486	550	614	678	742	806
679	870	934	998	062	125	189	253	317	381	445
680	832.509	573	637	700	764	828	892	956	019	083
681	833.147	211	275	338	402	466	530	593	657	721
682	784	848	912	975	039	103	166	230	293	357
683	834.421	484	548	611	675	738	802	866	929	993
684	835.056	120	183	246	310	373	437	500	564	627
685	691	754	817	881	944	007	071	134	197	261
686	836.324	387	451	514	577	640	704	767	830	893
687	957	020	083	146	209	273	336	399	462	525
688	837.588	652	715	778	841	904	967	030	093	156
689	838.219	282	345	408	471	534	597	660	723	786

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690	838.849	912	975	038	101	164	227	289	352	415
691	839.478	541	604	667	729	792	855	918	981	043
692	840.106	169	232	294	357	420	482	545	608	671
693	733	796	859	921	984	046	109	172	234	297
694	841.359	422	485	547	610	672	735	797	860	922
695	984	047	110	172	235	297	360	422	484	547
696	842.609	672	734	796	859	921	983	046	108	170
697	843.233	295	357	420	482	544	606	669	731	793
698	855	918	980	042	104	166	229	291	353	415
699	844.477	539	601	663	726	788	850	912	974	036
700	845.098	160	222	284	346	408	470	532	594	656
701	718	780	842	904	966	028	090	151	213	275
702	846.337	399	461	523	584	646	708	770	832	893
703	955	017	079	141	202	264	326	388	449	511
704	847.573	634	696	758	819	881	943	004	067	127
705	848.189	251	312	374	435	497	559	620	682	743
706	805	866	928	989	051	112	174	236	295	358
707	849.419	481	542	604	665	726	788	849	911	972
708	850.033	095	156	217	279	340	401	462	523	585
709	646	707	769	830	891	952	014	075	136	197
710	851.258	319	381	442	503	564	625	686	747	808
711	870	931	992	053	114	175	236	297	358	419
712	852.480	541	602	663	724	785	846	907	968	029
713	853.089	150	211	272	333	394	455	516	576	637
714	698	759	820	881	942	002	063	124	184	245
715	854.306	367	427	488	549	610	670	731	792	852
716	913	974	034	095	156	216	277	337	398	459
717	855.519	580	640	701	761	822	882	943	003	064
718	856.124	185	245	306	366	427	487	548	608	668
719	729	789	850	910	970	031	091	151	212	272
720	857.332	393	453	513	574	634	694	754	815	875
721	935	995	056	116	176	236	296	357	417	477
722	858.537	597	657	718	778	838	898	958	018	079
723	859.138	198	258	318	378	438	499	559	619	679
724	739	798	858	918	978	038	098	158	218	278
725	860.338	398	458	518	578	637	697	757	817	877
726	937	996	056	116	176	236	295	355	415	475
727	861.534	594	654	714	773	833	893	952	012	072
728	862.131	191	251	310	370	430	489	549	608	668
729	727	787	846	906	966	025	085	144	204	263
730	863.323	382	442	501	561	620	680	739	798	858
731	917	977	036	096	155	214	274	333	392	452
732	864.511	570	630	689	748	808	867	926	985	045
733	865.104	163	222	282	341	400	459	518	578	637
734	696	755	814	873	932	992	051	110	169	228
735	866.287	346	405	465	524	583	642	701	760	819
736	878	937	996	055	114	173	232	291	350	409
737	867.467	526	585	644	703	762	821	880	939	997
738	868.056	115	174	233	292	350	409	468	527	586
739	644	702	762	821	879	938	007	066	124	172

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740	859.232	250	349	408	466	525	584	642	701	760
741	818	877	935	994	053	111	170	228	287	345
742	870.404	462	521	579	638	696	755	813	872	930
743	989	047	106	164	223	281	339	398	456	515
744	871.573	631	690	748	806	865	923	981	040	098
745	872.156	215	273	331	389	449	506	564	622	681
746	739	797	855	913	972	030	088	146	204	262
747	873.321	378	437	495	553	611	669	727	785	843
748	502	960	017	076	134	192	250	308	366	424
749	874.482	540	598	656	714	772	830	887	945	003
750	875.061	119	177	235	293	351	409	466	524	582
751	640	698	756	813	871	929	987	045	102	160
752	876.218	276	333	391	449	506	564	622	680	737
753	795	853	910	968	026	083	141	198	256	314
754	877.371	429	486	544	602	659	717	774	832	889
755	947	004	062	119	177	234	292	349	407	464
756	878.522	579	637	694	751	809	866	924	981	038
757	879.096	153	211	268	325	383	440	497	555	612
758	669	726	784	841	898	956	013	070	127	185
759	880.242	299	356	413	471	528	585	642	699	756
760	814	871	928	985	042	099	156	213	270	328
761	881.385	442	499	556	613	670	727	784	841	898
762	955	012	069	126	183	240	297	354	411	468
763	882.524	581	638	695	752	809	866	923	980	036
764	883.093	150	207	264	321	377	434	491	548	605
765	661	718	775	832	888	945	002	059	115	172
766	884.229	285	342	399	455	512	569	625	682	739
767	795	852	909	965	022	078	135	191	248	305
768	885.361	418	474	531	587	644	700	757	813	870
769	926	983	039	096	152	209	265	321	378	434
770	886.491	547	603	660	716	773	829	885	942	998
771	887.054	111	167	223	280	336	392	448	505	561
772	617	674	730	786	842	898	955	011	067	123
773	888.179	236	292	348	404	460	516	573	629	685
774	741	797	853	909	965	021	077	134	190	246
775	889.302	358	414	470	526	582	638	694	750	806
776	862	918	974	030	085	141	197	253	309	365
777	890.421	477	533	589	644	700	756	812	868	924
778	980	035	091	147	203	259	314	370	426	482
779	891.537	593	649	705	760	816	872	927	983	039
780	892.095	150	206	262	317	373	428	484	540	595
781	651	707	762	818	873	929	985	040	096	151
782	893.207	262	318	373	429	484	540	595	651	706
783	762	817	873	928	984	039	094	150	205	261
784	894.316	371	427	482	538	593	648	704	759	814
785	870	925	980	036	091	146	201	257	312	367
786	895.422	478	533	588	643	699	754	809	864	919
787	975	030	085	140	195	251	306	361	416	471
788	896.526	581	636	691	747	802	857	912	967	022
789	897.077	132	187	242	297	352	407	462	517	572

*20 *The Logarithms of Natural Numbers to 8400.*

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770	897.627	682	737	792	847	902	957	012	067	122
791	898.176	231	286	341	396	451	506	561	615	670
792	725	780	835	890	944	999	054	109	164	218
793	829.273	328	383	437	492	547	602	656	711	766
794	820	875	930	985	039	094	149	203	258	312
795	900.357	422	476	531	586	640	695	749	804	858
796	913	968	022	077	131	186	240	295	349	404
797	901.458	513	567	622	676	731	785	840	894	948
798	902.003	057	112	166	220	275	329	384	438	492
799	547	601	655	710	764	818	873	927	981	036
800	903.090	144	198	253	307	361	415	470	524	578
801	632	687	741	795	849	903	958	012	066	120
802	904.174	228	283	337	391	445	499	553	607	661
803	715	770	824	878	932	986	040	094	148	202
804	905.256	310	364	418	472	526	580	634	688	742
805	796	850	904	958	012	065	119	173	227	281
806	906.335	339	443	497	550	604	658	712	766	820
807	873	927	981	035	089	142	196	250	304	358
808	907.411	465	519	573	626	680	734	787	841	895
809	948	002	056	109	163	217	270	324	378	431
810	908.485	539	592	646	699	753	807	860	914	967
811	909.021	074	128	181	235	288	342	395	449	502
812	556	609	663	716	770	823	877	930	984	037
813	910.090	144	197	251	304	358	411	464	518	571
814	624	678	731	784	838	891	944	998	051	104
815	911.158	211	264	317	371	424	477	530	584	637
816	690	743	797	850	903	956	009	063	116	169
817	912.222	275	328	381	435	488	541	594	647	700
818	753	806	859	913	966	019	072	125	178	231
819	913.284	337	390	443	496	549	602	655	708	761
820	814	867	920	973	026	079	131	184	237	290
821	914.343	396	449	502	555	608	660	713	766	819
822	872	925	977	030	083	136	189	241	294	347
823	915.400	453	505	558	611	664	716	769	822	874
824	927	980	033	085	138	191	243	295	349	401
825	916.454	507	559	612	664	717	770	822	875	927
826	930	033	085	138	190	243	295	348	400	453
827	917.505	558	610	663	715	768	820	873	925	978
828	918.030	083	135	188	240	292	345	397	450	502
829	554	607	660	712	764	816	869	921	973	026
830	919.078	130	183	235	287	340	392	444	496	549
831	601	653	705	758	810	862	914	967	019	071
832	920.123	175	228	280	332	384	436	489	541	593
833	645	697	749	801	853	906	958	010	062	114
834	921.165	218	270	322	374	426	478	530	582	634
835	686	738	790	842	894	946	998	050	102	154
836	922.206	258	310	362	414	466	518	570	622	674
837	725	777	829	881	933	985	037	088	140	192
838	923.244	296	348	399	451	503	555	607	658	710
839	762	814	865	917	969	021	072	124	176	228

	8	9
1	693	744
7	207	260
3	724	776
8	239	291
2	754	805
6	268	319
30	781	832
12	293	345
14	805	856
39	317	368
16	827	878
87	338	389
86	847	898
95	356	407
14	864	915
81	372	423
9	879	930
85	385	437
11	892	943
17	397	448
12	902	953
16	406	457
60	910	960
63	413	463
65	916	966
67	418	468
69	919	969
70	420	470
70	920	970
67	419	469
65	918	968
67	417	467
65	915	964
62	412	462
59	909	958
55	405	454
51	900	950
46	395	445
41	890	939
35	384	433
28	877	927
21	370	419
13	852	911
5	354	403
6	845	894
87	335	385
77	825	875
55	315	364
55	804	853
44	292	341

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890	949.390	439	488	536	585	634	683	731	780	829
891	878	926	975	024	073	121	170	219	267	316
892	950.365	413	462	511	560	608	657	705	754	803
893	851	900	949	997	046	095	143	192	240	289
894	951.337	386	435	483	532	580	629	677	726	774
895	283	872	920	969	017	066	114	163	211	259
896	952.308	356	405	453	502	550	599	647	696	744
897	792	841	889	938	986	034	083	131	180	228
898	953.276	325	373	421	470	518	566	615	663	711
899	760	808	856	905	953	001	049	098	146	194
900	954.242	291	339	387	435	484	532	580	628	677
901	725	773	821	869	918	966	014	062	110	158
902	955.206	255	303	351	399	447	495	543	592	640
903	688	736	784	832	880	928	976	024	072	120
904	956.168	216	264	312	361	409	457	505	553	601
905	649	697	744	792	840	888	936	984	032	080
906	957.128	176	224	272	320	368	416	464	511	559
907	607	655	703	751	799	847	894	942	990	038
908	958.086	134	181	229	277	325	373	420	468	516
909	564	612	659	707	755	803	850	898	946	994
910	959.041	084	137	184	232	280	328	375	423	471
911	518	566	614	661	709	757	804	852	900	947
912	995	042	090	138	185	233	280	328	376	423
913	960.471	518	566	613	661	709	756	804	851	899
914	946	994	041	089	136	184	231	279	326	374
915	961.421	469	516	563	611	658	706	753	801	848
916	896	943	990	038	085	132	180	227	275	322
917	962.369	417	464	511	559	606	653	701	748	795
918	843	890	937	985	032	079	126	174	221	268
919	963.315	363	410	457	504	552	599	646	693	741
920	788	835	882	929	977	024	071	118	165	212
921	964.260	307	354	401	448	495	542	590	637	684
922	731	778	825	872	919	966	013	060	108	155
923	965.202	249	296	343	390	437	484	531	578	625
924	672	719	766	813	860	907	954	001	048	095
925	966.142	189	236	283	329	376	423	470	517	564
926	611	658	705	752	798	845	892	939	986	033
927	967.080	127	173	220	267	314	361	408	454	501
928	548	595	642	688	735	782	829	875	922	969
929	968.016	062	109	156	203	249	296	343	389	436
930	483	530	576	623	670	716	763	810	856	903
931	950	996	043	090	136	183	229	276	323	369
932	969.416	462	509	556	602	649	695	742	788	835
933	882	929	975	021	058	114	161	207	254	300
934	970.347	393	440	486	533	579	626	672	719	765
935	812	858	904	951	997	044	090	137	183	229
936	971.276	322	369	415	461	508	554	600	647	693
937	740	786	832	879	925	971	018	064	110	156
938	972.203	249	295	342	388	434	480	527	573	619
939	665	712	758	804	851	897	943	989	035	082

No	0	1	2	3	4	5	6	7	8	9
940	973.128	174	220	266	313	359	405	451	497	543
941	590	636	682	728	774	820	866	913	959	005
942	974.051	097	143	189	235	281	327	373	420	466
943	512	558	604	650	696	742	788	834	880	926
944	972	018	064	110	156	202	248	294	340	386
945	975.432	478	524	570	616	661	707	753	799	845
946	891	937	983	029	075	121	166	212	258	304
947	976.350	396	442	487	533	579	625	671	717	762
948	808	854	900	946	991	037	083	129	175	220
949	977.266	312	358	403	449	495	541	586	632	678
950	724	769	815	861	906	952	997	043	089	135
951	978.180	226	272	317	363	409	454	500	546	591
952	637	683	728	774	819	865	911	956	002	047
953	979.093	138	184	230	275	321	366	412	457	503
954	548	594	639	685	730	776	821	867	912	958
955	980.003	049	094	140	185	231	276	322	367	412
956	458	503	549	594	640	685	730	776	821	867
957	912	957	003	048	093	139	184	229	275	320
958	981.365	411	456	501	547	592	637	683	728	773
959	819	864	909	954	000	045	090	135	181	226
960	982.271	316	362	407	452	497	543	588	633	678
961	723	769	814	859	904	949	994	040	085	130
962	983.175	220	265	310	356	401	446	491	536	581
963	626	671	716	762	807	852	897	942	987	032
964	984.077	122	167	212	257	302	347	392	437	482
965	527	572	617	662	707	752	797	842	887	932
966	977	022	067	112	157	202	247	292	337	382
967	985.426	471	516	561	606	651	696	741	786	830
968	875	920	965	010	055	100	144	189	234	279
969	986.324	369	413	458	503	548	593	637	682	727
970	772	816	861	906	951	995	040	085	130	174
971	987.219	264	309	353	398	443	487	532	577	622
972	666	711	756	800	845	890	934	979	024	068
973	988.113	157	202	247	291	336	381	425	470	514
974	559	603	648	693	737	782	826	871	915	960
975	989.005	049	094	138	183	227	272	316	361	405
976	450	494	539	583	628	672	717	761	806	850
977	895	939	983	028	072	117	161	206	250	294
978	990.339	383	428	472	516	561	605	650	694	738
979	783	827	871	916	960	004	049	093	137	182
980	991.226	270	315	359	403	448	492	536	580	625
981	669	713	757	802	846	890	934	979	023	067
982	992.111	156	200	244	288	333	377	421	465	509
983	553	598	642	686	730	774	818	863	907	951
984	995	039	083	127	172	216	260	304	348	392
985	993.436	480	524	568	612	657	701	745	789	833
986	877	921	965	009	053	097	141	185	229	273
987	994.317	361	405	449	493	537	581	625	669	713
988	757	801	845	889	933	977	021	064	108	152
989	995.156	240	284	328	372	416	460	504	547	591

N ^c .	0	1	2	3	4	5	6	7	8	9
990	995 035	679	723	767	811	854	898	942	986	030
991	996.074	117	161	205	249	293	336	380	424	468
992	512	555	599	643	687	730	774	818	862	905
993	949	993	037	080	124	168	212	255	299	343
994	997.386	430	474	517	561	605	648	692	736	779
995	823	867	910	954	998	041	085	128	172	216
996	998.259	303	346	390	434	477	521	564	608	652
997	695	739	782	826	869	913	956	000	043	087
998	999.130	174	218	261	305	348	392	435	478	522
999	565	609	652	696	739	783	826	870	913	957

A
T A B L E
O F

Artificial or Logarithmic
SINES and TANGENTS,

*To every Degree and Minute of the
Quadrant.*

T A B L E

O F

Artificial or Logarithmic

S I N E S.

To every Degree and Minute of the Quadrant.

<i>A Table of Logarithmic Sines.</i>						
Min.	0 Deg.	1 Deg.	2 Deg.	3 Deg.	4 Deg.	Min.
0	0	8.241855	8.542819	8.718800	8.843584	0
1	6.463726	49033	46432	21204	5387	1
2	6.764756	56094	49998	23195	7183	2
3	6.940847	63042	53539	25972	8971	3
4	7.065786	69881	57054	28337	8.850751	4
5	7.162696	76614	60540	8.730688	2524	5
6	7.241877	83243	63999	3027	4290	6
7	7.308824	89773	67431	5353	6049	7
8	7.366816	96207	70836	7667	7801	8
9	7.417968	8.302546	74214	9969	9546	9
10	7.463725	08794	77566	8.742259	8.861283	10
11	7.505118	14954	80892	4536	3014	11
12	7.542906	21027	84193	6801	4738	12
13	7.577668	27016	87469	9055	6454	13
14	7.609853	32924	90721	8.751297	8165	14
15	7.639816	38753	93948	3528	9868	15
16	7.667844	44504	97152	5747	8.871565	16
17	7.694173	50180	8.600932	7955	3255	17
18	7.718997	55783	03489	8.760151	4938	18
19	7.742477	61315	06623	2337	6615	19
20	7.764754	66777	09734	4511	8285	20

TABLE

OF

Artificial or Logarithmic

TANGENTS.

To every Degree and Minute of the Quadrant.

<i>A Table of Logarithmic Tangents.</i>						
Min.	0 Deg.	1 Deg.	2 Deg.	3 Deg.	4 Deg.	Min
0	0	8.841921	8.543083	8.719396	8.844644	0
1	6.463726	49101	46691	21806	6455	1
2	6.764756	56165	50268	24203	8260	2
3	6.940847	63115	52817	26588	8.850057	3
4	7.065786	69956	57336	28959	1846	4
5	7.162696	76691	60828	8.731317	3628	5
6	7.241878	83323	64291	3663	5403	6
7	7.308825	89856	67727	5996	7171	7
8	7.366817	95292	71337	8317	8932	8
9	7.417970	8.302633	74520	8.740626	8.860686	9
10	7.463727	08884	77877	2922	2433	10
11	7.505120	15046	81208	5207	4172	11
12	7.542909	21222	84514	7479	5905	12
13	7.577671	27114	87794	9740	7632	13
14	7.609857	33025	91051	8.751989	9351	14
15	7.639820	38856	94283	4227	8.871064	15
16	7.667849	44610	97492	6453	2770	16
17	7.694179	50289	8.600677	8668	4469	17
18	7.719003	55895	03839	8.760872	6162	18
19	7.742484	61430	06978	3055	7849	19
20	7.764761	66894	10094	5245	9529	20

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A Table of Logarithmic Tangents.

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Min.	0 Deg.	1 Deg.	2 Deg.	3 Deg.	4 Deg.	Min.
21	7.785951	8.372291	8.613189	8.767417	8.881202	21
22	7.806154	8. 77622	8. 6261	8. 9578	8. 2869	22
23	7. 25460	8. 82889	8. 9313	8.771727	8. 4530	23
24	7. 43944	8. 88092	8.622343	8. 3866	8. 6185	24
25	7. 61674	8. 93234	8. 5352	8. 5995	8. 7833	25
26	7. 78708	8. 98315	8. 8340	8. 8114	8. 9476	26
27	7. 95099	8.403338	8.631308	8.780222	8.891112	27
28	7.910894	8. 08304	8. 4256	8. 2320	8. 2742	28
29	7. 26134	8. 13213	8. 7185	8. 4108	8. 4366	29
30	7. 40858	8. 18068	8.640093	8. 6486	8. 5984	30
31	7. 55100	8. 22869	8. 2982	8. 8554	8. 7596	31
32	7. 68889	8. 27618	8. 5853	8.790613	8. 9203	32
33	7. 82253	8. 32315	8. 8704	8. 2662	8.900803	33
34	7. 95219	8. 36962	8.651537	8. 4701	8. 2398	34
35	8.007809	8. 41560	8. 4352	8. 6731	8. 387	35
36	8. 20044	8. 46110	8. 7149	8. 8752	8. 5570	36
37	8. 31945	8. 50613	8. 9928	8.800763	8. 7147	37
38	8. 43527	8. 55070	8.662689	8. 2765	8. 8719	38
39	8. 54809	8. 59481	8. 5433	8. 4758	8.910285	39
40	8. 65806	8. 63849	8. 8160	8. 6742	8. 1846	40
41	8. 76531	8. 68172	8.670870	8. 8717	8. 3401	41
42	8.86997	8. 72453	8. 3563	8. 0683	8. 4951	42
43	8. 97217	8. 76693	8. 6239	8. 2641	8. 6495	43
44	8.107202	8. 80892	8. 8900	8. 4589	8. 8034	44
45	8. 16963	8. 85050	8.681544	8. 6529	8. 9567	45
46	8. 26510	8. 89170	8. 4172	8. 8461	8.921096	46
47	8. 35851	8. 93250	8. 6784	8.820384	8. 2619	47
48	8. 44996	8. 97293	8. 9381	8. 2298	8. 4136	48
49	8. 53952	8.501298	8.691963	8. 4205	8. 5649	49
50	8. 62727	8. 05267	8. 4529	8. 6103	8. 7156	50
51	8. 71328	8. 09200	8. 7081	8. 7592	8. 8658	51
52	8. 79762	8. 13098	8. 9617	8. 9874	8.930155	52
53	8. 88036	8. 16961	8.702139	8.831748	8. 1647	53
54	8. 96156	8. 20790	8. 4646	8. 3613	8. 3134	54
55	8.204126	8. 24586	8. 7139	8. 4471	8. 4616	55
56	8. 11953	8. 28349	8. 9618	8. 7321	8. 6093	56
57	8. 19641	8. 32080	8.712083	8. 9163	8. 7565	57
58	8. 27195	8. 35779	8. 4534	8.840998	8. 9032	58
59	8. 34621	8. 39447	8. 6972	8. 2824	8.940494	59
60	8. 41921	8. 43084	8. 9396	8. 4644	8. 1952	60

30 *Logarithmic Sines, Degrees 5, 6, 7, 8, 9, 10, 11, 12. Logarithms*

D	0	1	2	3	4	5	6	7	8	9
5. 0	8.94.0296	1738	3174	4605	6034	7446	8874	0287	1696	3099
10	95.4499	5894	7284	8670	0052	1429	2801	4170	5534	6893
20	56.8249	9600	0947	2289	3628	4962	6293	7619	8941	0259
30	98.1573	2883	4189	5491	6789	8083	9374	0660	1943	3222
40	99.4497	5768	7036	8299	9560	0816	2069	3318	4563	5805
50	9.00.7040	8278	9510	0737	1926	3182	4400	5613	6824	8031
6. 0	01.9235	0435	1632	2825	4016	5203	6386	7567	8744	9918
10	03.1089	2256	3421	4584	5741	6895	8048	9197	0342	1485
20	04.2615	3762	4895	6026	7154	8279	9400	0519	1635	2748
30	05.3859	4956	6071	7172	8271	9367	0460	1551	2639	3723
40	06.4806	5855	6962	8036	9107	0176	1242	2305	3366	4424
50	07.5480	6833	7883	8931	9976	0719	1759	2797	3832	4864
7. 0	08.5894	6922	7947	8970	9990	1008	2024	3037	4047	5056
10	09.6061	7065	8066	9065	0061	1056	2047	3037	4025	5010
20	10.5992	6973	7951	8921	9901	0873	1842	2809	3774	4737
30	11.5698	6856	7812	8766	9719	0669	1617	2562	3506	4448
40	12.5187	6725	7660	8593	9525	0454	1381	2306	3230	4151
50	13.4470	5587	6503	7416	8327	9237	0144	1050	1954	2855
8. 0	14.3551	4453	5350	6243	7136	8026	8915	9801	0686	1569
10	15.2451	3330	4208	5083	5957	6830	7700	8569	9435	0300
20	16.1664	2025	2885	3743	4600	5454	6307	7159	8008	8855
30	16.9702	0446	1389	2230	3070	3908	4744	5578	6411	7242
40	17.8072	8900	9726	0551	1374	2196	3016	3834	4651	5466
50	18.6280	7092	7903	8712	9519	0325	1130	1933	2734	3534
9. 0	19.4332	5129	5925	6719	7511	8302	9091	9879	0666	1451
10	20.2234	3017	3797	4577	5354	6131	6906	7679	8452	9222
20	9992	0759	1526	2291	3055	3818	4579	5338	6097	6854
30	21.7609	8363	9116	9868	0618	1367	2115	2861	3606	4349
40	22.5092	5833	6572	7311	8048	8784	9518	0252	0984	1714
50	23.2444	3172	3899	4625	5349	6073	6795	7515	8235	8953
0. 0	9670	0386	1101	1814	2526	3237	3947	4656	5363	6069
10	24.6775	7478	8181	8883	9583	0282	0980	1677	2375	3067
20	25.3761	4453	5144	5834	6523	7211	7898	8583	9268	9951
30	26.0633	1314	1994	2673	3351	4027	4703	5377	6051	6723
40	7394	8065	8734	9402	0069	0735	1399	2063	2726	3388
50	27.4094	4708	5367	6024	6681	7337	7991	8644	9297	9948
1. 0	28.0599	1248	1897	2544	3190	3836	4480	5124	5766	6408
10	7048	7687	8326	8964	9600	0235	0870	1504	2137	2768
20	29.3399	4029	4658	5286	5913	6539	7164	7788	8411	9034
30	9655	0276	0895	1514	2132	2748	3364	3979	4593	5207
40	30.5819	6430	7041	7650	8259	8867	9474	0080	0685	1289
50	31.1893	2455	3097	3698	4297	4896	5495	6092	6688	7284
2. 0	7879	8473	9066	9658	0249	0840	1430	2019	2607	3194
10	32.3750	4366	4950	5534	6117	6700	7281	7862	8442	9021
20	9599	0176	0753	1328	1903	2478	3051	3624	4195	4766
30	33.5337	5906	6475	7043	7610	8176	8742	9306	9871	0434
40	34.0996	1556	2119	2679	3239	3797	4355	4912	5469	6024
50	6579	7134	7687	8240	8792	9343	9893	0443	0992	1540

Logarithmic Tangents Degrees, 5, 6, 7, 8, 9, 10, 11, 12, Index 9. 3

D	0	1	2	3	4	5	6	7	8	9
5. 0	8.94.1952	3404	4852	6295	7734	9168	0597	2021	3141	4856
10	95.6267	7673	9075	0473	1866	3254	4639	6019	7394	8766
20	97.0133	1496	2855	4209	5560	6906	8248	9586	0920	2251
30	98.3577	4893	6217	7532	8842	0149	1451	2750	4045	5337
40	99.6624	7905	9188	0465	1737	3007	4272	5534	6792	8047
50	00.9298	0546	1790	3031	4268	5502	6732	7959	9183	0403
6. 0	02.1620	2834	4044	5251	6455	7655	8852	0046	1237	2425
10	03.3606	4791	5959	7144	8316	9485	0651	1813	2973	4130
20	04.5284	6434	7582	8727	9869	1008	2144	3277	4407	5535
30	05.6659	7781	8900	0016	1130	2240	3348	4453	5556	6655
40	06.7752	8846	9938	1027	2113	3197	4278	5356	6432	7505
50	07.8576	9644	0710	1773	2833	3891	4947	6000	7050	8098
7. 0	08.9144	0187	1238	2266	3302	4335	5367	6395	7422	8446
10	09.9468	0487	1504	2519	3532	4542	5550	6556	7559	8560
20	10.9559	0556	1551	2543	3533	4521	5507	6491	7472	8452
30	11.9429	0404	1377	2348	3317	4284	5249	6211	7172	8130
40	12.9086	0041	0993	1944	2893	3839	4783	5726	6666	7605
50	13.8542	9476	0409	1340	2269	3196	4121	5044	5965	6885
8. 0	14.7802	8718	9632	0544	1454	2363	3269	4171	5077	5978
10	15.6877	7775	8671	9565	0457	1347	2236	3132	4038	4892
20	16.5774	6654	7532	8409	9284	0157	1029	1899	2767	3634
30	17.4499	5362	6224	7084	7942	8799	9655	0508	1360	2211
40	18.3059	3907	4752	5596	6439	7280	8119	8957	9794	0629
50	19.1462	2294	3124	3953	4780	5606	6430	7253	8074	8891
9. 0	9712	0529	1345	2159	2971	3782	4592	5400	6207	7012
10	20.7816	8619	9420	0220	1018	1815	2611	3405	4198	4985
20	21.5779	6568	7356	8141	8926	9710	0492	1272	2052	2830
30	22.3606	4382	5156	5929	6700	7471	8239	9007	9773	0535
40	23.1302	2065	2826	3586	4345	5103	5859	6614	7368	8120
50	8871	9622	0371	1118	1865	2610	3354	4097	4839	5575
10. 0	24.6319	7057	7794	8530	9264	9998	0730	1461	2191	2920
10	25.3648	4374	5100	5824	6547	7269	7990	8710	9428	0146
20	26.0862	1578	2292	3005	3717	4428	5138	5847	6555	7261
30	7967	8671	9375	0077	0779	1479	2178	2876	3573	4265
40	27.4964	6568	6351	7043	7734	8424	9113	9801	0489	1174
50	28.1858	2542	3225	3907	4588	5268	5947	6624	7301	7977
11. 0	8652	9326	9999	0671	1342	2013	2682	3350	4017	4684
10	29.1349	6013	6677	7339	8001	8662	9322	9980	0638	1291
20	30.1951	2607	3261	3914	4567	5218	5869	6519	7167	7811
30	8463	9109	9714	0398	1042	1685	2327	2967	3608	4241
40	31.4885	5523	6159	6795	7430	8064	8697	9329	9961	0591
50	32.1222	1851	2479	3106	3733	4358	4983	5607	6230	6851
12. 0	7474	8095	8715	9334	9953	0570	1187	1803	2418	3031
10	33.3646	4259	4871	5482	6093	6702	7311	7919	8527	9133
20	9739	0344	0948	1552	2155	2757	3358	3958	4558	5154
30	34.5755	6353	6949	7545	8141	8735	9329	9922	0514	1104
40	35.1697	2287	2876	3465	4053	4640	5227	5813	6398	6981
50	7566	8149	8731	9313	9893	0474	1053	1632	2210	2784

*32 *Logarithm Sines, Deg. 13, 14, 15, 16, 17, 18, 19, 20. Index 9.*

D	0	1	2	3	4	5	6	7	8	9
13. 0	35.2098	2635	3181	3726	4271	4815	5358	5901	6443	6984
10	7524	8064	8603	9141	9678	0215	0751	1287	1822	2356
20	36.2889	3422	3954	4485	5016	5546	6075	6604	7131	7659
30	8185	8711	9236	9751	0265	0808	1330	1852	2373	2894
40	37.3414	3933	4452	4970	5487	6003	6519	7035	7549	8063
50	8576	9089	9601	0113	0624	1134	1643	2152	2660	3168
14. 0	38.3675	4181	4687	5192	5697	6201	6704	7207	7709	8210
10	8711	9211	9711	0210	0708	1206	1703	2199	2695	3190
20	39.3685	4179	4673	5166	5658	6150	6641	7131	7621	8111
30	8600	9088	9575	0052	0539	1035	1520	2005	2489	2972
40	40.3455	3938	4423	4901	5381	5862	6341	6820	7299	7777
50	8754	9231	9707	0182	0657	1130	1601	2070	2538	3005
15. 0	41.2955	5127	5738	6348	6957	7565	8172	8778	9383	9987
10	7684	8144	8603	9061	9518	0007	0470	0933	1395	1857
20	42.3318	2778	3238	3697	4156	4615	5073	5530	5987	6443
30	6809	7354	7899	8453	8917	9370	9823	0075	0526	0978
40	43.1429	1879	2323	2778	3226	3675	4122	4569	5016	5462
50	5908	6353	6798	7242	7686	8129	8572	9014	9456	9897
16. 0	44.0330	0778	1215	1658	2095	2535	2973	3410	3847	4284
10	4720	5155	5590	6025	6459	6893	7326	7759	8191	8623
20	9054	9485	9915	0345	0775	1204	1632	2060	2488	2915
30	45.3342	3768	4194	4619	5044	5469	5893	6316	6739	7162
40	7584	8066	8542	8948	9268	9688	0108	0526	0945	1364
50	46.1742	2199	2616	3032	3448	3864	4279	4694	5108	5522
17. 0	5935	6348	6761	7173	7585	7997	8407	8817	9227	9637
10	47.0046	0455	0863	1271	1678	2086	2492	2898	3304	3710
20	4115	4519	4923	5327	5730	6133	6536	6938	7340	7741
30	8142	8542	8942	9342	9741	0140	0538	0937	1334	1731
40	48.2128	2525	2921	3316	3712	4107	4501	4895	5289	5682
50	6075	6467	6859	7251	7643	8033	8424	8814	9204	9593
18. 0	9982	0371	0759	1147	1534	1922	2308	2695	3081	3466
10	49.3851	4236	4621	5005	5388	5772	6154	6537	6919	7301
20	7682	8063	8444	8824	9204	9584	9963	0342	0721	1099
30	50.1476	1854	2231	2607	2984	3340	3735	4110	4485	4860
40	5234	5608	5981	6354	6727	7099	7471	7843	8214	8585
50	8256	9326	9696	0065	0434	0803	1172	1540	1907	2275
19. 0	51.2642	3009	3375	3741	4107	4472	4837	5202	5566	5930
10	6294	6657	7020	7382	7745	8107	8468	8829	9190	9551
20	9911	0271	0631	0990	1349	1707	2066	2423	2781	3138
30	52.3495	3852	4208	4564	4920	5275	5630	5984	6339	6693
40	7246	7403	7753	8105	8458	8810	9161	9513	9864	0215
50	53.0565	0915	1265	1614	1963	2312	2661	3009	3357	3704
20. 0	4052	4399	4745	5091	5437	5783	6129	6474	6818	7163
10	7527	7851	8194	8537	8880	9223	9565	9907	0249	0590
20	54.0931	1272	1613	1953	2293	2632	2971	3310	3649	3987
30	4325	4663	5000	5338	5674	6011	6347	6683	7019	7354
40	7689	8024	8358	8693	9026	9360	9693	0026	0359	0691
50	55.1024	1356	1687	2018	2349	2680	3010	3341	3670	4000

6	7	8	9
10	7382	7953	8524
99	3064	3629	4193
22	8681	9229	9797
682	4234	4786	5337
78	9724	0270	0815
14	5654	5694	6233
990	0524	1058	1591
08	5836	6364	6892
69	1092	1615	2137
75	6223	6810	7326
27	1440	1951	2463
27	6534	7041	7547
975	1577	2079	2580
73	6570	7067	7563
322	1514	2006	2497
923	6411	6858	7384
777	1260	1743	2225
86	6064	6542	7019
49	0823	1297	1770
969	5539	6008	6476
746	0211	0676	1141
381	4842	5303	5763
975	9432	9889	0345
529	3982	4435	4887
043	8492	8941	9390
519	2965	3410	3854
957	7399	7841	8282
359	1797	2235	2672
724	6159	6593	7027
054	0485	0916	1346
349	4777	5204	5631
610	9034	9458	9882
838	3259	3679	4100
033	7451	7868	8285
196	1611	2025	2439
328	5739	6150	6561
429	9837	0245	0653
500	3905	4310	4715
540	7943	8345	8747
552	1952	2351	2750
536	5933	6329	6721
491	9885	0279	0671
419	3811	4202	4592
320	7709	8097	8486
195	1581	1967	2351
044	5427	5810	6199
867	9248	9629	0008
665	3043	6422	3806

	0	1	2	3	4	5	6	7	8	9
21.0	4329	4658	4987	5315	5643	5971	6299	6626	6953	7280
10	7606	7932	8258	8583	8909	9234	9558	9883	0207	0531
20	56.0854	1178	1501	1824	2146	2468	2790	3112	3433	3755
30	4075	4396	4716	5036	5356	5676	5995	6314	6632	6951
40	7269	7587	7904	8222	8539	8855	9172	9488	9804	0120
50	57.0435	0751	1066	1380	1695	2009	2323	2636	2949	3263
22.0	3575	3888	4200	4512	4824	5136	5447	5758	6068	6379
10	6689	6999	7309	7618	7927	8236	8545	8853	9162	9469
20	9777	0084	0391	0699	1005	1312	1618	1924	2229	2534
30	58.2840	3144	3449	3753	4058	4361	4665	4968	5272	5574
40	5877	6179	6482	6783	7085	7386	7688	7988	8289	8590
50	8890	9190	9489	9789	0088	0387	0686	0984	1282	1580
23.0	59.1878	2175	2473	2770	3067	3363	3659	3955	4251	4547
10	4842	5137	5432	5727	6021	6315	6609	6903	7196	7490
20	7783	8075	8368	8660	8952	9244	9536	9827	0118	0409
30	60.0700	0990	1280	1570	1860	2150	2439	2728	3017	3305
40	3594	3881	4170	4457	4755	5032	5319	5606	5892	6178
50	6465	6751	7036	7321	7607	7892	8176	8461	8745	9029
24.0	9313	9597	9880	0163	0446	0729	1012	1294	1576	1858
10	61.2140	2421	2702	2983	3264	3545	3825	4105	4385	4665
20	4944	5223	5502	5781	6060	6338	6616	6894	7172	7450
30	7727	8004	8281	8558	8834	9110	9386	9662	9937	0213
40	62.0488	0763	1038	1313	1587	1861	2135	2409	2682	2956
50	3229	3502	3774	4047	4319	4591	4863	5135	5406	5677
25.0	5948	6219	6490	6760	7030	7300	7570	7840	8109	8378
10	8647	8916	9184	9453	9721	9989	0257	0524	0792	1059
20	63.1326	1593	1859	2125	2391	2658	2923	3189	3454	3719
30	3984	4249	4514	4778	5042	5306	5570	5833	6097	6360
40	6623	6886	7148	7411	7673	7935	8197	8458	8720	8981
50	9242	9503	9764	0024	0284	0544	0804	1064	1323	1583
26.0	64.1842	2101	2360	2618	2876	3135	3393	3650	3908	4165
10	4423	4679	4936	5193	5450	5706	5962	6218	6473	6729
20	6984	7239	7494	7749	8004	8258	8512	8766	9020	9274
30	9527	9781	0034	0287	0539	0792	1044	1297	1549	1800
40	65.2052	2303	2555	2806	3057	3307	3558	3808	4058	4308
50	4558	4808	5057	5307	5556	5805	6054	6302	6550	6799
27.0	7047	7295	7542	7790	8037	8284	8531	8778	9025	9271
10	9517	9763	0009	0255	0500	0746	0991	1236	1481	1726
20	66.1970	2214	2459	2703	2946	3190	3433	3677	3920	4163
30	4406	4648	4891	5133	5375	5617	5859	6100	6341	6583
40	6824	7065	7305	7546	7786	8026	8266	8506	8746	8986
50	9225	9464	9703	9942	0181	0419	0658	0896	1134	1372
28.0	67.1609	1847	2084	2321	2558	2795	3032	3268	3505	3741
10	3977	4213	4448	4684	4919	5155	5389	5624	5859	6094
20	6328	6562	6796	7030	7264	7497	7731	7964	8197	8430
30	8663	8895	9128	9360	9592	9824	0056	0287	0519	0750
40	68.0982	1213	1443	1674	1905	2135	2365	2595	2825	3055
50	2284	2514	2743	2972	3201	3430	3658	3887	4115	4343

6	7	8	9
39	6815	7190	7566
88	0562	0935	1308
014	4285	4656	5027
616	7985	8354	8722
96	1662	2029	2395
53	5317	5682	6046
88	8950	9312	9674
01	2561	2921	3281
93	6151	6509	6867
64	9720	0076	0432
15	3269	3623	3976
45	6797	7149	7501
56	0306	0655	1005
47	3795	4143	4490
018	7265	7611	7956
71	0716	1060	1404
06	4148	4490	4832
22	7562	7903	8243
20	0959	1297	1636
00	4337	4674	5011
63	7699	8034	8369
10	1043	1377	1710
39	4371	4703	5035
52	7682	8013	8343
49	0977	1306	1634
29	4257	4584	4910
94	7520	7846	8171
44	0768	1092	1416
78	4001	4323	4646
27	7219	7540	7861
03	0423	0742	1062
293	3612	3930	4248
170	6786	7103	7420
32	9947	0263	0578
780	3095	3409	3722
916	6228	6541	6853
37	9349	9660	9971
146	2456	2766	3076
242	5551	5859	6168
325	8633	8940	9248
396	1702	2008	2315
454	4759	5065	5365
501	7505	8109	8411
535	0338	1141	1444
558	3861	4162	4466
570	6871	7171	7471
570	9870	0469	0468
559	2853	3156	3456

D	0	1	2	3	4	5	6	7	8	9
29.0	68.5571	5799	6027	6254	6482	6709	6936	7163	7389	7616
10	7842	8069	8295	8521	8747	8972	9198	9423	9648	9873
20	59.0098	0323	0548	0772	0996	1220	1444	1668	1892	2115
30	2339	2562	2785	3008	3231	3453	3676	3898	4120	4342
40	4564	4786	5007	5229	5450	5671	5892	6113	6334	6554
50	6774	6995	7215	7435	7654	7874	8094	8313	8532	8751
30.0	8970	9189	9407	9626	9844	0062	0280	0498	0716	0933
10	70.1151	1368	1585	1802	2019	2236	2452	2669	2885	3101
20	3317	3533	3749	3964	4179	4395	4610	4825	5040	5254
30	5469	5683	5897	6112	6326	6539	6753	6967	7180	7393
40	7606	7819	8032	8245	8457	8670	8882	9094	9306	9518
50	9730	9941	0153	0364	0575	0786	0997	1208	1419	1629
31.0	71.1839	2049	2260	2469	2679	2889	3098	3308	3517	3726
10	3935	4144	4352	4561	4769	4978	5186	5394	5601	5809
20	6017	6224	6432	6639	6846	7053	7259	7466	7672	7879
30	8085	8291	8497	8703	8908	9114	9320	9525	9730	9935
40	72.0140	0345	0549	0754	0958	1162	1366	1570	1774	1978
50	2181	2385	2588	2791	2994	3197	3400	3603	3805	4007
32.0	4210	4411	4614	4815	5017	5219	5420	5622	5823	6024
10	6225	6426	6626	6827	7027	7228	7428	7628	7828	8027
20	8227	8427	8626	8825	9024	9223	9422	9621	9819	0018
30	73.0216	0415	0613	0811	1009	1206	1404	1601	1799	1996
40	2193	2390	2587	2784	2980	3177	3373	3569	3765	3961
50	4157	4353	4548	4744	4939	5134	5329	5525	5719	5914
33.0	6109	6303	6498	6692	6886	7080	7274	7467	7661	7854
10	8048	8241	8434	8627	8820	9013	9205	9398	9590	9783
20	9974	0157	0359	0550	0742	0934	1125	1316	1507	1699
30	74.1889	2080	2271	2462	2652	2842	3032	3223	3413	3602
40	3792	3982	4171	4361	4550	4739	4928	5117	5306	5490
50	5683	5871	6059	6248	6436	6624	6811	6999	7187	7374
34.0	7562	7749	7936	8123	8310	8497	8683	8870	9056	9242
10	9429	9615	9801	9987	0172	0358	0543	0728	0914	1099
20	75.1284	1469	1654	1838	2023	2207	2392	2576	2760	2944
30	3128	3312	3495	3679	3862	4046	4229	4412	4595	4778
40	4960	5143	5326	5508	5690	5872	6054	6236	6418	6600
50	6781	6963	7144	7326	7507	7688	7869	8049	8230	8411
35.0	8591	8772	8952	9132	9312	9492	9672	9851	0031	0211
10	76.0390	0569	0748	0927	1106	1285	1464	1642	1821	1999
20	2177	2356	2534	2712	2889	3067	3245	3422	3599	3777
30	3954	4131	4308	4485	4662	4838	5015	5191	5367	5543
40	5720	5896	6071	6247	6423	6598	6774	6949	7124	7299
50	7475	7649	7824	7999	8173	8348	8522	8696	8871	9045
36.0	9219	9392	9566	9740	9913	0087	0260	0433	0606	0779
10	77.0952	1125	1298	1470	1643	1815	1987	2159	2331	2503
20	2675	2847	3018	3190	3361	3532	3704	3875	4046	4216
30	4388	4558	4729	4899	5069	5240	5410	5580	5750	5920
40	6090	6259	6429	6598	6768	6937	7106	7275	7444	7613
50	7781	7950	8119	8287	8455	8623	8791	8959	9127	9295

7	8	9
5835	6132	6429
8801	9097	9393
1757	2052	2347
4703	4997	5291
7638	7931	8224
0564	0856	1148
3479	3770	4061
6385	6675	6965
9281	9570	9859
2168	2457	2745
5046	5333	5621
7915	8210	8487
0775	1060	1346
3625	3910	4195
6468	6752	7036
9302	9585	9868
2128	2410	2692
4945	5227	5508
7755	8036	8316
0557	0836	1116
3351	3630	3908
6137	6415	6693
8916	9193	9471
1687	1964	2241
4452	4728	5004
7209	7484	7759
9959	0234	0508
2703	2977	3250
5439	5713	5986
8170	8442	8715
0893	1165	1437
3611	3882	4154
6322	6593	6864
9027	9297	9568
1726	1996	2266
4420	4689	4958
7107	7376	7644
9790	0057	0325
2466	2733	3001
5137	5404	5671
7803	8059	8336
0464	0730	0995
3119	3385	3650
5770	6035	6300
8416	8680	8945
1057	1321	1585
3694	3957	4220
6326	6589	6851

38 Logar. Sines, Deg. 37, 38, 39, 40, 41, 42, 43, 44. Index 9.

D	0	1	2	3	4	5	6	7	8	9
37.0	77.9463	9531	9798	9965	0133	0300	0467	0634	0801	0968
10	78.1134	1301	1467	1634	1800	1966	2132	2298	2464	2630
20	2796	2961	3127	3292	3457	3623	3788	3953	4118	4282
30	4447	4612	4776	4941	5107	5269	5433	5597	5761	5925
40	6089	6252	6416	6579	6741	6905	7069	7232	7395	7557
50	7720	7883	8045	8208	8370	8532	8594	8856	9018	9180
38.0	9342	9540	9665	9826	9988	0149	0310	0471	0532	0793
10	79.0954	1115	1275	1436	1596	1757	1917	2077	2237	2397
20	2557	2716	2876	3035	3195	3354	3513	3673	3832	3991
30	4149	4308	4467	4626	4784	4942	5101	5259	5417	5575
40	5733	5891	6048	6206	6364	6521	6679	6836	6993	7150
50	7307	7464	7621	7777	7934	8091	8247	8403	8560	8716
39.0	8872	9028	9184	9339	9495	9651	9806	9962	0117	0272
10	80.0427	0582	0737	0892	1047	1201	1356	1511	1665	1819
20	1973	2128	2282	2435	2589	2743	2897	3050	3204	3357
30	3510	3664	3817	3970	4123	4276	4428	4581	4734	4886
40	5038	5191	5343	5495	5647	5799	5951	6103	6254	6406
50	6557	6709	6860	7011	7163	7314	7465	7615	7766	7917
40.0	8067	8218	8368	8519	8669	8819	8969	9119	9269	9419
10	9569	9718	9867	0017	0167	0316	0465	0614	0763	0912
20	81.1061	1210	1358	1507	1655	1804	1952	2100	2248	2396
30	2544	2692	2840	2987	3135	3283	3430	3578	3725	3872
40	4019	4166	4313	4460	4607	4753	4900	5046	5193	5339
50	5481	5631	5778	5923	6069	6215	6361	6507	6652	6797
41.0	6943	7088	7233	7378	7523	7668	7813	7958	8103	8247
10	8392	8536	8681	8825	8969	9113	9257	9401	9545	9689
20	9832	9976	0120	0263	0406	0550	0693	0836	0979	1122
30	82.1265	1407	1550	1693	1835	1977	2120	2262	2404	2546
40	2688	2830	2972	3114	3255	3397	3538	3680	3821	3963
50	4104	4245	4386	4527	4668	4808	4949	5089	5230	5370
42.0	5511	5651	5791	5931	6071	6211	6351	6491	6631	6770
10	6910	7049	7189	7328	7467	7606	7745	7884	8023	8162
20	8301	8439	8578	8716	8855	8993	9131	9269	9407	9545
30	9683	9821	9959	0097	0234	0372	0509	0646	0784	0921
40	83.1058	1195	1332	1469	1606	1742	1879	2015	2152	2288
50	2425	2561	2697	2833	2969	3105	3241	3377	3512	3648
43.0	3743	3919	4054	4189	4325	4460	4595	4730	4865	4999
10	5134	5269	5403	5538	5672	5806	5941	6075	6209	6343
20	6477	6611	6745	6878	7012	7145	7279	7412	7546	7679
30	7812	7945	8078	8211	8344	8477	8609	8742	8875	9007
40	9139	9272	9404	9536	9668	9800	9932	0064	0196	0327
50	84.0339	0591	0722	0854	0985	1116	1247	1378	1509	1640
44.0	1771	1902	2033	2163	2294	2424	2555	2685	2815	2945
10	3076	3206	3336	3465	3595	3725	3855	3984	4114	4243
20	4372	4502	4631	4760	4889	5018	5147	5276	5404	5533
30	5652	5790	5919	6047	6175	6304	6432	6560	6688	6816
40	6944	7071	7199	7327	7454	7582	7709	7836	7964	8091
50	8218	8345	8472	8599	8726	8853	8979	9106	9232	9359

D	0	1	2	3	4	5	6	7	8	9
37.0	87.7114	7377	7640	7903	8165	8428	8691	8953	9216	9478
10	9741	0003	0265	0528	0790	1052	1314	1576	1839	2101
20	88.2363	2625	2887	3148	3410	3672	3934	4196	4457	4719
30	4980	5242	5503	5765	6026	6288	6549	6810	7072	7333
40	7594	7855	8116	8377	8639	8900	9160	9421	9682	9943
50	89.0204	0465	0725	0986	1247	1507	1768	2028	2289	2549
38.0	2810	3070	3331	3591	3851	4111	4371	4632	4892	5152
10	5412	5672	5932	6192	6452	6712	6971	7231	7491	7751
20	8010	8270	8530	8789	9049	9308	9568	9827	0086	0346
30	90.0605	0864	1124	1383	1642	1901	2160	2419	2679	2938
40	3197	3455	3714	3973	4232	4491	4750	5008	5267	5526
50	5784	6042	6302	6560	6819	7077	7336	7594	7852	8111
39.0	8369	8627	8886	9144	9402	9660	9918	0177	0435	0693
10	91.0951	1209	1467	1724	1982	2240	2498	2756	3014	3271
20	3529	3787	4044	4302	4560	4817	5075	5332	5590	5847
30	6104	6362	6619	6876	7134	7391	7648	7905	8163	8420
40	8677	8934	9191	9446	9705	9962	0219	0476	0733	0990
50	92.1247	1503	1760	2017	2274	2530	2787	3044	3300	3557
40.0	3813	4070	4327	4583	4840	5096	5352	5609	5865	6121
10	6378	6634	6890	7147	7403	7659	7915	8171	8427	8683
20	8940	9196	9452	9708	9964	0219	0475	0731	0987	1243
30	93.1499	1755	2010	2266	2522	2778	3033	3289	3545	3800
40	4056	4311	4567	4822	5078	5333	5589	5844	6100	6355
50	6610	6866	7121	7376	7632	7887	8142	8397	8653	8908
41.0	9163	9418	9673	9928	0183	0438	0694	0949	1204	1458
10	94.1713	1968	2223	2478	2733	2988	3243	3498	3752	4007
20	4262	4517	4771	5026	5281	5535	5790	6045	6299	6554
30	6808	7063	7317	7572	7826	8081	8335	8590	8844	9099
40	9353	9607	9862	0116	0370	0625	0879	1133	1387	1642
50	95.1896	2150	2404	2659	2913	3167	3421	3675	3929	4183
42.0	4437	4691	4945	5199	5453	5707	5961	6215	6469	6723
10	6977	7231	7485	7739	7993	8246	8500	8754	9008	9262
20	9515	9769	0023	0277	0530	0784	1038	1291	1545	1799
30	96.2052	2306	2560	2813	3067	3320	3574	3827	4081	4335
40	4588	4842	5095	5349	5602	5855	6109	6362	6616	6869
50	7122	7376	7629	7883	8136	8389	8643	8896	9149	9403
43.0	9656	9909	0162	0416	0669	0922	1175	1429	1682	1935
10	97.2188	2441	2694	2948	3201	3454	3707	3960	4213	4466
20	4719	4973	5226	5479	5732	5985	6238	6491	6744	6997
30	7250	7503	7756	8009	8262	8515	8768	9021	9274	9527
40	9780	0033	0286	0538	0791	1044	1297	1550	1803	2056
50	98.2399	2562	2814	3067	3320	3573	3826	4079	4331	4584
44.0	4837	5090	5343	5596	5848	6101	6354	6607	6860	7112
10	7365	7618	7871	8123	8376	8629	8882	9134	9387	9640
20	9893	0145	0398	0651	0903	1156	1409	1662	1914	2167
30	99.2420	2672	2925	3178	3430	3683	3936	4189	4441	4694
40	4947	5199	5452	5705	5957	6210	6463	6715	6968	7221
50	7473	7726	7979	8231	8484	8737	8989	9242	9495	9747

D	0	1	2	3	4	5	6	7	8	9
45.0	84.9485	9611	9737	9864	9990	0116	0242	0367	0493	0619
10	85.0745	0870	0996	1121	1246	1372	1497	1622	1747	1872
20	1997	2123	2247	2371	2496	2620	2745	2869	2994	3118
30	3243	3366	3490	3614	3738	3862	3986	4109	4233	4356
40	4480	4603	4727	4850	4973	5096	5219	5342	5465	5588
50	5711	5833	5955	6078	6201	6323	6445	6568	6690	6812
46.0	6934	7056	7178	7300	7421	7543	7665	7786	7908	8029
10	8150	8272	8393	8514	8635	8756	8877	8998	9119	9239
20	9340	9460	9581	9701	9821	9942	0062	0182	0302	0422
30	86.0562	0682	0802	0921	1041	1161	1280	1399	1519	1638
40	1758	1877	1996	2115	2234	2353	2471	2590	2709	2827
50	2946	3064	3183	3301	3419	3537	3656	3774	3892	4010
47.0	4127	4245	4363	4481	4598	4715	4833	4950	5068	5185
10	5302	5419	5536	5653	5770	5887	6004	6120	6237	6353
20	6470	6586	6703	6819	6935	7051	7167	7283	7399	7515
30	7631	7747	7862	7978	8094	8209	8324	8439	8555	8670
40	8785	8900	9015	9130	9245	9360	9474	9589	9704	9818
50	87.9932	0047	0161	0276	0390	0504	0618	0732	0846	0960
48.0	1073	1187	1301	1414	1528	1641	1755	1868	1981	2094
10	2208	2321	2434	2547	2659	2772	2885	2997	3110	3223
20	3335	3448	3560	3672	3784	3896	4008	4120	4232	4344
30	4456	4568	4679	4791	4903	5014	5125	5237	5348	5459
40	5570	5681	5793	5904	6014	6125	6236	6347	6457	6568
50	6678	6789	6900	7010	7120	7230	7340	7450	7560	7670
49.0	7780	7889	7999	8109	8218	8328	8437	8547	8656	8765
10	8875	8984	9093	9202	9311	9420	9529	9637	9746	9855
20	9963	0072	0180	0289	0397	0505	0613	0721	0829	0937
30	88.1045	1153	1261	1369	1477	1584	1692	1799	1907	2014
40	2121	2228	2336	2443	2550	2657	2764	2871	2977	3084
50	3191	3297	3404	3510	3617	3723	3829	3936	4042	4148
50.0	4254	4360	4466	4572	4677	4783	4889	4994	5100	5205
10	5311	5416	5521	5627	5732	5837	5942	6047	6152	6257
20	6362	6466	6571	6675	6780	6885	6989	7093	7198	7302
30	7406	7510	7614	7718	7822	7926	8030	8133	8237	8341
40	8444	8549	8651	8755	8858	8961	9064	9167	9271	9374
50	9476	9579	9682	9785	9888	9990	0093	0195	0297	0400
51.0	890.503	605	707	809	911	013	115	217	319	402
10	891.523	624	726	827	929	030	132	233	334	435
20	892.536	637	738	839	940	041	142	242	343	444
30	893.544	645	745	846	946	046	146	246	346	446
40	894.546	646	746	846	945	045	144	244	343	443
50	895.542	641	741	840	939	038	137	236	335	433
52.0	896.532	631	729	828	926	025	123	222	320	418
10	897.516	624	712	810	908	006	104	201	299	397
20	898.494	592	689	787	884	981	078	176	273	370
30	899.467	564	660	757	854	951	047	144	240	337
40	890.433	529	626	722	818	914	010	106	202	298
50	901.394	489	585	681	776	872	967	063	158	253

D	0	1	2	3	4	5	6	7	8	9
45.0	00.0000	0253	0505	0758	1011	1263	1516	1769	2021	2274
10	2527	2779	3032	3285	3537	3790	4043	4295	4548	4801
20	5053	5306	5559	5811	6064	6317	6569	6822	7075	7328
30	7580	7833	8086	8338	8591	8844	9096	9349	9602	9855
40	01.0107	0360	0613	0866	1118	1371	1624	1877	2129	2382
50	2635	2888	3140	3393	3646	3899	4152	4404	4657	4910
46.0	5163	5416	5668	5921	6174	6427	6680	6932	7185	7438
10	7691	7944	8197	8450	8703	8956	9209	9461	9714	9967
20	02.0220	0473	0726	0979	1232	1485	1738	1991	2244	2497
30	2750	3003	3256	3509	3762	4015	4268	4521	4774	5027
40	5280	5534	5787	6040	6293	6546	6799	7052	7305	7559
50	7812	8065	8318	8571	8825	9078	9331	9584	9837	0091
47.0	03.0344	0597	0851	1104	1357	1611	1864	2117	2371	2624
10	2877	3131	3384	3638	3891	4144	4398	4651	4905	5158
20	5412	5665	5919	6172	6426	6680	6933	7187	7440	7694
30	7947	8201	8455	8708	8962	9215	9469	9723	9977	0231
40	04.0484	0738	0992	1246	1500	1753	2007	2261	2515	2769
50	3023	3277	3531	3785	4038	4292	4546	4800	5054	5308
48.0	5563	5817	6071	6325	6579	6833	7087	7341	7595	7850
10	8104	8358	8612	8867	9121	9375	9629	9884	0138	0392
20	05.0647	0901	1156	1410	1664	1919	2173	2428	2682	2937
30	3191	3446	3701	3955	4210	4464	4719	4974	5229	5483
40	5738	5992	6248	6502	6757	7012	7267	7522	7777	8032
50	8286	8541	8796	9051	9306	9561	9816	0072	0327	0582
49.0	06.0837	1092	1347	1602	1858	2113	2368	2623	2879	3134
10	3389	3643	3900	4156	4411	4666	4922	5177	5433	5689
20	5944	6200	6455	6711	6967	7222	7478	7734	7989	8245
30	8501	8757	9013	9269	9524	9780	0036	0292	0548	0804
40	07.1060	1316	1573	1829	2085	2341	2597	2853	3110	3366
50	3622	3878	4135	4391	4648	4904	5160	5417	5673	5930
50.0	6186	6443	6700	6956	7213	7470	7726	7983	8240	8497
10	8753	9010	9267	9524	9780	0038	0295	0552	0809	1066
20	08.1323	1580	1837	2094	2352	2609	2866	3123	3381	3638
30	3895	4153	4410	4668	4925	5183	5440	5698	5956	6213
40	6471	6728	6986	7244	7502	7760	8018	8275	8533	8791
50	9049	9307	9565	9823	0081	0340	0598	0856	1114	1372
51.0	09.1631	1889	2147	2406	2664	2923	3181	3440	3698	3957
10	4215	4474	4733	4991	5250	5509	5768	6026	6286	6544
20	6803	7062	7321	7580	7839	8099	8358	8617	8876	9135
30	9395	9654	9913	0173	0432	0692	0951	1211	1470	1730
40	10.1920	2249	2508	2769	3028	3288	3548	3808	4068	4328
50	4588	4848	5108	5368	5628	5889	6149	6409	6670	6930
52.0	7190	7451	7711	7971	8232	8493	8753	9013	9275	9535
10	9796	0057	0318	0579	0839	1100	1361	1622	1883	2145
20	11.2406	2667	2928	3189	3451	3712	3974	4235	4496	4758
30	5019	5281	5543	5804	6066	6328	6590	6852	7113	7375
40	7637	7899	8161	8423	8686	8948	9210	9472	9735	9997
50	12.0259	0522	0784	1047	1309	1572	1835	2097	2360	2623

8, 59, 60. In. g.

7	8	9
014	108	203
959	053	147
898	991	085
832	925	017
760	852	945
682	774	866
599	690	781
510	601	691
415	506	596
315	405	495
210	299	388
099	187	276
982	070	158
860	948	035
733	820	907
600	687	773
462	548	634
318	404	489
169	254	339
015	099	184
855	939	023
691	774	857
520	603	685
345	427	509
164	246	328
978	060	141
787	868	949
591	671	751
390	469	549
183	262	341
972	050	129
755	833	911
533	611	688
306	383	460
074	151	227
838	914	990
596	671	747
349	424	499
097	171	246
840	914	988
578	652	725
311	385	458
040	113	185
763	835	908
482	554	625
196	267	338
505	575	646
609	679	749

D	0	1	2	3	4	5	6	7	8	9
53.0	12.2886	3148	3411	3674	3937	4200	4463	4727	4990	5253
10	5516	5780	6043	6306	6570	6833	7097	7360	7624	7888
20	8151	8415	8679	8942	9207	9471	9735	9999	0263	0527
30	13.0791	1055	1320	1584	1848	2113	2377	2642	2906	3171
40	3435	3700	3965	4230	4495	4760	5024	5289	5555	5820
50	6085	6350	6615	6880	7146	7411	7677	7942	8208	8473
54.0	8739	9005	9270	9536	9802	0068	0334	0600	0866	1132
10	14.1398	1664	1931	2197	2463	2730	2996	3263	3529	3796
20	4062	4329	4596	4863	5130	5397	5663	5931	6198	6465
30	6732	6999	7266	7534	7801	8069	8336	8604	8871	9139
40	9407	9675	9942	0210	0478	0746	1014	1283	1551	1819
50	15.2087	2356	2624	2892	3161	3429	3698	3967	4236	4504
55.0	4773	5042	5311	5580	5849	6118	6387	6657	6926	7195
10	7.65	7734	8004	8273	8543	8813	9083	9352	9622	9892
20	16.0162	0432	0702	0973	1243	1513	1783	2054	2324	2595
30	2866	3136	3407	3678	3949	4220	4491	4762	5033	5304
40	5575	5846	6118	6389	6661	6932	7204	7475	7747	8019
50	8291	8563	8835	9107	9379	9651	9923	0195	0468	0740
56.0	17.023	1205	1558	1830	2103	2376	2649	2922	3195	3468
10	3741	4014	4287	4561	4834	5107	5381	5651	5928	0202
20	6476	6749	7023	7297	7571	7845	8120	8394	8668	8943
30	9217	9492	9766	0041	0316	0590	0865	1140	1415	1690
40	18.1965	2240	2516	2791	3066	3342	3618	3893	4169	4445
50	4720	4996	5272	5548	5824	6101	6377	6653	6930	7206
57.0	7453	7759	8036	8313	8589	8866	9143	9420	9697	9975
10	19.0252	0529	0807	1084	1362	1639	1917	2195	2473	2751
20	3029	3307	3585	3863	4141	4420	4698	4977	5255	5534
30	5813	6091	6370	6649	6928	7207	7487	7766	8045	8325
40	8864	9163	9443	9723	0003	0283	0563	0843	1123	1403
50	20.1403	1684	1964	2245	2525	2806	3087	3368	3649	3930
58.0	4211	4492	4773	5054	5336	5617	5899	6180	6462	6744
10	7026	7308	7590	7872	8154	8436	8719	9001	9284	9566
20	9849	0132	0415	0698	0981	1264	1547	1830	2114	2397
30	21.2681	2964	3248	3532	3816	4100	4384	4668	4952	5236
40	5521	5805	6090	6374	6659	6944	7229	7514	7799	8084
50	8369	8654	8940	9225	9511	9797	0082	0368	0654	0940
59.0	22.1226	1512	1799	2085	2371	2658	2945	3231	3518	3805
10	4092	4379	4667	4954	5241	5529	5816	6104	6392	6679
20	6967	7255	7543	7832	8120	8408	8697	8985	9274	9563
30	9851	0140	0429	0719	1008	1297	1586	1876	2166	2455
40	23.2745	3035	3325	3615	3905	4195	4486	4776	5067	5357
50	5648	5939	6230	6520	6812	7103	7394	7685	7977	8268
60.0	8561	8852	9144	9436	9728	0021	0313	0605	0898	1190
10	24.1483	1776	2069	2362	2655	2948	3241	3535	3828	4122
20	4415	4709	5003	5297	5592	5885	6180	6474	6769	7063
30	7358	7653	7948	8243	8538	8833	9128	9424	9719	0015
40	25.0311	0607	0903	1199	1495	1791	2087	2384	2681	2977
50	3274	3571	3868	4165	4462	4760	5057	5355	5652	5950

*44 Log. Sines Deg. 61, 62, 63, 64, 65, 66, 67, 68. In g.

D	0	1	2	3	4	5	6	7	8	9
61. 0	941.819	889	959	029	099	169	239	308	378	448
10	942.517	587	656	725	795	864	933	003	072	141
20	943.201	279	348	417	486	555	624	692	761	830
30	898	967	036	104	172	241	309	377	446	514
40	944.582	650	718	786	854	922	990	058	125	193
50	945.261	328	396	464	531	598	666	733	800	868
62. 0	935	002	069	136	203	270	337	404	471	538
10	946.604	671	738	804	870	937	004	070	136	203
20	947.269	335	401	467	533	599	665	731	797	863
30	929	995	060	126	192	257	323	388	453	519
40	948.584	649	715	780	845	910	975	040	105	170
50	949.235	300	364	429	494	558	623	688	752	816
63. 0	881	945	009	074	138	202	266	330	394	458
10	950.522	586	650	714	777	841	905	968	032	096
20	951.159	222	286	349	412	476	539	602	665	728
30	791	854	917	980	043	105	168	231	294	356
40	952.419	481	544	606	668	731	793	855	917	979
50	953.042	104	166	228	290	351	413	475	537	598
64. 0	660	722	783	845	906	967	029	090	152	213
10	954.274	335	396	457	518	579	640	701	762	823
20	883	944	005	065	126	186	247	307	368	428
30	955.488	548	609	669	729	789	849	909	969	029
40	956.089	148	208	268	328	387	447	506	566	625
50	684	744	804	862	921	980	040	099	158	217
65. 0	957.276	335	393	452	511	570	628	687	746	804
10	862	921	979	038	096	154	212	271	329	387
20	958.455	503	561	619	677	734	792	850	908	965
30	959.023	080	138	195	253	310	367	425	482	539
40	595	653	711	768	825	881	938	995	052	109
50	960.165	222	279	335	392	448	505	561	618	674
66. 0	730	786	845	899	955	011	067	123	179	235
10	961.290	340	402	458	513	569	624	680	735	791
20	846	902	957	012	067	123	178	233	288	343
30	962.398	453	508	562	617	672	727	781	836	891
40	945	999	054	108	162	217	271	325	379	434
50	963.488	542	596	650	704	757	811	865	919	972
67. 0	964.056	080	133	187	240	294	347	400	454	507
10	560	613	666	719	773	826	878	931	984	037
20	965.090	143	195	248	301	353	406	458	511	563
30	615	668	720	772	824	879	928	980	032	085
40	966.136	188	240	292	344	395	447	499	550	602
50	653	705	755	807	858	910	961	012	064	115
68. 0	967.166	217	268	319	369	420	471	522	573	623
10	674	725	775	826	876	927	977	027	078	128
20	968.178	228	278	328	379	429	478	528	578	628
30	678	728	777	827	877	926	976	025	075	124
40	969.173	223	272	321	370	420	469	518	567	616
50	665	714	762	811	860	909	957	006	055	103

gar. tang. Deg. 61, 62, 63, 64, 65, 66, 67, 68, In. to. 45

D	1	2	3	4	5	6	7	8	9	
61.0	85.6248	6546	6844	7142	7441	7739	8038	8336	8635	8934
10	9233	9532	9831	0130	0430	0730	1029	1329	1629	1928
20	20.2229	2529	2829	3129	3430	3731	4031	4332	4633	4934
30	5285	5537	5838	6140	6442	6743	7045	7347	7649	7952
40	8254	8556	8859	9162	9465	9767	0070	0374	0677	0980
50	27.1284	1588	1891	2195	2499	2803	3107	3412	3716	4021
62.0	4326	4630	4935	5240	5546	5851	6156	6462	6768	7073
10	7379	7685	7991	8298	8604	8911	9217	9524	9831	0138
20	28.0445	0752	1060	1367	1673	1983	2291	2599	2907	3215
30	3523	3832	4140	4449	4758	5067	5376	5685	5995	6304
40	6614	6924	7234	7544	7854	8164	8475	8785	9096	9407
50	9718	0029	0340	0651	0963	1274	1586	1898	2210	2522
63.0	29.2834	3146	3459	3772	4084	4397	4710	5023	5337	5650
10	5964	6277	6591	6905	7219	7534	7848	8163	8477	8792
20	9167	9482	9797	0053	0368	0684	0999	1315	1631	1947
30	30.2264	2580	2897	3213	3530	3847	4164	4482	4799	5117
40	5434	5752	6070	6388	6707	7025	7323	7662	7981	8300
50	8619	8938	9258	9577	9897	0217	0537	0857	1177	1498
64.0	31.1818	2139	2460	2781	3102	3423	3745	4066	4388	4710
10	5032	5354	5676	5999	6321	6644	6967	7292	7613	7937
20	8260	8584	8908	9232	9556	9880	0205	0529	0854	1179
30	32.1504	1829	2154	2480	2806	3131	3457	3783	4110	4436
40	4763	5089	5416	5743	6071	6398	6725	7053	7381	7709
50	8037	8365	8694	9023	9351	9680	0009	0339	0668	0998
65.0	33.1327	1657	1987	2318	2648	2979	3309	3640	3971	4302
10	4634	4965	5297	5629	5961	6293	6625	6958	7291	7623
20	7957	8290	8623	8957	9290	9624	9958	0292	0627	0961
30	34.1296	1631	1966	2301	2636	2972	3308	3644	3980	4316
40	4652	4989	5326	5662	6000	6337	6674	7012	7350	7688
50	8026	8364	8702	9041	9380	9719	0058	0398	0737	1077
66.0	35.1417	1757	2097	2438	2778	3119	3460	3801	4142	4484
10	4826	5168	5510	5852	6194	6537	6880	7223	7566	7909
20	8253	8596	8940	9284	9629	9973	0318	0662	1008	1353
30	36.1698	2044	2389	2735	3081	3428	3774	4121	4468	4815
40	5162	5510	5857	6205	6553	6901	7250	7598	7947	8296
50	8645	8995	9344	9694	0044	0394	0745	1095	1446	1797
67.0	37.2148	2499	2851	3203	3555	3907	4259	4612	4965	5317
10	5671	6024	6377	6731	7085	7439	7793	8148	8503	8858
20	9213	9568	9924	0279	0535	0992	1348	1705	2061	2418
30	38.2776	3133	3491	3849	4207	4565	4923	5282	5641	6000
40	6359	6719	7079	7438	7799	8159	8519	8880	9241	9603
50	9964	0326	0689	1050	1412	1775	2137	2500	2863	3226
68.0	39.3590	3954	4318	4683	5047	5412	5777	6142	6507	6873
10	7239	7605	7971	8337	8704	9071	9438	9806	0173	0541
20	40.0909	1277	1646	2015	2384	2753	3122	3492	3862	4232
30	4602	4973	5344	5715	6086	6458	6829	7201	7574	7946
40	8319	8692	9055	9418	9812	0186	0560	0934	1309	1684
50	41.2050	2434	2809	3185	3561	3938	4314	4691	5068	5445

46 *Logar. Sines. Deg.* 69, 70, 71, 72, 73, 74, 75, 76. *In. 9.*

D	0	1	2	3	4	5	6	7	8	9
69.0	970.152	200	248	297	345	394	442	490	538	586
10	635	683	731	779	826	874	922	970	018	065
20	971.113	161	208	256	303	351	398	446	493	540
30	588	635	682	729	776	823	870	917	964	011
40	972.058	105	151	198	245	291	338	384	431	477
50	524	570	617	662	709	755	802	848	894	940
70.0	986	032	078	124	169	215	261	307	352	398
10	973.443	489	535	580	625	671	716	761	807	852
20	897	942	987	032	077	122	167	212	257	302
30	974.347	391	436	481	525	570	614	659	703	747
40	792	836	881	925	969	013	057	101	145	189
50	975.233	277	321	365	408	452	496	539	583	626
71.0	670	713	757	800	844	887	930	974	017	060
10	976.103	146	189	232	275	318	361	404	446	489
20	532	574	617	660	702	745	787	830	872	914
30	957	999	041	083	125	167	209	251	293	335
40	977.377	419	461	503	544	586	628	669	711	752
50	794	835	877	918	959	001	042	083	124	165
72.0	978.206	247	288	329	370	411	452	493	533	574
10	615	655	696	736	777	817	858	898	939	979
20	979.019	059	100	140	180	220	260	300	340	380
30	420	460	500	539	579	618	658	697	737	776
40	816	855	895	934	973	012	052	091	130	169
50	980.208	247	286	325	364	403	441	480	519	558
73.0	596	635	673	712	750	789	827	866	904	943
10	980	019	057	095	133	171	209	247	285	323
20	981.361	399	436	474	512	549	587	624	662	699
30	737	774	812	849	886	924	961	998	035	072
40	982.109	146	183	220	257	294	331	367	404	440
50	477	514	551	587	623	660	696	733	769	805
74.0	842	878	914	950	986	022	058	094	130	166
10	983.202	238	273	309	345	380	416	452	487	523
20	558	594	629	664	700	735	770	805	840	875
30	910	945	980	015	050	085	120	155	189	224
40	984.259	293	328	363	397	432	466	500	535	569
50	603	637	672	706	740	774	808	843	876	910
75.0	944	978	011	045	079	112	146	180	213	247
10	985.280	314	347	380	414	447	480	513	547	580
20	613	646	679	712	745	778	811	843	876	909
30	945	974	007	039	072	104	137	169	202	234
40	986.266	299	331	363	395	437	459	491	523	555
50	587	619	651	683	714	746	778	809	841	873
76.0	904	936	967	998	030	061	092	124	155	186
10	987.217	248	279	310	341	372	403	434	465	495
20	526	557	587	618	649	679	710	740	771	801
30	831	862	892	922	953	983	013	043	073	803
40	988.133	163	193	222	252	282	312	341	371	401
50	430	460	490	519	548	578	607	636	665	695

Logar. Tang. Deg. 69, 70, 71, 72, 73, 74, 75, 76. In. 10. ° 47

D	0	1	2	3	4	5	6	7	8	9
69.0	41.5823	6200	6578	6956	7335	7714	8093	8472	8851	9231
10	9611	9991	0371	0752	1133	1514	1896	2277	2659	3041
20	42.3424	3807	4190	4573	4956	5340	5724	6108	6493	6877
30	7262	7648	8033	8419	8805	9191	9578	9964	0352	0739
40	43.1126	1514	1902	2291	2679	3068	3458	3847	4237	4627
50	5017	5407	5798	6189	6581	6972	7364	7756	8148	8541
70.0	8934	9327	9721	0115	0509	0903	1297	1692	2087	2483
10	44.2879	3274	3671	4067	4464	4861	5258	5656	6054	6452
20	6851	7250	7649	8048	8448	8847	9248	9648	0049	0450
30	45.0851	1253	1655	2057	2459	2862	3265	3669	4072	4476
40	4881	5285	5690	6095	6501	6906	7312	7719	8125	8532
50	8939	9347	9755	0163	0571	0980	1389	1798	2208	2618
71.0	46.3028	3439	3849	4261	4672	5084	5496	5908	6321	6734
10	7147	7561	7975	8389	8804	9219	9634	0049	0465	0881
20	47.1298	1715	2132	2549	2967	3385	3803	4222	4641	5060
30	5480	5900	6320	6741	7162	7583	8005	8427	8849	9272
40	9695	0118	0542	0966	1390	1814	2239	2665	3090	3516
50	48.3942	4369	4796	5223	5651	6079	6507	6936	7365	7794
72.0	8224	8654	9084	9515	9946	0378	0809	1241	1674	2107
10	49.2540	2973	3407	3841	4276	4711	5146	5582	6018	6454
20	6891	7328	7765	8203	8641	9080	9519	9958	0397	0837
30	50.1278	1718	2159	2601	3043	3485	3927	4370	4813	5257
40	5701	6145	6590	7035	7481	7927	8373	8820	9267	9714
50	51.1062	0610	1059	1508	1957	2407	2857	3307	3758	4209
73.0	4661	5113	5565	6018	6471	6925	7379	7833	8288	8743
10	9199	9655	0111	0568	1025	1483	1941	2399	2858	3317
20	52.3777	4237	4697	5158	5619	6081	6543	7005	7468	7931
30	8395	8859	9324	9789	0254	0720	1186	1653	2120	2587
40	53.3055	3523	3992	4461	4931	5401	5871	6342	6814	7285
50	7758	8230	8703	9177	9651	0125	0600	1075	1551	2027
74.0	54.2504	2981	3458	3936	4414	4893	5372	5852	6332	6813
10	7294	7775	8257	8740	9223	9706	0190	0674	1159	1644
20	55.2130	2616	3102	3589	4077	4565	5053	5542	6031	6521
30	7012	7502	7994	8485	8978	9471	9964	0457	0951	1446
40	56.1941	2437	2933	3430	3927	4424	4922	5421	5920	6419
50	6920	7420	7921	8423	8925	9427	9930	0434	0938	1442
75.0	57.1947	2453	2959	3466	3973	4481	4989	5497	6006	6516
10	7026	7537	8048	8560	9072	9585	0099	0613	1127	1642
20	58.2157	2673	3190	3707	4225	4743	5262	5781	6301	6821
30	7342	7863	8385	8908	9431	9955	0479	1003	1529	2055
40	59.2581	3108	3636	4164	4692	5222	5751	6282	6813	7344
50	7876	8409	8942	9476	0010	0545	1081	1617	2154	2691
76.0	60.3229	3767	4306	4846	5386	5927	6469	7011	7553	8097
10	8640	9185	9730	0276	0822	1369	1916	2464	3013	3562
20	61.4112	4663	5214	5766	6318	6871	7425	7979	8534	9090
30	9646	0203	0761	1319	1877	2434	2997	3558	4119	4681
40	62.5244	5807	6371	6935	7501	8067	8633	9201	9768	0337
50	63.0906	1476	2047	2618	3190	3762	4336	4910	5484	6060

83, 84. m. 9.

7	8	9
927	956	985
214	243	271
497	525	553
777	804	832
052	079	107
324	351	377
591	618	644
855	881	908
115	141	167
372	397	422
624	649	674
873	897	992
117	142	166
358	382	406
596	619	643
829	852	875
058	081	104
284	307	329
506	528	550
725	746	768
939	960	981
150	171	191
357	377	397
560	580	600
759	779	798
955	974	993
140	165	184
334	353	372
519	537	555
699	717	735
876	894	911
049	066	083
218	235	252
384	400	417
546	562	578
704	720	735
858	874	889
009	024	039
156	170	185
299	313	327
439	452	466
574	588	601
706	719	732
835	847	860
959	972	984
080	092	104
197	209	220
311	322	333

Logar Tang. Deg. 77, 78, 79, 80, 81, 82, 83, 84. Index IO, II. *49

D	0	1	2	3	4	5	6	7	8	9
77. 0	63.6636	7213	7790	8368	8947	9526	0106	0687	1269	1851
10	64.2434	3018	3602	418	4773	5360	5947	6535	7124	7713
20	8303	3894	9486	0078	0671	1265	1859	2455	3051	3647
30	65.4245	4843	5442	6042	6642	7243	7845	8448	9052	9656
40	66.0261	0867	1473	2081	2689	3298	3907	4518	5129	5741
50	6354	6967	7582	8197	8813	9430	0047	0665	1285	1905
78. 0	67.2525	3147	3769	4393	5017	5642	6267	6894	7521	8149
10	8778	9408	0039	0670	1303	1936	2570	3205	3841	4477
20	68.5115	5753	6392	7032	7673	8315	8958	9601	0246	0891
30	69.1537	2184	2832	3481	4131	4782	5433	6086	6739	7393
40	8049	8705	9362	0020	0678	1338	1999	2660	3323	3987
50	70.4651	5316	5983	6650	7318	7987	8658	9329	0001	0674
79. 0	71.1348	2023	2699	3375	4053	4732	5412	6093	6775	7458
10	8141	8826	9512	0199	0887	1576	2266	2957	3647	4342
20	72.5036	5731	6427	7124	7822	8521	9221	9923	0625	1329
30	73.2033	2739	3445	4153	4862	5572	6283	6995	7708	8422
40	9137	9854	0571	1290	2010	2731	3453	4176	4900	5626
50	74.6352	7080	7809	8539	9270	0002	0736	1470	2206	2943
80. 0	75.3681	4421	5161	5903	6646	7390	8135	8881	9629	0378
10	76.1128	1880	2632	3386	4141	4897	5656	6414	7174	7935
20	8698	9461	0226	0993	1761	2529	3300	4071	4844	5618
30	77.6393	7170	7948	8728	9508	0290	1074	1858	2644	3432
40	78.4220	5011	5802	6595	7389	8185	8982	9780	0580	1381
50	79.2183	2987	3793	4600	5408	6217	7029	7841	8655	9471
81. 0	80.0287	1106	1926	2747	3570	4394	5220	6047	6876	7706
10	8538	9371	0206	1042	1880	2720	3561	4403	5247	6093
20	81.6940	7789	8640	9492	0345	1201	2057	2916	3776	4638
30	82.5501	6366	7233	8101	8971	9843	0716	1591	2468	3346
40	83.4226	5108	5992	6877	7764	8653	9543	0435	1329	2225
50	84.3123	4022	4923	5826	6730	7637	8546	9456	0368	1282
82. 0	85.2197	3115	4034	4956	5879	6804	7731	8660	9591	0524
10	86.1458	2395	3333	4274	5216	6161	7107	8056	9006	9959
20	87.0913	1870	2828	3789	4751	5716	6683	7652	8623	9596
30	88.0571	1548	2528	3509	4493	5479	6467	7457	8449	9444
40	89.0441	1440	2441	3444	4450	5458	6468	7481	8495	9513
50	90.0532	1554	2578	3604	4633	5664	6698	7734	8772	9813
83. 0	91.0856	1902	2950	4000	5053	6109	7167	8227	9290	0356
10	92.1424	2495	3567	4644	5722	6803	7887	8973	0062	1153
20	93.2248	3345	4444	5547	6652	7760	8870	9984	1100	2219
30	94.3340	4465	5593	6723	7856	8992	0131	1273	2418	3566
40	95.4716	5870	7027	8187	9349	0515	1684	2856	4031	5209
50	96.6391	7575	8763	9953	1148	2345	3545	4749	5956	7166
84. 0	97.8380	9597	0817	2041	3267	4498	5732	6969	8210	9454
10	99.0702	1953	3208	4466	5728	6993	8262	9535	0812	2192
20	11.00.3376	4663	5955	7250	8549	9851	1158	2468	3783	5101
30	01.6423	7749	9079	0413	1752	3094	4440	5791	7145	8504
40	02.9867	1234	2606	3981	5361	6745	8131	9527	0925	2326
50	04.3733	5144	6560	7979	9403	0832	2266	3705	5148	6596

8	9
31	42
37	47
39	49
37	47
32	41
23	32
10	19
94	02
74	81
50	57
22	29
91	98
56	62
18	24
75	81
29	35
80	81
26	31
69	74
09	12
44	48
76	79
04	07
29	32
50	52
68	69
81	82
91	92
97	98
99	99

The Remainder of the Logarithm Tangents to 90.

*5

Min.	85 Deg.	86 Deg.	87 Deg.	88 Deg.	89 Deg.	Min.
0	11.058048	11.155356	11.280604	11.456916	11.758078	0
1	9506	11. 7175	11. 3028	11. 60553	11. 65379	1
2	11.060968	11. 9092	11. 5465	11. 64221	11. 72805	2
3	11. 2435	11.160837	11. 7917	11. 67920	11. 80359	3
4	11. 3907	11. 2679	11.293381	11. 71651	11. 88047	4
5	11. 5384	11. 4529	11. 2860	11. 75414	11. 95874	5
6	11. 6866	11. 6387	11. 5353	11. 79210	11.803844	6
7	11. 8353	11. 8152	11. 7861	11. 83039	11. 11964	7
8	11. 9845	11.170126	11.300383	11. 86902	11. 20237	8
9	11.071342	11. 2008	11. 2919	11. 90800	11. 28672	9
10	11. 2844	11. 3897	11. 5471	11. 94733	11. 37273	10
11	11. 4351	11. 5795	11. 8037	11. 98702	11. 46048	11
12	11. 5864	11. 7702	11.310618	11.502707	11. 55004	12
13	11. 7381	11. 9616	11. 3216	11. 06750	11. 64149	13
14	11. 8904	11.181539	11. 5828	11. 10830	11. 73490	14
15	11.080432	11. 3471	11. 8456	11. 14949	11. 83037	15
16	11. 1966	11. 5410	11.321100	11. 19108	11. 92797	16
17	11. 3505	11. 7359	11. 3761	11. 23307	11.902783	17
18	11. 5049	11. 9317	11. 6437	11. 27546	11. 13003	18
19	11. 6599	11.191283	11. 9730	11. 31827	11. 23469	19
20	11. 8154	11. 3258	11.331848	11. 36151	11. 34194	20
21	11. 9715	11. 5242	11. 4566	11. 40519	11. 45191	21
22	11.091281	11. 7235	11. 7311	11. 44930	11. 56473	22
23	11. 2853	11. 9236	11.340072	11. 49387	11. 68055	23
24	11. 4430	11.201248	11. 2851	11. 53890	11. 79955	24
25	11. 6013	11. 3269	11. 5648	11. 58440	11. 92191	25
26	11. 7602	11. 5299	11. 8462	11. 63078	12.004781	26
27	11. 9197	11. 7338	11.351296	11. 67685	12. 17746	27
28	11.100797	11. 9387	11. 4147	11. 72382	12. 31111	28
29	11. 2404	11.211446	11. 7017	11. 77131	12. 44900	29
30	11. 4016	11. 3514	11. 9907	11. 81932	12. 59142	30
31	11. 5634	11. 5592	11.362815	11. 86787	12. 73866	31
32	11. 7258	11. 7680	11. 5744	11. 91696	12. 89106	32
33	11. 8888	11. 9778	11. 8692	11. 96662	12.104901	33
34	11.110524	11.221886	11.371660	11.601685	12. 21292	34
35	11. 2167	11. 4005	11. 4648	11. 06766	12. 38326	35
36	11. 3815	11. 6133	11. 7657	11. 11908	12. 56056	36
37	11. 5470	11. 8272	11.380687	11. 17111	12. 74540	37
38	11. 7131	11.230422	11. 3738	11. 22378	12. 93845	38
39	11. 8798	11. 2582	11. 6811	11. 27708	12.214049	39
40	11.120471	11. 473	11. 9906	11. 33105	12. 35239	40

52 The Remainder of the Logarithmic Tangents, to 90 Degrees.

Min.	85 Deg.	86 Deg.	87 Deg.	88 Deg.	89 Deg.	Min.
41	11.122151	11.236935	11.393022	11.638570	12.257516	41
42	11. 3838	11. 9128	11. 6161	11. 44105	12. 80997	42
43	11. 5531	11.241332	11. 9323	11. 49710	12.305821	43
44	11. 7230	11. 3547	11.402508	11. 55379	12. 32151	44
45	11. 8936	11. 5773	11. 5717	11. 61144	12. 60180	45
46	11.130649	11. 8011	11. 8949	11. 66975	12. 90143	46
47	11. 2368	11.250260	11.412205	11. 72886	12.422328	47
48	11. 4094	11. 2521	11. 5486	11. 78878	12. 57091	48
49	11. 5827	11. 4793	11. 8792	11. 84954	12. 94880	49
50	11. 7567	11. 7078	11.422123	11. 91116	12.536273	50
51	11. 9314	11. 9374	11. 5480	11. 97366	12. 82030	51
52	11.141068	11.261683	11. 8863	11.703708	12.633183	52
53	11. 2829	11. 4004	11.432272	11. 10144	12. 91175	53
54	11. 4597	11. 6337	11. 5709	11. 16677	12.758122	54
55	11. 6372	11. 8683	11. 9172	11. 23309	12.837304	55
56	11. 8154	11.271041	11.442664	11. 30044	12.934214	56
57	11. 9943	11. 3412	11. 6183	11. 36885	13.059153	57
58	11.151740	11. 5796	11. 9732	11. 43835	13.235241	58
59	11. 3545	11. 8194	11.453309	11. 50898	13.536274	59
60	11. 5356	11.280604	11. 6916	11. 58078		60

Mr *STREET*'s
T A B L E
O F

Logistical Logarithms

To every $\left\{ \begin{array}{l} \text{Minute} \\ \text{Second} \end{array} \right\}$ of a $\left\{ \begin{array}{l} \text{Degree.} \\ \text{Minute.} \end{array} \right\}$

This Day is Published,

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7	8	9	11
112	26532	26021	0
259	3010	2775	10
249	1091	0939	20
881	9765	9652	30
842	8751	8661	40
8004	7929	7855	50
302	238	175	60
698	642	587	70
168	118	069	80
695	651	607	90
269	229	189	100
881	844	808	110
525	491	457	120
196	165	133	130
890	860	831	140
604	576	549	150
336	310	284	160
083	059	034	170
845	821	798	180
618	556	574	190
403	382	362	200
198	178	159	210
003	984	965	220
816	797	779	230
636	615	601	240
464	447	430	250
298	282	266	260
138	123	107	270
984	969	954	280
835	821	806	290
692	678	663	300
552	539	525	310
418	404	391	320
287	274	261	330
160	147	135	340
036	024	012	350
916	905	893	360
800	784	777	370
686	675	664	380
575	564	553	390
467	456	446	400
362	351	341	410
259	249	238	420
158	148	138	430
060	050	041	440
964	954	945	450
870	861	851	460
778	769	760	470

"	0	1	2	3	4	5	6	7	8	9	1
8.0	8. 751	742	733	724	715	706	697	688	679	670	480
10	661	652	643	635	626	617	608	599	591	582	490
20	573	565	556	547	539	530	522	513	504	496	500
30	487	479	470	462	453	445	437	428	420	411	510
40	403	395	386	378	370	361	353	345	337	328	520
50	320	312	304	296	288	279	271	263	255	247	530
9.0	82. 39	31	23	15	07	99	91	83	75	67	540
10	81. 59	52	44	36	28	20	12	04	97	89	550
20	80. 81	73	66	58	50	43	35	27	20	12	560
30	04	97	89	81	74	66	59	51	44	36	570
40	79. 29	21	14	06	99	91	84	77	69	62	580
50	78. 55	47	40	32	25	18	11	03	96	89	590
10.0	77. 82	74	67	60	53	45	38	31	24	17	600
10	03	96	88	81	74	67	60	53	46	39	610
20	76. 39	32	25	18	11	04	97	90	83	77	620
30	75. 70	63	56	49	42	35	28	22	15	08	630
40	01	94	88	81	74	67	61	54	47	41	640
50	74. 34	27	21	14	07	01	94	87	81	74	650
11.0	73. 68	61	54	48	41	35	28	22	15	09	660
10	02	96	89	83	76	70	64	57	51	44	670
20	72. 38	32	25	19	12	06	00	93	87	81	680
30	71. 75	68	62	56	49	43	37	31	24	18	690
40	12	06	00	93	87	81	75	69	63	57	700
50	70. 50	44	38	32	26	20	14	08	02	96	710
12.0	69. 90	84	78	72	66	60	54	48	42	36	720
10	30	24	18	12	06	00	94	88	82	77	730
20	68. 71	65	59	53	47	41	36	30	24	18	740
30	13	07	01	95	89	84	78	72	66	61	750
40	67. 55	49	43	38	32	26	21	15	09	04	760
50	66. 96	92	87	81	76	70	64	59	53	48	770
13.0	42	37	31	25	20	14	09	03	98	92	780
10	65. 87	81	76	70	65	59	54	48	43	38	790
20	32	27	21	16	10	05	00	94	89	84	800
30	64. 78	73	67	62	57	51	46	41	35	30	810
40	25	20	14	09	04	98	93	88	83	77	820
50	63. 72	67	62	57	51	46	41	36	31	25	830
14.0	20	15	10	05	00	94	89	84	79	74	840
10	62. 69	64	59	54	48	43	38	33	28	23	850
20	18	13	08	03	98	93	88	83	78	73	860
30	61. 68	63	58	53	48	43	38	33	28	23	870
40	18	13	08	03	99	94	89	84	79	74	880
50	60. 69	64	59	55	50	45	40	35	30	25	890
15.0	21	16	11	06	01	97	92	87	82	77	900
10	59. 73	68	63	58	54	49	44	39	35	30	910
20	25	20	16	11	06	02	97	92	88	83	920
30	58. 78	74	69	64	60	55	50	46	41	36	930
40	32	27	23	18	13	09	04	00	95	90	940
50	57. 86	81	77	72	68	63	58	54	49	45	950

1	11	0	1	2	3	4	5	6	7	8	9	11
16.	0	57.40	36	31	27	22	18	13	09	04	00	960
	10	56.95	91	86	82	77	73	69	64	60	55	970
	20	51	46	42	37	33	29	24	20	15	11	980
	30	07	02	98	94	89	85	80	76	72	67	990
	40	55.63	59	54	50	46	41	37	33	28	24	1000
	50	20	16	11	07	03	98	94	90	86	81	1010
17.	0	54.77	73	69	64	60	56	52	47	43	39	1020
	10	34	30	26	22	18	14	09	05	01	97	1030
	20	53.93	89	84	80	76	72	68	64	59	55	1040
	30	51	47	43	39	35	31	26	22	18	14	1050
	40	10	06	02	98	94	90	85	81	77	73	1060
	50	52.69	65	61	57	53	49	45	41	37	33	1070
18.	0	29	25	21	17	13	09	05	01	97	93	1080
	10	51.89	85	81	77	73	69	65	61	57	53	1090
	20	49	45	41	37	33	29	25	22	18	14	1100
	30	10	06	02	98	94	90	86	82	79	75	1110
	40	50.71	67	63	59	55	51	48	44	40	36	1120
	50	32	28	25	21	17	13	09	05	02	98	1130
19.	0	49.94	90	86	83	79	75	71	67	64	60	1140
	10	56	52	49	45	41	37	33	30	26	22	1150
	20	18	15	11	07	03	00	96	92	89	85	1160
	30	48.81	77	74	70	66	63	59	55	52	48	1170
	40	44	41	37	33	30	26	22	19	15	11	1180
	50	08	04	00	97	93	89	86	82	78	75	1190
20.	0	47.71	68	64	60	57	53	50	46	42	39	1200
	10	35	32	28	24	21	17	14	10	07	03	1210
	20	46.99	96	93	89	85	82	78	75	71	68	1220
	30	64	60	57	53	50	46	43	39	36	32	1230
	40	29	25	22	18	15	11	08	04	01	97	1240
	50	45.94	90	87	84	80	77	73	70	66	63	1250
21.	0	59	56	52	49	46	42	39	35	32	28	1260
	10	25	22	18	15	11	08	05	01	98	94	1270
	20	44.91	88	84	81	77	74	71	67	64	60	1280
	30	57	54	50	47	44	40	37	34	30	27	1290
	40	24	20	17	14	10	07	04	00	97	94	1300
	50	43.90	87	84	80	77	74	70	67	64	60	1310
22.	0	58	54	51	47	44	41	38	34	31	28	1320
	10	25	21	18	15	11	08	05	02	98	95	1330
	20	42.92	89	85	82	79	76	73	69	66	63	1340
	30	60	56	53	50	47	44	40	37	34	31	1350
	40	28	24	21	18	15	12	09	05	02	99	1360
	50	41.96	93	89	86	83	80	77	74	71	67	1370
23.	0	64	61	58	55	52	49	45	42	39	36	1380
	10	33	30	27	24	20	17	14	11	08	05	1390
	20	02	99	96	92	89	86	83	80	77	74	1400
	30	40.71	68	65	62	59	55	52	49	46	43	1410
	40	40	37	34	31	28	25	22	19	16	13	1420
	50	10	07	04	01	98	95	91	88	85	82	1430

"	0	1	2	3	4	5	6	7	8	9	"
24.0	39.79	76	73	70	67	64	61	58	55	52	1440
10	49	46	43	40	37	34	31	28	25	22	1450
20	19	17	14	11	08	05	02	99	96	93	1460
30	38.90	87	84	81	78	75	72	69	66	63	1470
40	60	57	54	51	49	46	43	40	37	34	1480
50	31	28	25	22	20	17	14	11	08	05	1490
25.0	03	99	96	93	91	88	85	82	79	76	1500
10	37.73	70	68	65	62	59	56	53	50	47	1510
20	45	42	39	36	33	30	27	25	22	19	1520
30	16	13	10	08	05	02	99	96	93	91	1530
40	36.88	85	82	79	77	74	71	68	65	63	1540
50	60	57	54	51	49	46	43	40	37	35	1550
26.0	32	29	26	23	21	18	15	12	10	07	1560
10	04	01	98	96	93	90	87	85	82	79	1570
20	35.76	74	71	68	65	63	60	57	55	52	1580
30	49	46	44	41	38	35	33	30	27	25	1590
40	22	19	16	14	11	08	06	03	00	97	1600
50	34.95	92	89	87	84	81	79	76	73	71	1610
27.0	68	65	63	60	57	54	52	49	46	44	1620
10	41	38	36	33	31	28	25	23	20	17	1630
20	15	12	09	07	04	01	99	96	93	91	1640
30	33.88	86	83	80	77	73	72	70	67	65	1650
40	62	59	57	53	51	49	46	44	41	38	1660
50	36	33	31	28	25	23	20	18	15	13	1670
28.0	10	07	05	02	00	97	94	92	89	87	1680
10	32.84	82	79	76	74	71	69	66	64	61	1690
20	59	56	53	51	48	46	43	41	38	36	1700
30	33	31	28	25	23	20	18	15	13	10	1710
40	08	05	03	00	98	95	93	90	88	85	1720
50	31.83	80	78	75	73	70	68	65	63	60	1730
29.0	58	55	53	50	48	45	43	40	38	35	1740
10	33	30	28	25	23	20	18	15	13	10	1750
20	08	05	03	01	98	96	93	91	88	86	1760
30	83	81	78	76	73	71	69	66	64	61	1770
40	59	56	54	52	49	47	44	42	39	37	1780
50	34	32	30	27	25	22	20	18	15	13	1790
30.0	10	08	05	03	01	98	96	93	91	89	1800
10	29.86	84	81	79	77	74	72	69	67	65	1810
20	62	60	58	55	53	50	48	46	43	41	1820
30	39	36	34	31	29	27	24	22	20	17	1830
40	15	12	10	08	05	03	01	98	96	94	1840
50	28.91	89	87	84	82	80	77	75	73	70	1850
31.0	68	66	63	61	59	56	54	52	49	47	1860
10	45	42	40	38	35	33	31	28	26	24	1870
20	21	19	17	15	12	10	08	05	03	01	1880
30	98	96	94	92	89	87	85	82	80	78	1890
40	75	73	71	69	66	64	62	60	57	55	1900
50	53	50	48	46	44	41	39	37	35	32	1910

Street's Logist. Logar. $\frac{1}{11}$, 32, 33, 34, 35, 36, 37, 38, 39. *59

"	0	1	2	3	4	5	6	7	8	9	"
32.0	27.30	28	25	23	21	19	16	14	12	10	1920
10	37	05	03	01	98	96	94	92	89	87	1930
20	26.85	83	81	78	76	74	72	69	67	65	1940
30	63	60	58	56	54	52	49	47	45	43	1950
40	40	38	36	34	32	29	27	25	23	21	1960
50	18	16	14	12	10	07	05	03	01	99	1970
33.0	25.96	94	92	90	88	85	83	81	79	77	1980
10	74	72	70	68	66	44	41	39	37	35	1990
20	53	51	48	46	44	42	40	38	35	33	2000
30	31	29	27	25	22	20	18	16	14	12	2010
40	10	07	05	03	01	99	97	94	92	90	2020
50	24.88	86	84	82	80	77	75	73	71	69	2030
34.0	67	65	62	60	58	56	54	52	50	48	2040
10	45	43	41	39	37	35	33	31	29	26	2050
20	24	22	20	18	16	14	12	10	08	05	2060
30	03	01	99	97	95	93	91	89	87	84	2070
40	23.82	80	78	76	74	72	70	68	66	64	2080
50	61	59	57	55	53	51	49	47	45	43	2090
35.0	41	39	37	35	33	31	28	26	24	22	2100
10	20	18	16	14	12	10	08	06	04	02	2110
20	00	98	96	94	91	89	87	85	83	81	2120
30	22.79	77	75	73	70	69	67	65	63	61	2130
40	59	57	55	53	51	49	47	45	43	41	2140
50	39	37	35	33	31	29	27	25	23	20	2150
36.0	18	16	14	12	10	08	06	04	02	00	2160
10	21.98	96	94	92	90	88	86	84	82	80	2170
20	78	76	74	72	70	69	67	65	63	61	2180
30	59	57	55	53	51	49	47	45	43	41	2190
40	39	37	35	33	31	29	27	25	23	21	2200
50	19	17	15	13	11	09	07	05	03	01	2210
37.0	20.99	98	96	94	92	90	88	86	84	82	2220
10	80	78	76	74	72	70	68	66	64	62	2230
20	60	59	57	55	53	51	49	47	45	43	2240
30	41	39	37	35	33	32	30	29	26	24	2250
40	22	20	18	16	14	12	10	09	07	05	2260
50	03	01	92	97	95	93	91	89	87	86	2270
38.0	19.84	82	80	78	76	74	72	70	68	67	2280
10	65	63	61	59	57	55	53	51	50	48	2290
20	46	44	42	40	38	36	34	33	31	29	2300
30	27	25	23	21	19	18	16	14	12	10	2310
40	08	06	04	03	01	99	97	95	93	91	2320
50	89	88	86	84	82	80	78	76	75	73	2330
39.0	71	69	67	65	63	62	60	58	56	54	2340
10	52	50	49	47	45	43	41	39	38	36	2350
20	34	32	30	28	27	25	23	21	19	17	2360
30	16	14	12	10	08	06	05	03	01	99	2370
40	97	95	94	92	90	88	86	85	83	81	2380
50	79	77	75	74	72	70	68	66	65	63	2390

3, 44, 45, 46, 47.

8	9	11
46	45	2400
28	27	2410
11	09	2420
92	91	2430
75	73	2440
57	55	2450
40	38	2460
22	20	2470
03	03	2480
87	85	2490
70	68	2500
52	51	2510
35	34	2520
18	16	2530
01	99	2540
84	82	2550
67	65	2560
50	49	2570
33	32	2580
17	16	2590
00	98	2600
83	82	2610
67	65	2620
50	49	2630
34	32	2640
17	16	2650
01	00	2660
85	83	2670
69	68	2680
53	51	2690
37	35	2700
21	19	2710
05	03	2720
89	87	2730
73	71	2740
57	56	2750
41	40	2760
26	24	2770
10	09	2780
95	93	2790
79	78	2800
64	62	2810
48	47	2820
33	31	2830
18	16	2840
02	01	2850
87	86	2860
72	71	2870

9	11
56	2880
41	2890
26	2900
11	2910
96	2920
82	2930
66	2940
53	2950
37	2960
22	2970
08	2980
93	2990
79	3000
64	3010
50	3020
36	3030
21	3040
07	3050
93	3060
79	3070
65	3080
51	3090
37	3100
23	3110
09	3120
95	3130
81	3140
68	3150
54	3160
40	3170
26	3180
13	3190
99	3200
86	3210
72	3220
59	3230
46	3240
32	3250
19	3260
06	3270
92	3280
79	3290
66	3300
53	3310
40	3320
27	3330
14	3340
01	3350

Hours	Deg.	M.	Deg.	M.	Deg.
I	2 30	1	0 3	31	1 18
II	5 00	2	0 5	32	1 20
III	7 30	3	0 8	33	1 23
IV	10 00	4	0 10	34	1 25
V	12 30	5	0 13	35	1 28
VI	15 00	6	0 15	36	1 30
VII	17 30	7	0 18	37	1 33
VIII	20 00	8	0 20	38	1 35
IX	22 30	9	0 23	39	1 38
X	25 00	10	0 25	40	1 40
XI	27 30	11	0 28	41	1 43
XII	30 00	12	0 30	42	1 45
XIII	32 30	13	0 33	43	1 48
XIV	35 00	14	0 35	44	1 50
XV	37 30	15	0 38	45	1 53
XVI	40 00	16	0 40	46	1 55
XVII	42 30	17	0 43	47	1 58
XVIII	45 00	18	0 45	48	2 00
XIX	47 30	19	0 48	49	2 3
XX	50 00	20	0 50	50	2 5
XXI	52 30	21	0 53	51	2 8
XXII	55 00	22	0 55	52	2 10
XXIII	57 30	23	0 58	53	2 13
XXIV	60 00	24	1 00	54	2 15
		25	1 3	55	2 18
		26	1 5	56	2 20
		27	1 8	57	2 23
		28	1 10	58	2 25
		29	1 13	59	2 28
		30	1 15	60	2 30